Quantum Mechanics 3   2001/2002

Problem set 4

(1) [2000 degree exam, short question] A quantum system has a set of eigenstates $u_n(x)$, with energies E_n. The system is placed in a state ψ that is not an eigenstate; use the fact that the u_n are a complete set to show that the expectation value of the Hamiltonian, $\langle\psi|H|\psi\rangle$, always overestimates the ground-state energy.

(2) Show that, for the 1D wavefunction

$$\psi = \begin{cases} (2a)^{-1/2} & |x| < a \\ 0 & |x| > a \end{cases}$$

the rms uncertainty in momentum is infinite (hint: you need to Fourier transform ψ. The mean momentum is zero by symmetry, so you only need $\langle p^2\rangle$). Comment on the relation of this result to the uncertainty principle.

(3) Show that the following relation applies for any operator O that lacks an explicit dependence on time:

$$\frac{\partial}{\partial t}\langle O\rangle = \frac{i}{\hbar}\langle [H, O]\rangle.$$

(hint: remember that the Hamiltonian, H, is a Hermitian operator, and that H appears in the time-dependent Schrödinger equation).

Use this result to derive Ehrenfest's relations, which show that classical mechanics still applies to expectation values:

$$m\frac{\partial}{\partial t}\langle \mathbf{x}\rangle = \langle \mathbf{p}\rangle$$
$$\frac{\partial}{\partial t}\langle \mathbf{p}\rangle = -\langle \boldsymbol{\nabla} V\rangle$$