A new radiation hypothesis;
by Max Planck.

(Presented at the meeting of 3 February 1911.)

(See above p.120.)

Gentlemen! A good ten years ago I had the honour of developing the basic principles of a theory of thermal radiation here, one of the essential prerequisites of which is that in the generation of radiant heat certain finite, indivisible energy quanta or energy elements of the size $E = h\nu$, where ν is the oscillation frequency of a monochromatic ray per second, h, the elementary quantum of action, a universal natural constant of the value $6.55.10^{27}$ erg.sec, play a certain characteristic role[1]).

As strangely as this assumption contrasts with the previously tried and tested concepts of electrodynamics and electron theory, some of the conclusions drawn from them have been preserved to such an extent, not only for the laws of black-body radiation, but also for the determination of elementary quanta for electricity and matter, and furthermore, thanks to the research of A. EINSTEIN and W. NERNST, also for the specific heat of solid and liquid bodies, that the attempt to continue along the path taken and to lift a little of the thick veil that still lies over the energy quanta seems justified.

Naturally, from the beginning I have been constantly trying to develop the ideas that one must form about the processes of absorption and emission of radiant heat on the basis of the assumption of energy quanta, unfortunately for a long time without any notable success. Because, as has already been pointed out by various parties, difficulties arise at every turn – difficulties whose significance can best be understood when one considers that even the validity of the MAXWELL-HERTZ basic electrodynamic equations, according to which every local electrodynamic disturbance propagates as a spherical wave in all directions of space, has been called into question. In my opinion, however, one need not go that far for the time being, but rather should try everything beforehand, and not shy away from hypotheses that seem daring, in order to get by with the principles of MAXWELL's electrodynamics, which have proven themselves even in the finest optical measurements.

[1]) M. PLANCK, Verh. d. D. Phys. Ges. 2, 237, 1900. In modified form Ann. d. Phys. (4) 4, 553, 1901.

It is this consideration that encourages me to now tell you about a new radiation hypothesis. I have formed it partly on the basis of the criticisms of my theory made by other investigators, of which I particularly emphasise the most recent one by H.A. LORENTZ[1]), and I ask you to consider it as an experiment which, I believe, could perhaps be fruitful in some way.

For a better explanation, I would like to briefly refer to the train of thought of my previous theory. I assumed that the centres of absorption and emission of radiant heat were structures of the type of a linear HERTZ oscillator. The excitation of such an oscillator with a natural frequency ν is caused by the electric field component E_z falling in its direction. The time average of the square of E_z is:

$$\bar{E}_z^2 = J,$$

and the quantity J is decomposed spectrally according to FOURIER:

$$J = \int_0^\infty \mathcal{J}_\nu \, d\nu,$$

the quantity $\mathcal{J}_\nu$, which I have called the "intensity of the oscillation exciting the oscillator", which is determined by the HERTZ Oscillator absorbing the following energy in a time dt:

$$\frac{3\, c^3\, \sigma}{16\, \pi^2\, \nu} \cdot \mathcal{J}_\nu \cdot dt. \qquad 1)$$

Where c is the speed of light, and the constant σ, a small number, is the logarithmic damping decrement of the oscillation amplitudes of the oscillator.

In the case of black-body radiation that is isotropic, the spatial radiation density u_ν, the oscillation frequency ν, are related to $\mathcal{J}_\nu$ by the relationship:

$$u_\nu = \frac{3}{4\pi}\, \mathcal{J}_\nu. \qquad 2)$$

On the other hand, in the time dt the HERTZ oscillator emits energy:

$$2\sigma\nu U \, .\, dt, \qquad 3)$$

[1]) H.A. LORENTZ, Phys. ZS. 11, 1248, 1910.

where U is the oscillation energy of the oscillator.

In a field of black-body radiation, the absorbed energy is equal to the emitted energy,

$$u_\nu = \frac{3}{4\pi}\mathcal{J}_\nu = \frac{8\pi\nu^2}{c^3}\cdot U. \qquad 4)$$

In order to get from this equation to the laws of blackbody radiation, it is necessary to introduce the concept of temperature.

This can be obtained from the general thermodynamic relationship between temperature T, energy U and entropy S:

$$\frac{1}{T} = \frac{dS}{dU}, \qquad 5)$$

in connection with the equally general relationship between entropy and probability:

$$S = k\,.\,\log W, \qquad 6)$$

where W is the probability that the oscillator has the energy U, and $k = 1.346.10^{16}$ erg/degree.

The whole task then boils down to calculating the probability that an oscillator with a certain oscillation frequency ν has a certain energy U. I tried to solve this task by considering U as a statistical mean value and accordingly examining the distribution of a very large energy quantum $N\,.\,U$ over N similar oscillators. In order to arrive at a certain finite value of this probability, I considered NU as the sum of a large number of equal, indivisible energy elements of the size $\epsilon = h\nu$, i.e.:

$$N\,.\,U = P\,.\,\epsilon \qquad 7)$$

and assumed that for each possible type of distribution (configuration) a certain number of energy elements, which can also be zero, is allocated to each oscillator. The number of all possible different configurations set equal to W_N then results in:

$$W_N = \frac{(N+P)!}{N!P!} \qquad 8)$$

and the corresponding entropy:

$$S_n = k\log W_N$$

provides the entropy for a single oscillator:

$$S = \frac{S_N}{N} = k\left\{\left(1+\frac{P}{N}\right)\log\left(1+\frac{P}{N}\right) - \frac{P}{N}\log\frac{P}{N}\right\}, \tag{9)}$$

from this with the help of 7):

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)\log\left(1+\frac{U}{h\nu}\right) - \frac{U}{h\nu}\log\frac{U}{h\nu}\right\}, \tag{10)}$$

finally by substitution in 5) the energy of the oscillator:

$$U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}, \tag{11)}$$

which then, using 4), gives the formula for the spatial density of black radiation:

$$u_\nu = \frac{3}{4\pi}\mathcal{J}_\nu = \frac{8\pi\nu^2}{c^3}\cdot\frac{1}{e^{\frac{h\nu}{kT}} - 1}. \tag{12)}$$

The above derivation would of course be immediately obvious if one could assume that the real energy U of every oscillator is actually a whole multiple of ϵ at every moment, and can therefore only change in steps in nature. I have also tried to develop this assumption further and expressed the hope a year ago[1]) that it would be possible to implement it. However, there are serious doubts about this. One of the most difficult questions is: How does such an oscillator manage to absorb an energy element ϵ when it is hit by heat rays? It must absorb it from the radiation falling on it and exciting it, and suddenly, in full. If the exciting radiation, which can have an arbitrarily small intensity, is therefore too weak, it cannot do anything at all. This suggests the idea that there is a certain threshold of stimulation for the oscillator, below which it is not capable of any excitation at all, while above this threshold absorption begins with an entire energy element. Incidentally, as I would like to emphasise here, the concept of such a threshold of stimulation had already been realised earlier by M. REINGANUM in his oscillator model[1]).

However, this does not eliminate the difficulties. The absorption of a finite energy quantum from a finite radiation intensity can only take

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[1]) M. PLANCK, Ann. d. Phys. (4) 31, 758, 1910.
[1]) M. REINGANUM, Phys. ZS. 10, 351, 1909.

place in a finite time, which is greater the smaller the intensity of the exciting oscillation $\mathcal{J}$ is in comparison to the energy element ϵ. Now, with increasing oscillation frequency, the energy element $\epsilon = h\nu$ becomes ever larger, while the intensity, on the other hand, drops so rapidly that the time mentioned would ultimately be enormously long for short waves. And this directly contradicts the assumption made; for if the oscillator has once started to absorb energy and then the incident radiation should break off, it would be absolutely prevented from taking possession of its full energy quantum, which it necessarily needs from time to time to produce the statistical average U.

These considerations suggest, it seems to me, that the process of absorption should be regarded as a completely continuous one and that expression 1) for the absorbed energy should therefore be regarded as exactly valid.

However, this removes the previous assumption of the absolute discontinuity of the oscillator's energy; the oscillator's energy U does not need to be a whole multiple of the energy element ϵ, but can take on any value between 0 and ∞. Furthermore, the possibility of linking any probability considerations to the absorbed energy is also eliminated. Rather, it is directly given by the value of 1).

As a complement, however, the hypothesis that the emission of energy from the oscillator occurs in jumps, according to energy quanta and according to the laws of chance, is now presented, completely independent of the simultaneous absorption. I would like to formulate this hypothesis as follows. The emission of energy occurs spontaneously, in terms of certain quanta of the size $\epsilon = h\nu$, and the probability that an elementary quantum of energy is emitted by an oscillator with the oscillation frequency ν in a sufficiently small[1]) time element dt is equal to

$$\eta \, . \, n \, . \, dt. \qquad 13)$$

Here, η is a constant that depends only on the nature of the oscillator and is determined below, n is the number of whole energy elements ϵ that the oscillator currently possesses, i.e. n is the positive

[1]) Namely small compared to the mean time interval between two successive emissions.

integer (including zero) that makes $\frac{U}{\epsilon} - n$ a positive proper fraction (< 1). Then the expression can be written:

$$U = n\epsilon + \rho, \tag{14)}$$

where $0 < \rho < \epsilon$.

If, for example, U is smaller than ϵ, then $n = 0$ and the oscillator does not emit at all. For large values of n, however, one can neglect ρ compared to $n\epsilon$ and set the emitted energy, as before, proportional to U.

Let us now ask about the stationary oscillation state of the oscillator when it is in a field of black-body radiation. We cannot then set the energy absorbed in a time element dt equal to the energy emitted in the same time element dt; because the former is continuously changing, the latter is discontinuously changing. The equilibrium is rather a static one and refers to the average amounts of energy absorbed and emitted over large periods of time. Assuming this, the condition of the stationary state results from 1), 13) and 14) in an easily understandable notation:

$$\frac{3c^3\sigma}{16\pi^2\nu} \cdot \mathcal{J}_\nu = \eta \,.\, \bar{n} \,.\, \epsilon = \eta(\bar{U} - \bar{\rho}).$$

The mean value $\bar{\rho}$ is obviously $\frac{\epsilon}{2}$, so it follows that:

$$\mathcal{J}_\nu = \frac{16\pi^2\nu\eta}{3c^3\sigma}\left(\bar{U} - \frac{\epsilon}{2}\right).$$

For large $\bar{U}$ this equation must be identical with 4); this gives the emission coefficient:

$$\eta = 2\sigma\nu, \tag{15)}$$

and the last equation is combined with 2):

$$u_\nu = \frac{3}{4\pi}\mathcal{J}_\nu = \frac{8\pi\nu^2}{c^3}\left(\bar{U} - \frac{h\nu}{2}\right), \tag{16)}$$

in remarkable contrast to 4).

Now it is time to determine the temperature again. For this purpose we proceed in exactly the same way as above, i.e. we use the general thermodynamic equations 5) and 6) and ask about the probability that the oscillator has the average energy $\bar{U}$. This again results from considering the distribution of a very large energy quantum $N.\bar{U}$ over N similar oscillators. But now, in contrast to the previous consideration, the energy U of an oscillator can also have values other than total multiples of ϵ. This is because the energy U of the oscillator at any particular time t is determined uniquely from its energy U_0 at time $t = 0$ and the energy absorbed and emitted by it in the period t. If t is large enough, the energy U_0 will no longer be taken into account for the probability of the energy U and can therefore be assumed to be arbitrarily fixed. Likewise, the absorbed energy has a very specific value given by 1) and is the same for all oscillators present in the field of black-body radiation[1]). Thus, the probability considerations refer only to the emitted energy, and this is, according to our hypothesis, a complete multiple of ϵ. Therefore, in the expressions 14) for the energy of the N oscillators:

$$U_1 = n_1\epsilon + \rho_1, \quad U_2 = n_2\epsilon + \rho_2, \ldots$$

only the integers $n_1, n_2 \ldots, n_N$ are to be subject to probability considerations. However, since the total energy:

$$U_1 + U_2 + \cdots = N\,.\,\bar{U}$$

is given, so is the sum of the integers:

$$n_1 + n_2 + \cdots = P = \frac{(U_1 + U_2 + \cdots) - (\rho_1 + \rho_2 + \cdots)}{\epsilon},$$

$$P = \frac{N\left(\bar{U} - \frac{\epsilon}{2}\right)}{\epsilon} \qquad 17)$$

and it is therefore, just as before, a matter of distributing a large number P of energy elements over N oscillators of the same nature. Hence we obtain for S again the equation 9), and further, using 17):

$$S = \left.\begin{array}{l} k\left\{\left(\frac{\bar{U}}{h\nu} + \frac{1}{2}\right)\log\left(\frac{\bar{U}}{h\nu} + \frac{1}{2}\right)\right. \\ \left.-\left(\frac{\bar{U}}{h\nu} - \frac{1}{2}\right)\log\left(\frac{\bar{U}}{h\nu} - \frac{1}{2}\right)\right\} \end{array}\right\}. \qquad 18)$$

[1]) Fluctuations in the exciting radiation intensity, spatial and temporal, are present, but, as a closer consideration shows, have no influence here.

The substitution in 5) now yields:

$$\bar{U} = \frac{h\nu}{2} \cdot \frac{e^{\frac{h\nu}{kT}} + 1}{e^{\frac{h\nu}{kT}} - 1}, \tag{19)}$$

which differs from the previous value 11) by the additive constant $\frac{h\nu}{2}$.

The laws of blackbody radiation result from 19) and 16) again as well as above in 12).

The consequences of the new hypothesis therefore do not require any modification for blackbody radiation, but they do require modification for the energy of a resonating oscillator. For $T = 0$, $\bar{U}$ is not equal to 0, but to $\frac{h\nu}{2}$. This residual energy remains on average with an oscillator even at absolute zero temperature. It cannot lose it because, if U is less than $h\nu$, it emits no energy at all. On the other hand, for high temperatures and long waves, in the range of the validity of the JEANS-RAYLEIGH law, the new formula for U changes to the old formula.

A. EINSTEIN[1]) introduced the further assumption that in solid bodies (crystals) the vibration energy U of the oscillators, multiplied by the numerical factor 3 due to the three possible directions of vibration in space, simultaneously represents the total heat energy of the body, and W. NERNST, in the measurement of the specific heat carried out in association with his colleagues, not only found this assumption essentially confirmed, but also, in accordance with the new heat theorem he developed, extended it to liquid bodies[2]) The measurement of the specific heat does not provide a decision between formulas 11) and 19), because when differentiating from U to T the additive constant term $\frac{h\nu}{2}$ is eliminated. Thus, a direct experimental test of the new expression of $\bar{U}$ does not seem to be possible for the time being. On the other hand, there are some other phenomena which, I believe, 18) speak in favour of the hypothesis put forward here that the absorption and the emission of radiant energy are two completely independent processes, namely that the absorption is determined in each time element by the energy which is incident at the time, whereas the emission occurs suddenly, spontaneously, after certain quanta, in intervals which depend only on

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[1]) A. EINSTEIN, Ann. d. Phys. (4) 22, 180, 1907.

[2]) W. NERNST, Session report of the Pruss. Akad. of the Wise., p. 247, 262,

the state of the emitting structure, regardless of whether it is irradiated or not.

The remarkable observations on the Doppler effect of cathode rays have already been discussed from the standpoint of quantum theory[3]). But one can go further. Since the temperature equalisation inside a body takes place not only through radiation but also through the conduction of heat, it is reasonable to assume that not only in the exchange of radiant heat but also in the exchange of the energy of corpuscular movements, the emission takes place according to certain energy quanta, so that, for example, if an oscillator with the oscillation frequency ν is struck by electrons, it emits these electrons not according to a kind of law of reflection but independently of the speed with which they impact, at a very specific speed that depends only on the oscillation frequency ν, with a frequency that depends only on its state (energy, electrical charge).

Perhaps this explains the question, which is so difficult for the kinetic theory, of why the "free" electrons of a metal do not make a noticeable contribution to the specific heat. According to the view described here, the electrons do not have any independent degrees of freedom, since their speeds are quite specific, and their movements are therefore not taken into account when distributing the energy of the entire metal to the various independent degrees of freedom. However, I would like to express this assumption with great reservation, especially with regard to the fact that the completely different DRUDE Theory, according to which the average energy of an electron is proportional to the absolute temperature, has led to results that agree remarkably well with experience.

If the emission of electrons is caused by radiant energy, as in the photoelectric effect or when X-rays hit, the speed of the electrons must also depend only on the nature of the excited oscillator, but not on the temperature and the intensity of the exciting radiation, which seems to be generally confirmed by experience, sometimes even quantitatively[1]). However, it must be noted that one cannot always directly deduce the oscillation frequency of the excited oscillators from the wavelength of the

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[3]) J. STARK, Phys. ZS. 9, 767, 1908.

[1]) See A. EINSTEIN, Ann. d. Phys. (4) 17, 145, 1905; O. v. BAEYER and A. GEHRTS, Verh. d. D. Phys. Ges. 12, 870, 1910; R. Pout and P. PRINGSHEIM, ibid.

exciting radiation (luminescence phenomena).

From the point of view represented here, the question of whether the energy of the emitted electrons comes from the incident radiation or from the emitting molecule can obviously be answered by the fact that the emitted energy always comes primarily from the energy of the oscillator, which in turn is, however, also caused by energy absorption from the incident radiation.

Finally, it could be pointed out that the phenomena of radioactivity also seem to fit in seamlessly with the hypothesis of "quantum emission". One only needs to assume that the vibrations of the oscillators from which the emitted rays originate are completely different and occur completely independently of the vibrations on which the temperature and the specific heat of the radioactive substances are based. The fact that one and the same atom can accommodate completely different vibrations at the same time is already generally suggested by the great variety of lines in many spectra, including the phosphorescence spectra. The fact that not only the α-rays have a certain speed, but also that, as recent experience has made probable, the β-rays, provided they originate from a certain atom, travel at a very different speed, is also in good agreement with quantum emission[1]).

Although the problem of the absorption and emission of radiant heat is by no means completely solved by the hypothetical experiment described, but rather pushed back even further – because the application of the laws of chance always means giving up the completeness of the causal connection – the hypothesis of quantum emission seems to me to be suitable for the time being not only for eliminating serious contradictions between the radiation theory and the most important principles of MAXWELL's electrodynamics, but also for shedding a clearer light on certain other phenomena which have not yet been properly connected. One would certainly not want to reject the kinetic theory of gases on the grounds that it is still unable to account for the most interesting processes in a gas, the collisions between two molecules.

[1]) O. HAHN and L. MEITNER, Phys. ZS. 10, 741, 948, 1909; O. V. BAEYER and O. HAHN, ibid. 11, 488, 1910. 11.