Radiation and Matter

Tutorial Questions 2

1. Show that if you treat the radiative interaction Hamiltonian as a classical potential energy, then its gradient gives the expression for the Lorentz force in the form

$$\frac{d}{dt}(\mathbf{p} - q\mathbf{A}) = q(\mathbf{E} + \mathbf{v}_\wedge \mathbf{B})$$

2. Check that the operator on atomic states $\mathbf{p}\,.\,\mathbf{A} = \mathbf{A}\,.\,\mathbf{p}$ *if* $\nabla\,.\,\mathbf{A} = 0$.

3. Demonstrate that the radiative transition between two atomic states involves only one photon (in first order theory).

4. Write the spontaneous transition rates in a given direction and for a given polarization – for electric dipole, magnetic dipole, and electric quadrupole transitions – in the format below, set so that the factor in the modulus is of order unity:

$$w(\alpha,\Omega) = \frac{1}{2\pi}\frac{\omega^3}{\hbar c^3}\frac{(...)^2}{4\pi\epsilon_0}\left|\mathbf{e}_{\mathbf{k},\alpha}\cdot\frac{(...)_{YX}}{(...)}\right|^2$$

For example, the electric dipole rate is

$$w(\alpha,\Omega) = \frac{1}{2\pi}\frac{\omega^3}{\hbar c^3}\frac{(ea_0)^2}{4\pi\epsilon_0}\left|\mathbf{e}_{\mathbf{k},\alpha}\cdot\frac{\mathbf{r}_{YX}}{a_0}\right|^2 ;$$

That is, show that the three missing terms are $e\hbar/2mc$, $\mathbf{L}$ and $\hbar$ for magnetic dipole transitions, and $(\frac{ka_0}{2})(ea_0)$, $\mathbf{rr}.\mathbf{n}$ and a_0^2 for electric quadrupole radiation.

5. At what wavelengths will the approximation in which the electric dipole transition rate dominates (*pace* selection rules) cease to be good?

6. Confirm that $N_{\mathbf{k},\alpha} = 1/(e^{h\nu/kT}-1)$ in black body radiation, and that its specific intensity is $B_\nu(T) = (2h\nu^3/c^2)/(e^{h\nu/kT}-1)$ and its energy density is $(8\pi h\nu^3/c^3)/(e^{h\nu/kT}-1)$.

7. Show that if a source is masering, its level populations appear to be those at equilibrium at a negative temperature. Show that the specific intensity in such a source grows initially exponentially; what limits this growth?

8. Show that the brightness temperature (the temperature of a black body giving an identical specific intensity) of a masering source can be *much* greater than the kinetic temperature in the source.

9. A non-masering source emits photons from a transition between two levels, u and l. The levels have populations in the ratio $R = n_u/n_l$. Show that the specific intensity cannot exceed the black body specific intensity at a temperature at which the levels

would be at thermodynamic equilibrium in the ratio R, *ie* at a temperature given by $(E_u - E_l)/kT = -\ln(Rg_l/g_u)$ [Hint: write down the specific intensity in the optically thick case, and express $\mathcal{E}_\nu$ and κ_ν in terms of n_u, n_l].

10. A line is emitted from atoms in a gas at temperature T which collisionally excites the levels u, l to Boltzmann equilibrium. Describe the changing shape of the line profile as the optical depth at the centre of the line is increased from small values.