Radiation and Matter

Tutorial questions 1

1. Find a prescription for a gauge transformation for $\mathbf{A}$ and ϕ that will make the transform of the expression $\nabla.\mathbf{A} + c^{-2}\partial\phi/\partial t$ equal to zero.

2. Show that the Lorentz gauge ($\nabla.\mathbf{A} + c^{-2}\partial\phi/\partial t = 0$) is Lorentz invariant

3. Show that it is possible to choose a gauge for Maxwell's equations in which $\nabla.\mathbf{A} = 0$ and that the consequence is (1) that ϕ obeys the Poisson equation, and is given by the charge distribution without retardation, ie as if all charges were at rest or else transmitted information about their position at infinite speed, and (2) that in the wave zone, $\dot{\mathbf{A}}$ is perpendicular to the direction of the wave motion.

4. How do you find a gauge in which $\phi = 0$ and $\nabla.\mathbf{A} = 0$, and under what conditions is it possible?

5. Verify (a)

$$\int f(x)\delta(g(x))\, dx \equiv (f(x)\, /\, dg/dx)_{g=0}\,,$$

(b)

$$\int \sin^3\alpha\, d\alpha\, d\phi = 8\pi/3$$

6. Use Maxwell's equations – without introducing potentials – to show that in an electromagnetic wave in free space (a) $\mathbf{E}, \mathbf{B}, \mathbf{k}$ are mutually perpendicular, and (b) $|\mathbf{E}| = c|\mathbf{B}|$.

7. Show that the Poynting vector represents momentum density as well as energy flux.

8. Show that the wave equation in empty space (no sources)

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\partial t^2} = 0$$

has solutions $\phi = \phi(t - \mathbf{r}.\mathbf{n}/c)$, and in particular $\exp i(\omega t - \mathbf{k}.\mathbf{r})$ and $\sin(\omega t - \mathbf{k}.\mathbf{r})$. Show in these cases that $\omega = kc$.

9. A point charge moves non-relativistically in a small circle at constant angular velocity. Find the rate of emission of energy, and compute the rate of decay of a classical electron in a classical Bohr orbit. Estimate how quickly a classical atom would collapse.

10. Sketch the *polar diagram* *i.e.* radiated power as a function of direction, for a particle oscillating in simple harmonic motion, and for a particle moving uniformly in a circle. Assume you are in the wave zone. What is the spectrum of the emission?

11. Show that a free (classical) electron scatters radiation with a cross section equal to

$$\frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)^2$$

12. Show that the result for the oscillator Hamiltonian in section 1.4 of the notes is correct (this is long ... but get it started so that you can see what happens!).

13. Do the same calculation as 11. above, but for the Poynting vector rather than the energy density of the electromagnetic field. Hence show that each quantum carries momentum $\hbar\mathbf{k}$.