Matter in the universe

Masses from orbits

The mass content of the universe is what determines fundamentals such as the global geometry, and whether the expansion will continue for ever. Unfortunately, weighing the universe is not so easy to do. The best that can be done is to determine the mass of individual objects such as galaxies or clusters of galaxies, and then count how many such objects there are in the universe:

$$\boxed{\text{density of mass} = \text{number density of galaxies} \times \text{mass per galaxy.}}$$

This strategy will miss any smoothly-distributed mass between galaxies, so the density estimate is really only a lower limit.

The classical way of measuring mass is to use 'the only equation in astronomy':

$$\boxed{v^2 = GM/r.}$$

This is the orbital velocity of material in circular motion at a radius r from a body of mass M. The material can be stars, but the 21-cm emission from neutral hydrogen often continues further out (the gas hasn't turned into stars yet). What tends to be found is that the orbital velocity is independent of radius out to as much as 30 kpc from the centre of galaxies like the Milky Way. The orbital equation only counts the mass inside r, so we see that $M(<r) \propto r$, which turns out to require a density falling as $\rho \propto 1/r^2$. The Milky Way has a rotation speed of $220\,\mathrm{km\,s^{-1}}$, so the mass inside 30 kpc is $10^{11.5}$ solar masses (1 solar mass is 2×10^{30} kg). Redshift surveys of galaxies measure the number density of bright galaxies to be about $0.003\,\mathrm{Mpc^{-3}}$, so the average density is 10^9 solar masses per $\mathrm{Mpc^3}$. The critical density in these units is $1.2 \times 10^{11} h^2$, so we learn that $\Omega_{\rm galaxies} \gtrsim 0.01$.

A safer method is to look where there are many galaxies orbiting each other – a **cluster of galaxies**. Rich clusters have several hundred galaxies orbiting each other at up to $1000\,\mathrm{km\,s^{-1}}$ (as deduced from the differences in redshift between different galaxies). In this way, total masses of up to 10^{15} solar masses have been measured, suggesting a mass of perhaps $10^{12.5}$ solar masses per galaxy. This figure is 10 times larger than the figure we had before, so a better lower limit for the density parameter is $\Omega \gtrsim 0.1$.

Since a galaxy like the Milky Way contains about 10^{11} stars, $10^{12.5}$ solar masses is a surprisingly large mass. Depending on the exact stellar population in a galaxy, the average mass of its stars should be about one solar mass, not 30. The mass in galaxies is therefore dominated by **dark matter**; the big unanswered question is what the physical nature of this material could be.

Masses from gravitational lensing

An alternative means of measuring mass comes from a prediction of general relativity: that even light should 'fall' in a gravitational field. A ray of light passing within a distance r of a body of mass M should be deflected through an angle

$$\boxed{\text{deflection angle} = \frac{4GM}{c^2 r}.}$$

This equation was shown to work for the sun (deflection angle about 1 arcsecond) in the 1919 total eclipse. The deflection angle is small for most bodies (black holes are the exception).

Over cosmological distances, however, these small deflections can add up to cause significant distortions. Looking at a distant galaxy through a foreground massive object such as a cluster of galaxies, the cluster acts as a distorted magnifying glass. When a galaxy is directly behind the cluster, light can be bent around the cluster on all sides (if the cluster is spherical), to give an image in the form of an **Einstein ring**. The radius of this ring gives a very direct measure of the mass within the cluster. These measurements are generally in good agreement with the masses estimated from galaxy orbits.

An important by-product of these studies is to compute how much of a cluster's mass is due to normal baryonic matter. Clusters shine in X-rays because of hot gas between the galaxies. The amount of gas turns out to far outweigh the stars in the galaxies, making up about 10% of the total. We might expect this figure to be a measure of the fraction of the total mass of the universe that is in the form of baryons. Since nucleosynthesis tells us that $\Omega_{\rm B} \simeq 0.03$ (depending on H), this suggests that the total density parameter is $\Omega \simeq 0.3$.

The nature of dark matter

Many candidates for the dark matter have been suggested. Observationally, there is no problem with putting it in the form of very cold and compact baryonic bodies: planets, comets, neutron stars would do. The main reason that these alternatives are rejected is through the nucleosynthesis limit on the baryon density.

The most likely possibility is therefore that the dark matter is in the form of exotic elementary particles. This is a safe bet at some level, because there will generally exist a large number of **frozen-out relics** from the big bang: particles that one existed in equilibrium, but whose rate of interaction then dropped to the point where it could no longer keep pace with the expansion. This mechanism was discussed in the context of nuclear reactions, but a cleaner example is the photons of the microwave background, which have not interacted since $z = 1000$. There are about 400 of these in every cm^3 of the universe, and a similar density would be expected for any particle that became frozen out while the universe was hotter than $kT = mc^2$. So, there should be a background of **neutrinos**, the weakly-interacting particles involved in nuclear reactions. If these have a mass of about 10 eV, that would make the dark-matter density. Massive neutrinos are called **Hot Dark Matter (HDM)**, since the neutrinos still have substantial residual thermal speeds. Alternatively, the particles involved could be much heavier, $\gtrsim$ 10 GeV, having frozen out after their numbers were reduced by annihilations. This kind of particle would be called **Cold Dark Matter (CDM)**. The particles involved would have to be of a kind not yet seen, but there are hopes that they might be seen in experiments at the **CERN** particle accelerator in the next few years.

Vacuum energy and antigravity

Everything we have said so far has neglected one possible contributor to the density of the universe: nothing. It may seem like common sense to say that the density of a vacuum is zero; life is not so simple in the world of quantum mechanics, where the **uncertainty principle** forbids us ever to have perfect knowledge of a physical system. Consider electromagnetic waves inside a box: can we specify exactly zero energy for these? Think of the electromagnetic modes as being like oscillating elastic strings: for the strings to be perfectly still, we would need to know their position and velocity exactly. However, Heisenberg (1925) told us that the product of the uncertainties (Δ) in position and momentum (mv) must exceed a minimum amount:

$$\boxed{\Delta(x)\Delta(mv) > h/4\pi,}$$

where h is Planck's constant ($E = h\nu$). If we know exactly where a particle is, we have no idea of its speed, and vice-versa. The quantum vacuum therefore has to weigh something,

Having believed one impossible thing already, it is not hard to add another: the gravitational properties of vacuum energy are opposite to those of normal matter: whereas a density of dark matter or radiation tends to decelerate the expanding universe, vacuum energy will tend to speed it up. We can see this by looking at the formula for gravitational potential energy: $V = -GM/R$. For a sphere of normal matter, M stays constant as the sphere expands. Since R goes down, V becomes closer to zero – i.e. the sphere is less strongly bound. The energy to achieve this unbinding has to come from somewhere, and it is at the expense of the sphere's kinetic energy of expansion. Now imagine expanding a sphere of vacuum. The vacuum doesn't really expand – we just get more of it as the sphere gets bigger. The mass of the sphere is therefore proportional to the vacuum density times the sphere volume, which increases as R^3. Thus, as the sphere increases, its binding energy scales as $V \propto -R^2$; the sphere becomes more bound as it expands, liberating kinetic energy.

To sum up, it is possible that the vacuum could have some constant non-zero mass density (called the **cosmological constant** by Einstein), giving a contribution $\Omega_{\rm vacuum}$ to the total density parameter, and causing the expanding universe to accelerate.

Supernovae and accelerating universes

The conclusion of the previous section sounds more like science fiction, but it can be tested. In the previous lecture, we introduced the idea that the brightness of distant objects could depend on the acceleration of the universe, with objects at a given redshift being bright for high Ω, and faint for low Ω. By this argument, objects in an accelerating universe with a big vacuum density would appear fainter still.

For decades, cosmologists were unable to apply this test, for want of a suitable **standard candle**. This is an ideal object of constant output, L, for which a plot of flux against redshift should show the expected cosmological curvature. Recently, however, a candidate for this role has emerged. **Supernovae, or SNe,** are stars that burn a large fraction of their nuclear fuel in an almost instantaneous explosion. About 10^{45} Joules of energy is released (although only about 1% goes into light). There are two main type of supernovae: type I & II. The latter correspond to very massive stars at the end of their lives. The origin of the former (SNeI) is still uncertain, although a common theory is that they are a nuclear flash that results from a white dwarf in a binary orbit swallowing more mass from the primary star than is good for it.

In any case, the important point about the SNeI explosions is that they are all very similar outbursts. They have similar **lightcurves**, consisting of a rise to maximum over a few weeks, a symmetric fall, and then a slower decline. They also have similar peak luminosities, to within about 30%. In the past few years, it was realized that we can make even better standard candles, because the timescale of the main 'hump' correlates with the brightness: the most luminous objects take longer to rise and fall. Presumably, for a bigger explosion, there is a larger cloud of debris, which takes longer to cool. Whatever the explanation, it can be exploited: by measuring the timescale and applying a correction, we get standard objects, whose peak brightnesses are all the same to better than 10%. Systematic searches have now picked out nearly 100 faint supernovae at redshifts up to 1, so this gives an excellent basis for the cosmological distance-redshift test.

As a by-product of this work, we can also prove that the universe is expanding. The supernovae are standard objects in their rest frames, so the high-redshift examples should display **time-dilated** lightcurves if the redshift results from expansion: a supernova at $z = 1$ should last for twice as long as one nearby. This is observed to work: supernovae are only standard objects if we correct for cosmological time dilation.

This is a very fine scientific achievement, but it is dwarfed by the radical implications of the distance-redshift test. In a result first published in 1998, two groups showed that the supernovae follow very well a cosmological model that is dominated by vacuum energy. Supernovae at $z = 1$ are too faint by about a factor 1.5 to be consistent with an $\Omega = 1$ critical-density universe. Even going to the limit of $\Omega = 0$ does not cure the problem. The only thing that works is to have a large amount of vacuum energy, and an accelerating universe:

$$\boxed{\Omega_{\rm matter} \simeq 0.3; \quad \Omega_{\rm vacuum} \simeq 0.7.}$$

The matter result is consistent with what we had before, but the vacuum result is more than twice as large.

Interestingly, the total matter + vacuum density appears to be close to critical, so the universe is balanced between being open and being closed. However, the dominant vacuum energy means that there is no doubt as to the future of the expanding universe: it will continue forever, with ever-increasing speed, driven by the **gravitational repulsion** of the vacuum.

It is no exaggeration to say that the above conclusion was one of the most important scientific results of the 20th century. It transforms our vision of the kind of universe we inhabit.