Structure formation

Galaxy clustering

As well as proving that galaxies were large systems of stars at great distances, Hubble was also the first to show that galaxies were clustered – i.e. they are not sprinkled randomly on the sky. Simple images of the sky, without distance information, shows that there are some special regions containing very large numbers of galaxies. With the aid of distances measured from **redshift surveys** (using $d = cz/H$), we now know that galaxies come in a variety of families:

Groups These are systems of just a handful of galaxies. The **local group** consists of the Milky Way and the Andromeda Nebula (M31), plus about a dozen much smaller galaxies. Most galaxies live in systems of this sort of size.

Clusters About 1% of galaxies live in very rich systems, where hundreds of galaxies the size of the Milky Way live within 1 Mpc of each other.

Superclusters There are larger-scale patterns in the galaxy distribution, hinted at in pictures of the sky, but fully revealed by redshift surveys. There are **voids**: regions of up to 50 Mpc diameter that contain no galaxies. There are also **filaments** and wall-like features that connect the rich clusters and bound the voids. However, all this structure makes up a single connected web, where the density of galaxies is only a few times larger than average. Clusters are the largest single clearly-defined systems in the universe.

It is worth bearing in mind that pictures from redshift surveys give a distorted picture of the universe, because redshifts measure only the total recessional velocity, which we can write as

$$\boxed{v = Hd + \delta v.}$$

The first term on the right is the ideal velocity from a uniformly expanding universe. The second term represents extra velocities, e.g. from orbital motions within a cluster. The effects of these velocities is to appear to stretch clusters radially, giving the effect called **fingers of God**.

Gravitational instability

The **large-scale structure** of the universe can arise because of the action of gravity. Imagine two spherical regions of the universe, one less dense than average, one denser. By the same argument that we used to understand the dynamics of the whole universe, the evolution of these spherical regions depends only on the density inside them: each behaves like a little universe. The low-density sphere will therefore expand faster than average, since its expansion will suffer less deceleration, and it will evolve into a void. The high-density sphere, conversely, will behave like a closed universe: gravity will eventually halt its expansion, and it will fall in on itself. This evolution can be solved exactly, and it divides into three regimes:

(1) **Linear growth** If the density inside the sphere exceeds the mean density by a factor $(1+\delta)$, then δ grows as a universal function of time while it is small.

(2) **Turnround** When the density inside the sphere is about 6 times the mean density, the sphere ceases to expand.

(3) **Virialization** As the sphere falls in on itself, the orbits of the galaxies that it contains become randomized, and it settles into a group or cluster where the galaxies simply orbit each other. At this point, the density is about 200 times the mean.

This process can repeat itself on different scales, in a **hierarchical** fashion: galaxies form from the collapse of small clumps of matter, and clusters form later by collapse of systems of galaxies.

Primordial fluctuations and dark matter

Of course, the universe does not contain neat little spherical systems. Rather, the density is perturbed by a superposition of residual quantum ripples with different wavelengths, analogous to the disturbance of the water on the surface of a swimming pool. How do all these ripples affect the density as a function of scale? Suppose we take a box of side L, place it at random in the universe, and measure the overall density within it. If we repeat the process putting the box in a different place, we will get a different answer. The prediction of inflation is that these fluctuations in density depend on the side of the box:

$$\boxed{\text{density variation} \propto 1/L^2,}$$

so that the universe becomes uniform only when viewed on the largest scales.

This story is complicated a little by the effects of dark matter. Depending on what the dark matter actually is, the small-scale fluctuations can be erased. For example, massive neutrinos (hot dark matter) have a random velocity of around $100\,\mathrm{km\,s^{-1}}$: they are frozen-out relics of particles that were once in thermal equilibrium, and the residual thermal velocity is high because the particles are very light (a few eV). Over the age of the universe, these random velocities smear out the dark-matter distribution over scales of about 1 Mpc: there are no galaxy-sized fluctuations left, so galaxies can only form after clusters have collapsed. This means that we wouldn't expect to see galaxies at high redshifts, because clusters wouldn't yet have collapsed. The existence of galaxies in apparently unchanged numbers up to $z > 4$ therefore tells us that the dark matter is probably not massive neutrinos. Conversely, very heavy cold dark matter (**CDM**) particles have very little damping effect. It is therefore commonly believed that the dark matter is most likely to be in the form of CDM.

N-body simulations

Many of these conclusions have been tested in detail by making a DIY universe in a box. A computer is used to store a large set of numbers that represent the positions and velocities of a set of imaginary dark-matter particles. The force on one particle comes from adding up the gravitational attraction of all the other particles. The orbits of all these particles can then be followed, simulating the evolution of structure in the universe. A few of the key issues are:

(1) **Comoving coordinates** The box has to be expanded so that it keeps pace with the mean expansion of the universe – otherwise all the particles would be lost from the box.

(2) **Periodic volume** Even so, some particles will move off the edge of the box. This is fixed by re-injecting ones that move off the top of the box through the bottom – so the infinite universe is treated as a repeating set of identical boxes.

(3) **Resolution** The fictitious particles are much more massive than the real dark-matter particles. The best we can do is to require that they are at least no bigger than a galaxy. How many particles do we then need? The density of the universe is about $10^{10.5}\, M_\odot Mpc^{-3}$, and we saw that the effective mass for a galaxy like the Milky Way was perhaps $10^{12.5}\, M_\odot$. A 100-Mpc cube would contain $10^{16.5}\, M_\odot$ of dark matter, so $N = 10^4$ particles is a very bare minimum. A practical limit is roughly $N = 10^6$ particles in a volume this size, so galaxies are represented by clumps of about 100 particles. The state of the art is $N = 10^9$ particles, allowing volumes of side > 1000 Mpc to be simulated (a good fraction of the whole universe).

Using these techniques, it is possible to find a impressively close match between the patterns seen in the local galaxy distribution, and the predictions of gravitational instability. The next step would be to see if we can prove that the structure really does grow with time.

Microwave anisotropies

The best probe of early structure is to look as far back in time as possible – to the last-scattering surface where the microwave background originates. At this time, the precursors of clusters and superclusters should be present at a very low level. We can work out the angle these should subtend by assuming that the distance to the last-scattering surface is roughly the Hubble distance $c/H = 4600$ Mpc. A length L Mpc therefore subtends an angle θ:

$$\boxed{(\theta/\mathrm{radians}) \simeq L/4600\,\mathrm{Mpc} \quad \mathrm{or} \quad (\theta/\mathrm{degrees}) \simeq L/80\,\mathrm{Mpc}.}$$

The microwave sky therefore probes supercluster scales at angles of 1 degree; larger angles measure structures too big to study easily today.

The existence of fluctuations in density at last scattering makes the microwave sky non-uniform, mainly because of **gravitational redshift**. Photons that originate in the centre of a dense blob will lose energy as they climb out. The magnitude of the effect can be worked out as follows. The gravitational potential energy of a particle of mass m near a system of mass M and radius r is $-GMm/r$. For a photon, $m = h\nu/c^2$, so the fractional shift in frequency is

$$\boxed{\frac{\delta\nu}{\nu} = -\frac{GM}{c^2 r} = -\frac{v^2}{c^2},}$$

where v is the orbital velocity about the body. The biggest systems we know about are clusters, with $v \simeq 1000\,\mathrm{km\,s^{-1}}$, so the fractional redshift is about 10^{-5}.

This effect was measured in 1992 by the **COBE** satellite mentioned earlier. It mapped the sky at frequencies between 30 GHz and 90 GHz, at 7-degree resolution, and found an intensity that fluctuated at about the 10^{-5} level (a tremendous experimental achievement, given the need to remove foreground emission such as that from the Milky Way). Is seems that we understand the evolution of structure in the universe between $z = 1000$ and $z = 0$.

The future

Our understanding of the universe has changed incredibly in only 70 years, but the general framework now seems relatively stable. In many cases, the key problems lie in the area of particle physics: we need a particle candidate for the dark matter and we need a detailed mechanism for the generation of the matter-antimatter asymmetry.

Perhaps the biggest challenge for the future will be to find a way of testing inflation. The universe grew its structure from density fluctuations like those predicted by inflation, but it is always possible that the fluctuations originated in some other way. The real prediction of inflation is that quantum fluctuations should leave behind not only non-uniform density, but also a background of **gravity waves**. These will add extra temperature fluctuations to the microwave background, in a way that can be measured, if a high-resolution map of the microwave sky can be made. Over the next decade, a number of balloon and satellite experiments will attempt to achieve this, mapping the whole sky to 100 times the resolution of COBE. Being optimistic, the answer to the origin of everything might therefore possibly be known in time for the 2029 Centenary of Hubble's Law – but whatever happens, this will be a period of the greatest scientific excitement.

Further information The following books may be helpful if you are interested in following up any of the topics in more detail:

Bothun, *Modern cosmological observations and problems*, Taylor & Francis
Gribbin, *In search of the big bang*, Heinemann
Guth, *The inflationary universe*, Addison-Wesley
Rowan-Robinson, *Cosmology*, Oxford University Press
Weinberg, *The first three minutes*, Basic Books

There is also a lot of interesting information (and spectacular pictures) available on the web. You may like to explore some of the following addresses:

http://www.damtp.cam.ac.uk/user/gr/public
http://panisse.lbl.gov
http://map.gsfc.nasa.gov/html/web_site.html
http://www.cern.ch