Traversable Wormholes and Scalar Fields

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Background

Traversable wormholes
- would allow FTL communications & time travel,
- require exotic matter (negative energy) in GR.

Topological Censorship Theorem:
ANEC $\Rightarrow$ No Traversable Wormholes

Average Null Energy Condition (ANEC):

$$\langle \rho \rangle_\Gamma \equiv \int_\Gamma T_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda}\mathrm{d}\lambda \geq 0,$$

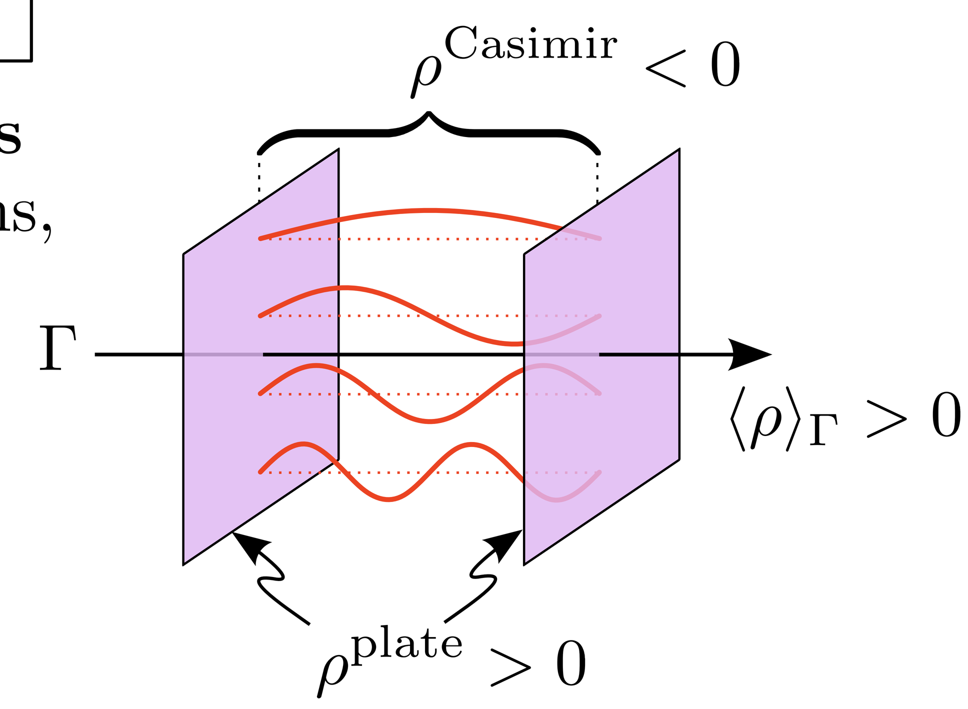
for all complete null geodesics Γ (affine parameter λ).

However, the ANEC can be violated by quantum fields.

The Casimir Effect

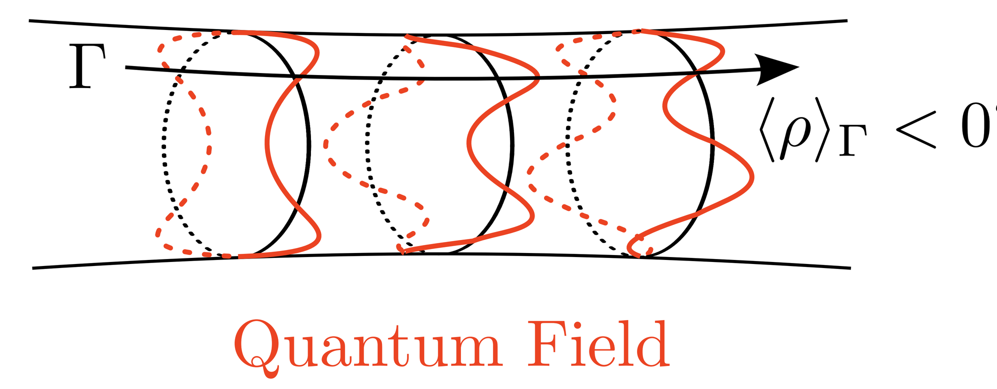
Typically, **conductive plates**
- impose boundary conditions,
- induce $\rho^{\text{Casimir}} < 0$,
- but $\rho^{\text{plate}} > 0$ dominates.

$\Rightarrow$ ANEC is obeyed.



However...

If ρ^{Casimir} is generated by **topology** and **curvature**, the ANEC can be violated.



A Long Thin Wormhole, supported by its own Casimir Energy

Seek traversable wormhole spacetime that solves

$$G_{\mu\nu} = \kappa T^{\text{Casimir}}_{\mu\nu} + \kappa T^{\text{ord.}}_{\mu\nu},$$

- $T^{\text{Casimir}}_{\mu\nu}$ induced by the wormhole throat,
- $T^{\text{ord.}}_{\mu\nu}$ obeys all energy conditions.

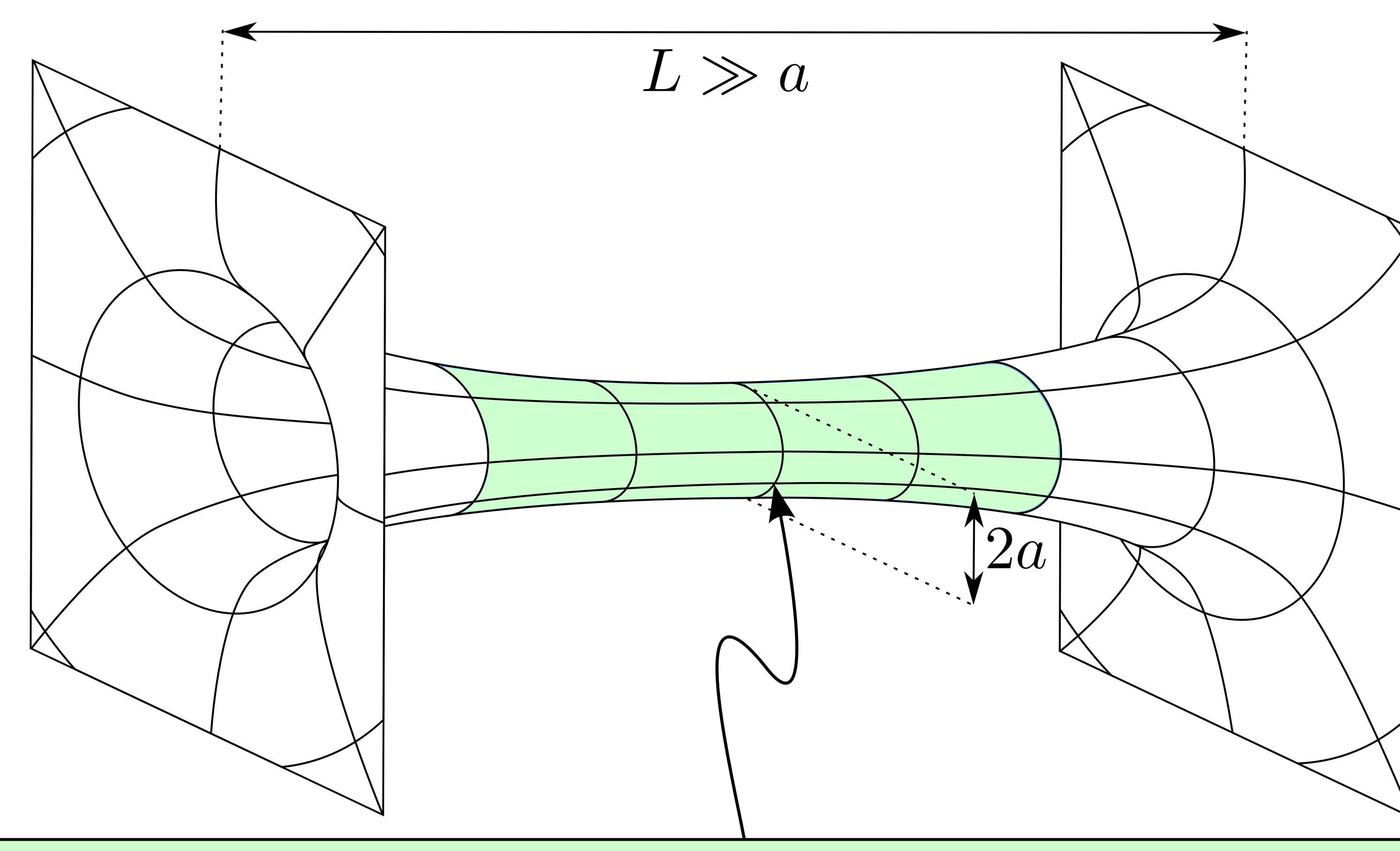
A long thin wormhole $L \gg a$
- generates a large Casimir energy: $|\rho| \propto 1/a^4$,
- requires less exotic matter: $|\rho| \propto 1/La$,

$\Rightarrow$ can have both $a, L \gg l_p$ (plank length).

Results obtained for a conformally coupled massless scalar field. Methods:
- mode sum approach with Abel-Plana formula,
- Pauli-Villars regularisation,
- renormalisation (introduces unknown a_0).

$T^{\text{Casimir}}_{\mu\nu}$ cannot stabilise the wormhole, but slows the collapse significantly.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \left[\sqrt{L^2+z^2} - L + a\right]^2 (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$$

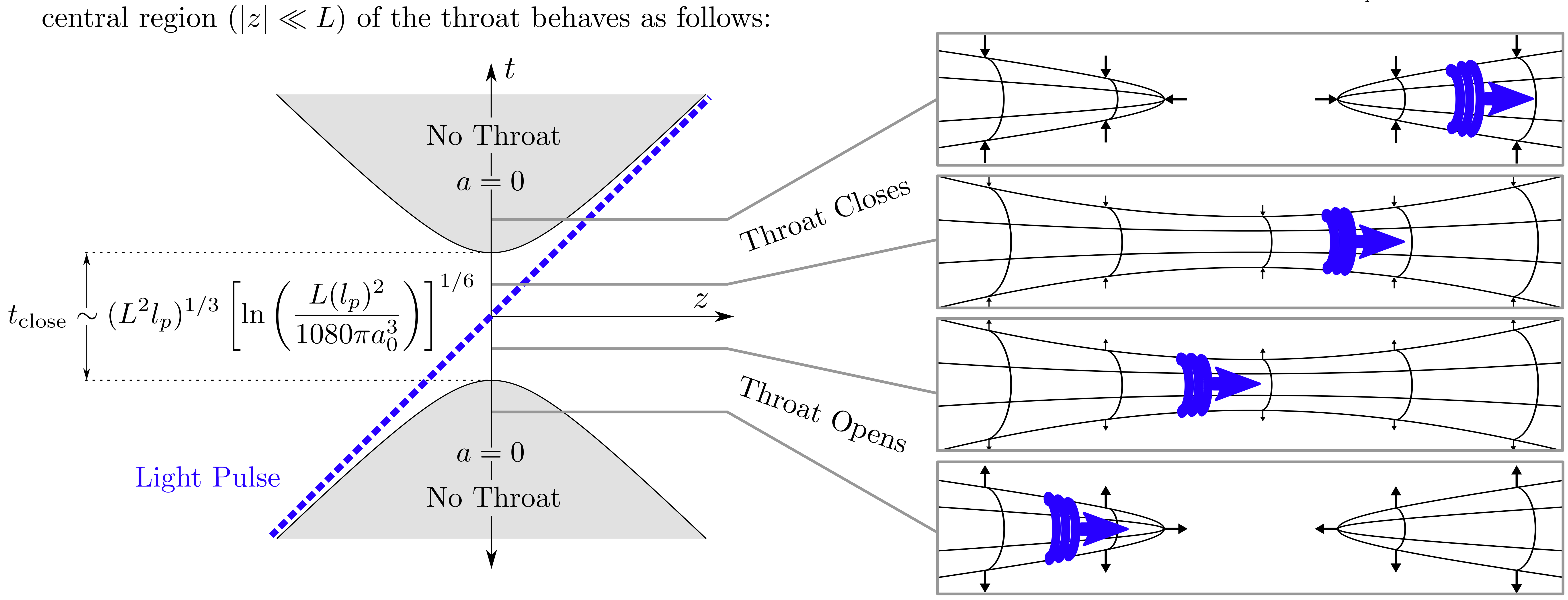


Generates: $$T^{\text{Casimir}}_{\hat\mu\hat\nu} = \frac{1}{2880\pi^2 a^4}\Big[\mathrm{diag}(-1,1,-1,-1)2\ln(a/a_0) + \mathrm{diag}(0,0,1,1)\Big]$$

Requires: $$\rho^{\text{exotic}} = \frac{-2L^2}{\kappa a (L^2+z^2)^{3/2}}$$

Slow Collapse and Light Crossing

Promote $a \to a(t)$, then Einstein field equations $\Rightarrow \ddot{a} = -1/L$ while $a \geq a_0 e^{1/2}$ and $a^{-3}\ln(a/a_0) \geq \frac{360\pi}{L l_p^2}$. Hence, the central region ($|z| \ll L$) of the throat behaves as follows:



Supported by its own Casimir energy, a long thin wormhole collapses sufficiently slowly that light can traverse the throat!