Recovering Shear with Elliptical Gauss Laguerre Method

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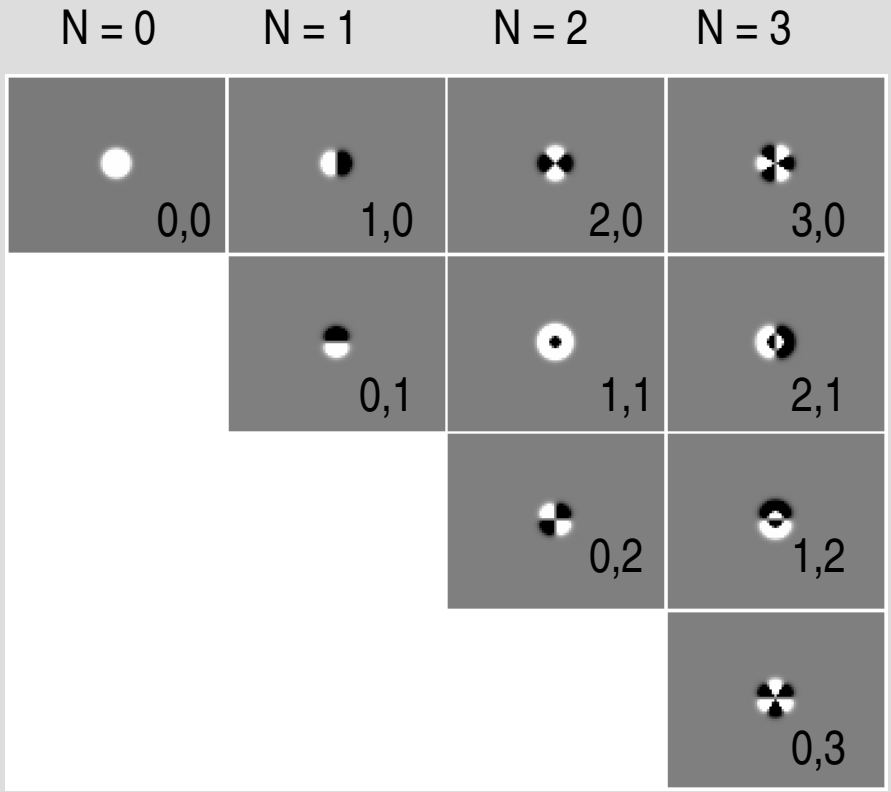
Testing Shear Recovery

- Under ideal testing conditions*, gravitational shear can be recovered to ~3% accuracy, using the **EGL technique**
- *Ideal testing conditions:
  - better than 99% convergence of fitting procedure
  - no overlap of individual galaxy images
  - galaxy shapes evenly distributed
- Realism:
  - image noise, finite sampling, asymmetric galaxies, PSF

Outline

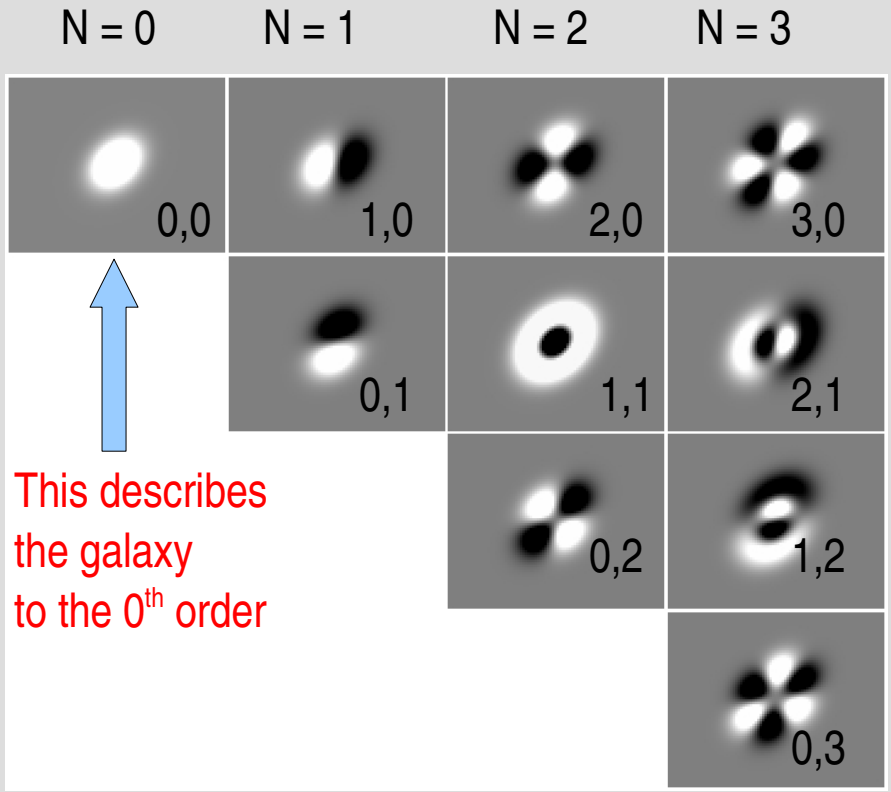
- EGL method
  - shape measurements &
  - shear recovery
- EGL Fitting Procedure
  - multifit (fitting multiple exposures at once)
- Testing shear recovery
  - boring stuff to watch out for (Nakajima & Bernstein 2007)
- PSF interpolation
  - with less noise
    (Schrabback et al. 2006 + Jarvis & Jain 2004)

Gauss-Laguerre = Polar Shapelets



- Orthogonal functions $\Psi_{pq}(\mathbf{x})$
  - Quantum number: (p,q)
  - Order: N = p + q
- Want to describe most of the galaxy image by the N = 0 term

GL + Elliptical Basis

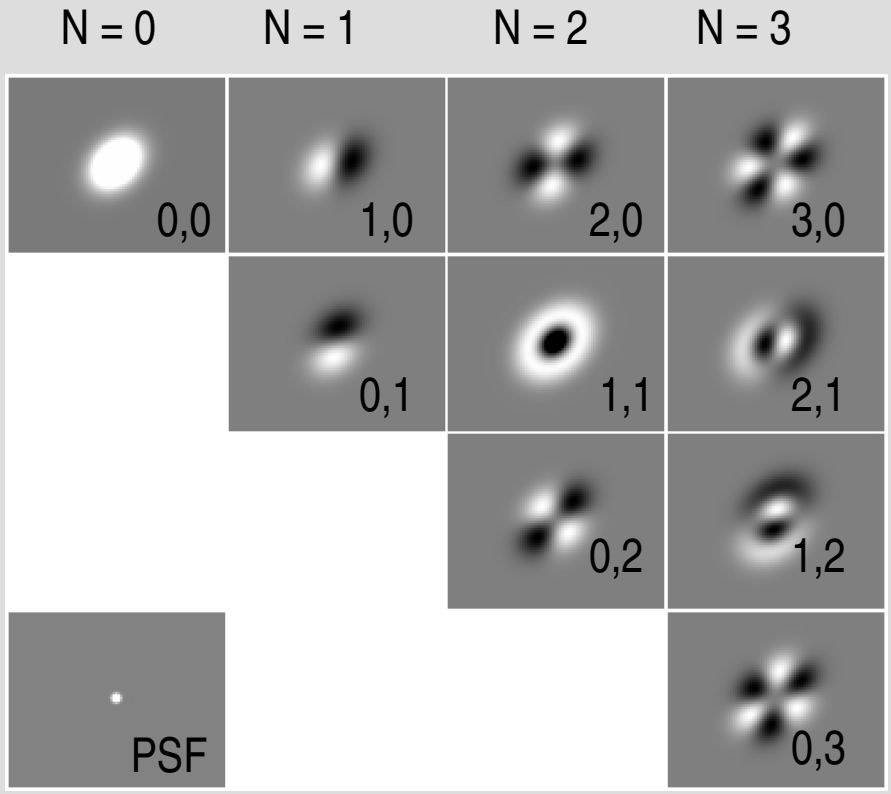


- Orthogonal functions $\Psi^{E}_{pq}(\mathbf{x})$
- basisE = (e_1, e_2, σ, x_0, y_0)
- Decompose image:

$$I(\mathbf{x}) = \sum_{pq} b_{pq} \Psi^{E}_{pq}(\mathbf{x})$$

  - centroid: $b_{10} = b_{01} = 0$
  - size: $b_{11} = 0$
  - shear: $b_{20} = b_{02} = 0$

EGL – with PSF



- Convolve EGL basis function with PSF

$$\Phi^{E}_{pq} = PSF * \Psi^{E}_{pq}$$

- Find coefficients for the basis functions Φ^{E}_{pq}

$$I(\mathbf{x}) = \sum_{pq} b_{pq} \Psi^{E}_{pq}(\mathbf{x})$$

$$PSF * I(\mathbf{x}) = \sum_{pq} b_{pq} \Phi^{E}_{pq}(\mathbf{x})$$

EGL Fitting Procedure

$$\chi^2 = \sum_{\text{pixel } p} \frac{[I_p - \sum_i b_i \Phi_i(\mathbf{x}_p)]^2}{\sigma_p^2}$$

- Standard χ^2 minimization

$$\mathbf{b} = \alpha^{-1} \beta$$

$$\alpha_{ij} \equiv \sum_{pix} \Phi_i^E(\mathbf{x}_p) \Phi_j^E(\mathbf{x}_p) / \sigma_p^2$$

$$\beta_j \equiv \sum_{pix} I_p \Phi_j^E(\mathbf{x}_p) / \sigma_p^2$$

- Find the matrix $\frac{db}{dE}$

$$\Psi_i^{E+\delta E} = \Psi_i^E + \sum_k \delta E_k \sum_j \mathbf{G}_{kij} \Psi_j^E$$

($\mathbf{G}_{kij}$: generator for δE_k)

EGL Fitting Procedure

$$\chi^2 = \sum_{\text{pixel } p} \frac{[I_p - \sum_i b_i \Phi_i(\mathbf{x}_p)]^2}{\sigma_p^2}$$

$$+ \sum_{\text{pixel } p'} \frac{[I'_{p'} - \sum_i b_i \Phi'_i(\mathbf{x}_{p'})]^2}{\sigma_{p'}^2}$$

for additional image

with image I', pixel p'

- Standard χ^2 minimization

$$\mathbf{b} = \alpha^{-1} \beta$$

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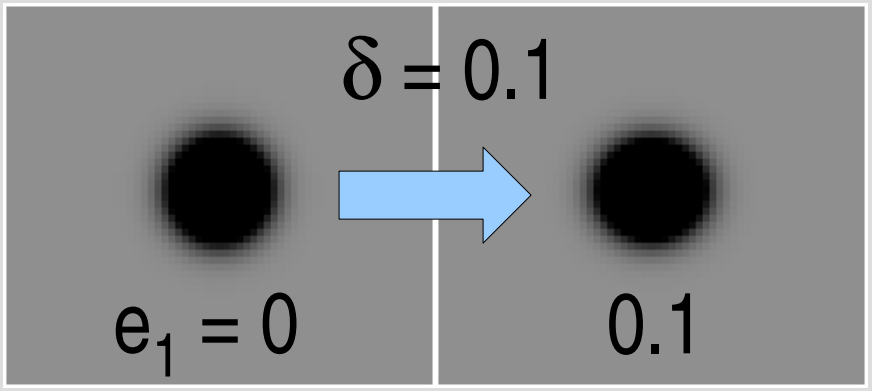
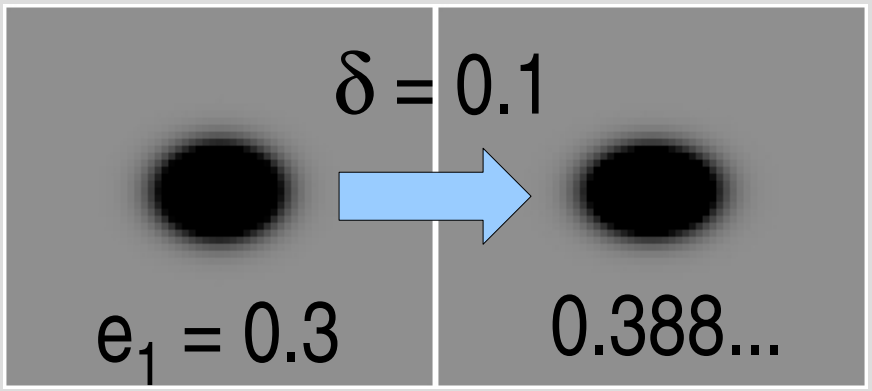
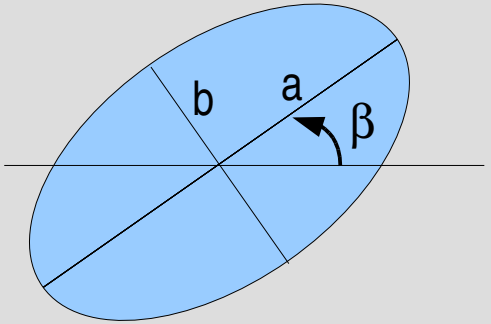
Estimating Shear = Averaging Shapes





$$\delta \oplus e_1 = e_1 + (1 - e_1^2)\delta + O(\delta^2)$$

- shape e



$$e = \frac{a^2 - b^2}{a^2 + b^2}$$

$$e_1 = e \cos 2\beta, \quad g = \frac{a - b}{a + b}$$

- distortion δ

$$\delta \approx 2g \qquad g = \frac{\gamma}{1 - \kappa}$$

- Responsivity

$$R_i = 1 - \langle e_i^2 \rangle$$

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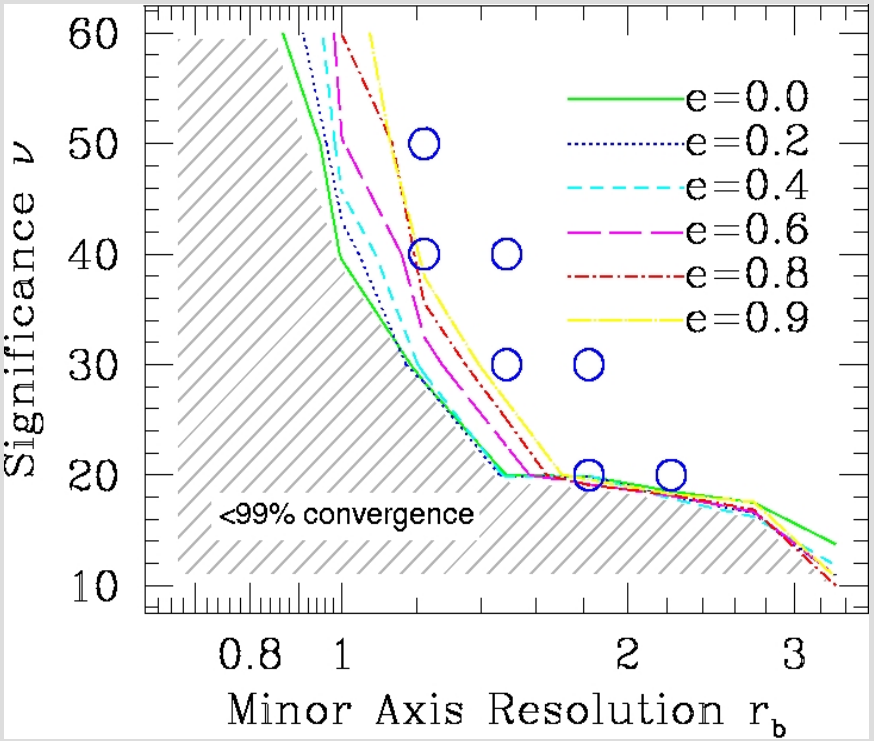
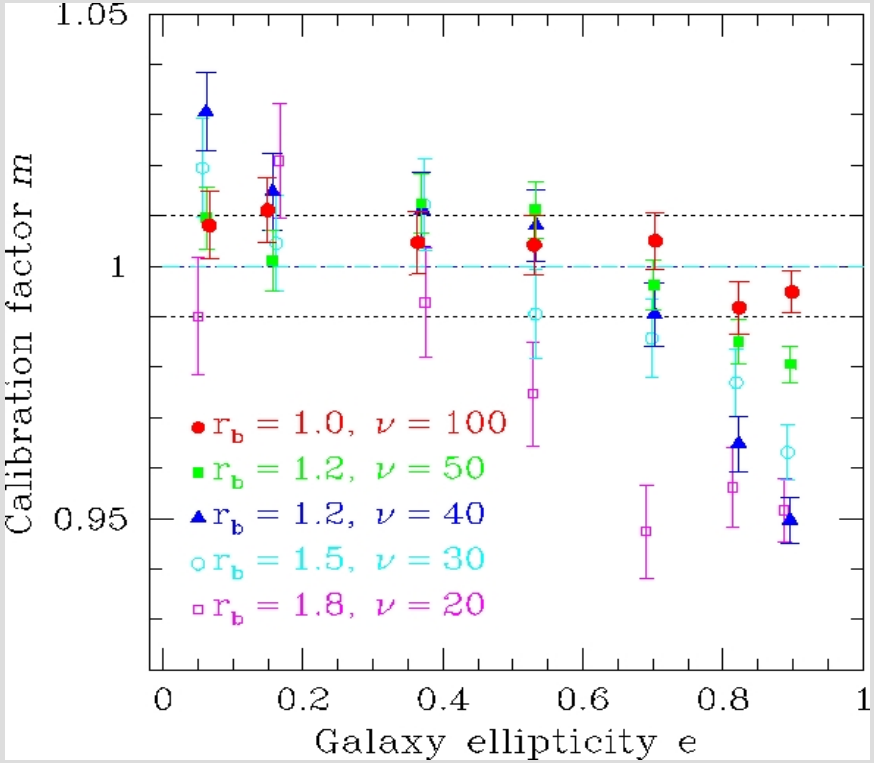
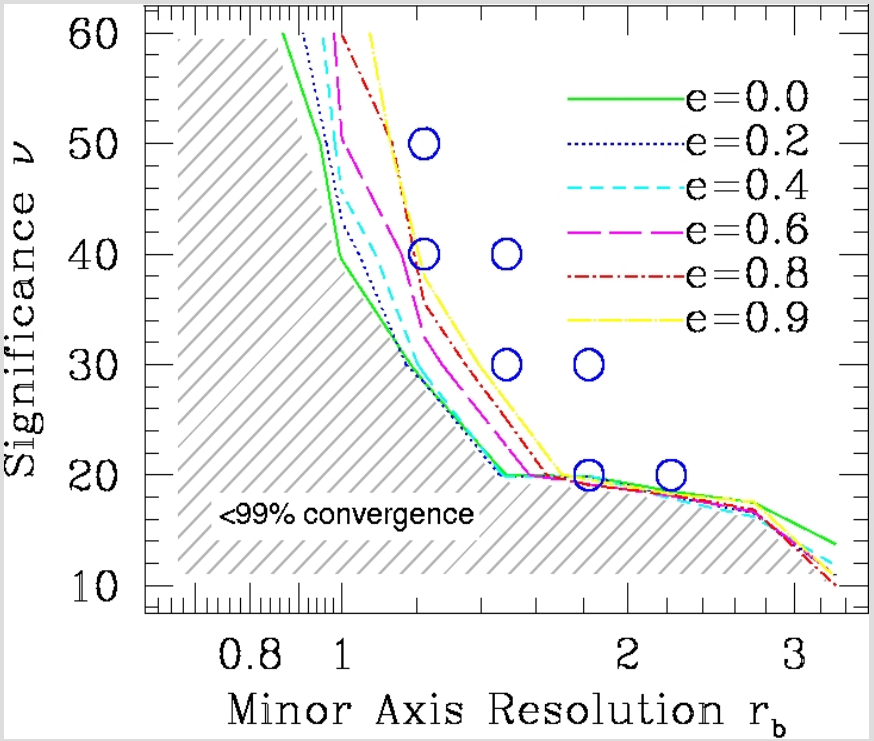
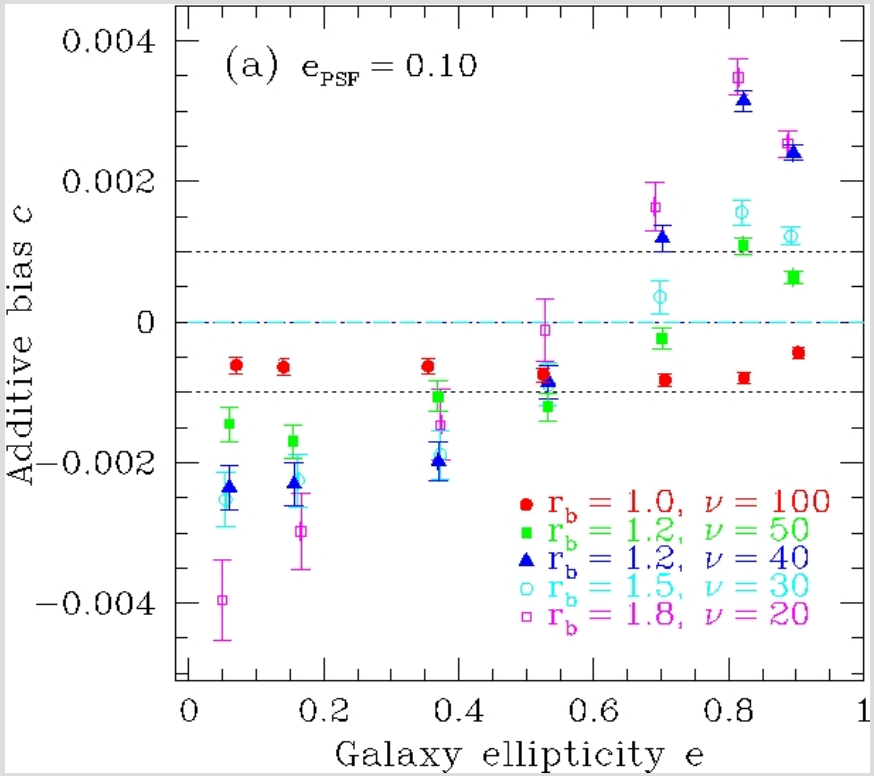




Calibration Bias across different galaxy ellipticity

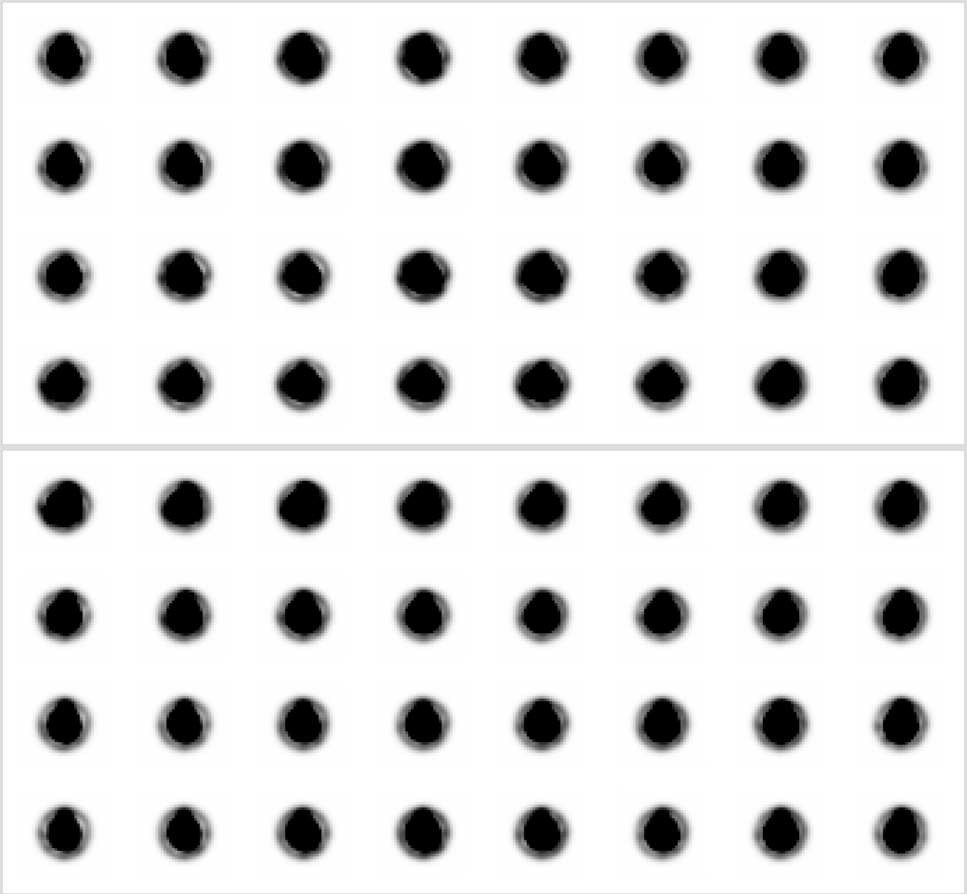
Testing Shear Recovery





PSF shape suppression

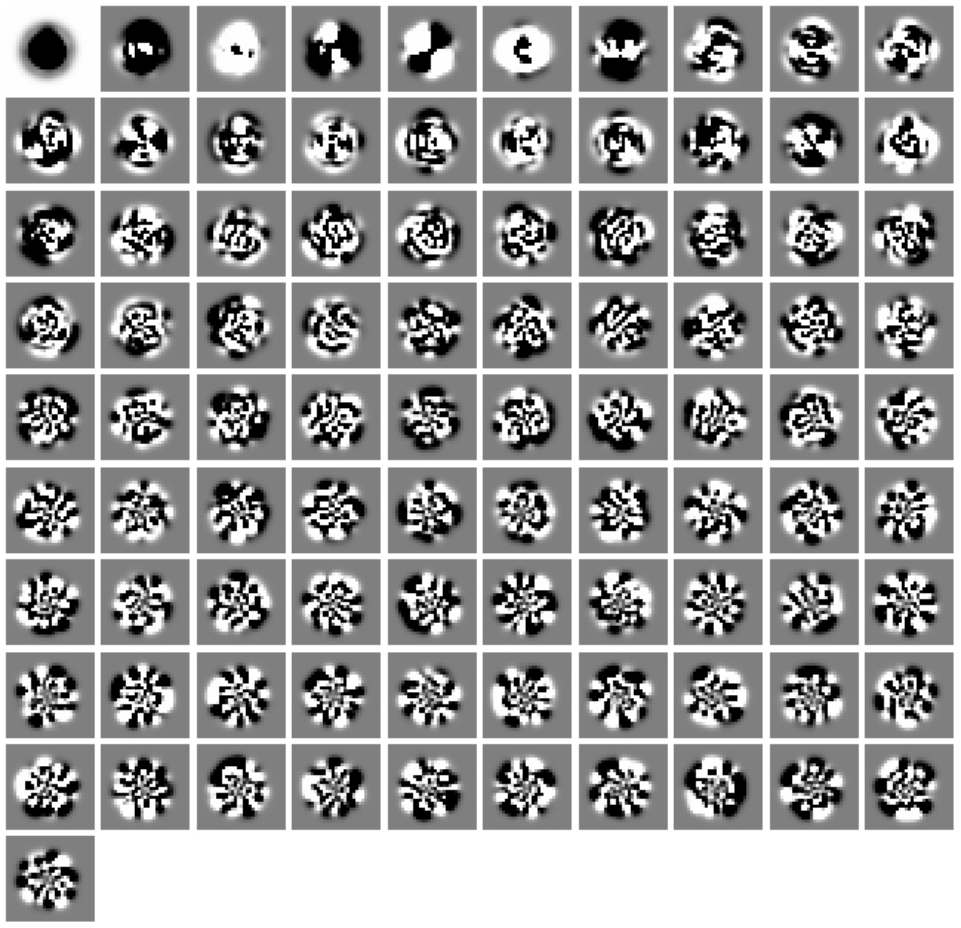
PSF Interpolation by PCA



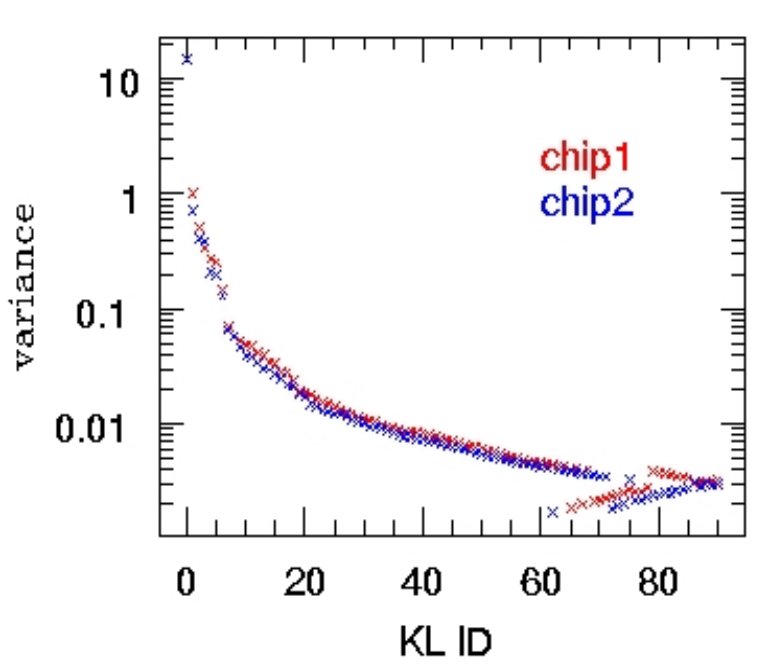
PSF across the HST ACS field

- Start with ACS PSF stellar fields from Schrabback et al. 2006
- Principal Component Analysis from PSF samples across all fields (PCA:Jarvis & Jain 2004)

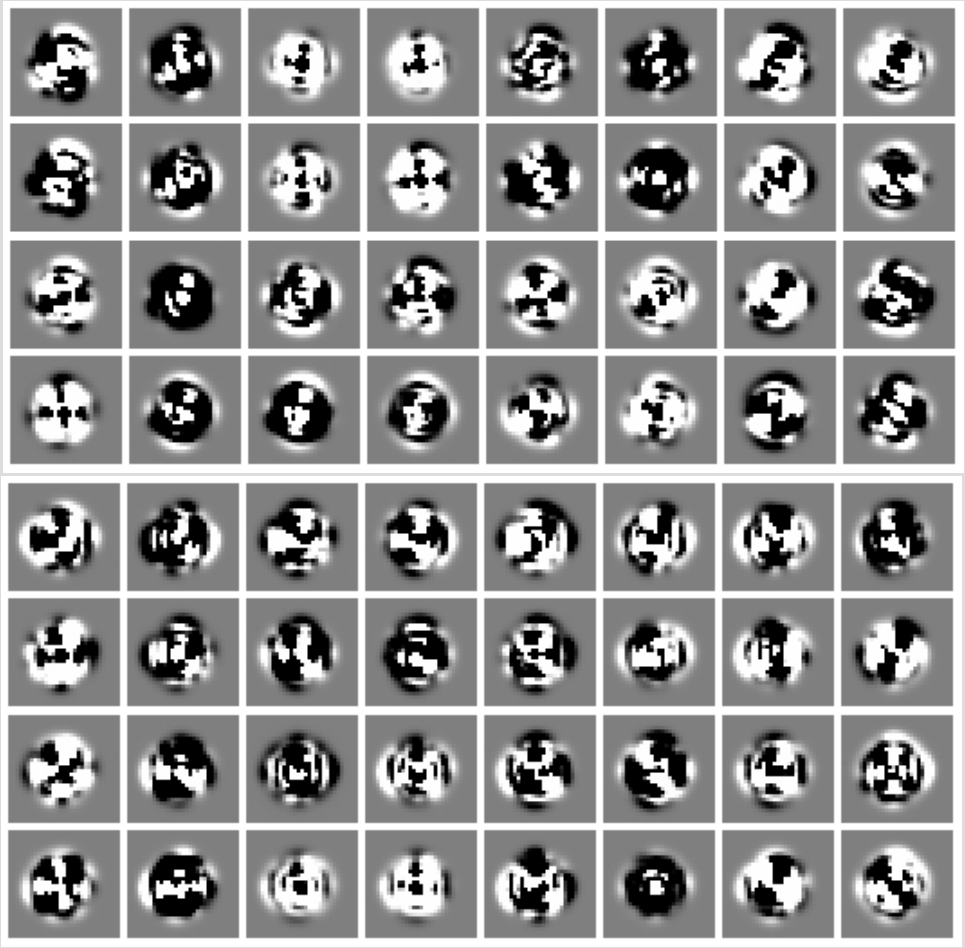
PSF Interpolation by PCA



- Decomposed into Principal Components



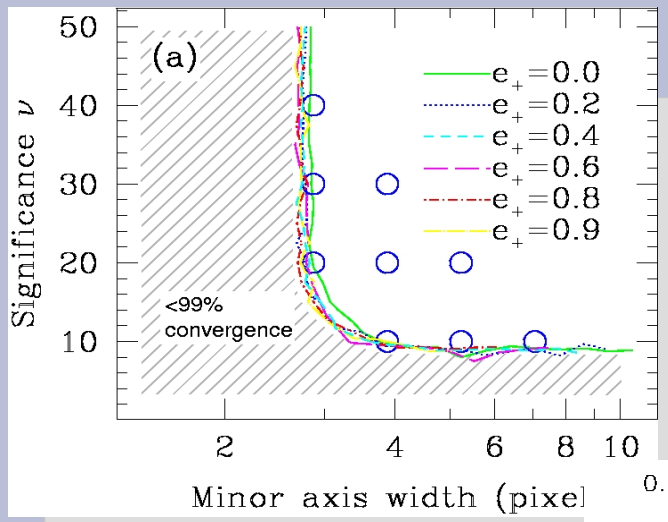
PSF Interpolation by PCA

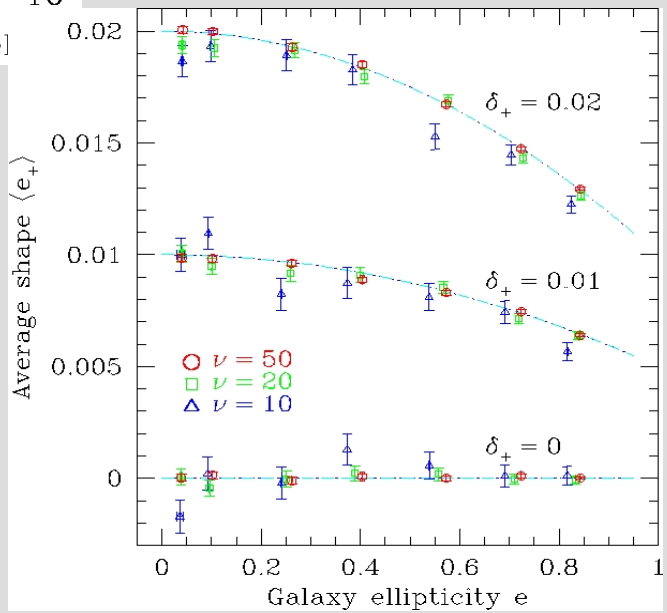


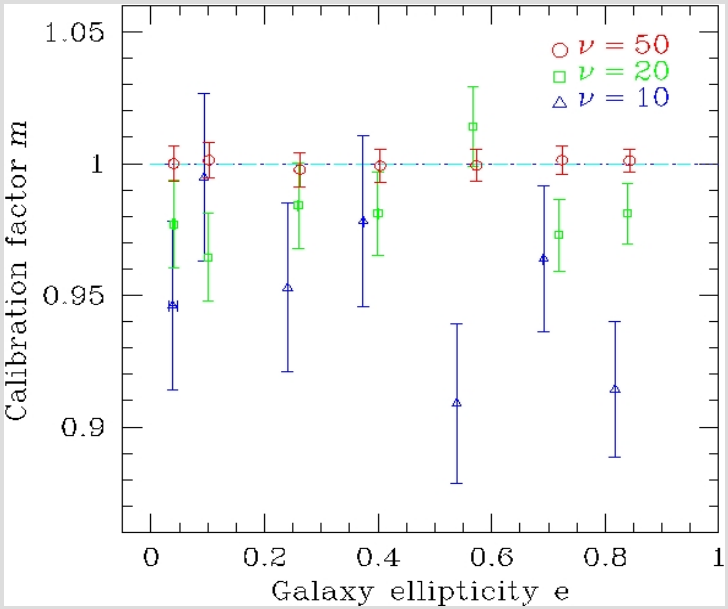
- Take PCA coefficients of each PSF image
- 2D Interpolate over each field

Residual after 2d polynomial fit of PCA components (maximum amplitude is ~1% of original flux)

Testing Shear Recovery







Why?

- Weak Lensing is great for probing matter distribution
  - sensitive only to gravitational deflection of light
- However, it is prone to many systematic errors
  - need a background image of galaxies
    - galaxies have random shapes, images are dim
  - shapes of galaxies used as estimator of the shear field
    - telescope PSF modifies galaxy images
  - other effects
    - sampling, crowding; intrinsic alignments, redshift distribution
- To understand WL as a quantitative probe, its accuracy of estimating gravitational shear must be quantified