Quantum Mechanics 3 2001/2002

Problem set 1

(1) Consider a particle in an infinitely deep one-dimensional potential well, where $V = 0$ for $|x| < L$. The wave function is of the form $\psi = A \sin kx + B \cos kx$.

(a) Apply the boundary conditions on the wavefunction to deduce that $k = n\pi/(2L)$, $n = 1, 2, 3, \ldots$, with $A = 0$ for odd n, $B = 0$ for even n.

(b) Consider the same problem with a shift of origin, so that the well now runs from $x = 0$ to $x = 2L$. What is the form of the wavefunction in this case?

(c) The probability density for finding the particle at a given value of x is $\propto |\psi|^2$. Use this fact to *normalize* the wavefunction; i.e. find the constant of proportionality A.

(d) Using the result of part (c), compute the mean and rms values of x as a function of n. Show that, in the limit of large n, the results tend to the classical values for a particle bouncing backwards and forwards in the well at uniform speed: $\langle x \rangle = 0$, $\langle x^2 \rangle^{1/2} = L/\sqrt{3}$.

(e) The time-independent Schrödinger equation may be written as $(p^2/2m + V)\psi = E\psi$, where $p = -i\hbar\partial/\partial x$ in 1D. Show that a particle in state n has energy $E_n = \hbar^2\pi^2 n^2/(8mL^2)$.

(f) The first transition in the Lyman series of Hydrogen has a wavelength of 121.6 nm. Using the 1D well model, estimate a characteristic size for the Hydrogen atom.

[PTO]

(2) The quantum flux density of probability is

$$\mathbf{j} = \frac{i\hbar}{2m}(\psi \boldsymbol{\nabla} \psi^* - \psi^* \boldsymbol{\nabla} \psi);$$

it is related to the probability density $\rho = |\psi|^2$ by $\boldsymbol{\nabla} \cdot \mathbf{j} + \dot{\rho} = 0$.

(a) Consider the case where ψ is a stationary state. Show that ρ and $\mathbf{j}$ are then independent of time. Show that, in one spatial dimension, $\mathbf{j}$ is also independent of position.

(b) Consider a 3D plane wave $\psi = A \exp(i\mathbf{k} \cdot \mathbf{x})$. What is $\mathbf{j}$ in this case? Give the physical interpretation.