Shear TEsting Programme: Weak lensing analysis of Shapelet simulations

1 Method

In the second STEP project we have analysed two sets of images produced using Shapelet morphologies [Massey et al., 2004]. For each of the 6 different PSFs we have used an 'original' set of 64 images and a second 'rotated' set where the galaxies in the original images are rotated by 90 degrees before gravitational shear and PSF smearing is applied.
For a perfect ellipse with axial ratio β at position angle θ, measured counter-clockwise from the x axis, we can define the following ellipticity parameters [Bonnet and Mellier, 1995]

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1-\beta}{1+\beta} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}, \tag{1}$$

and the complex ellipticity $e = e_1 + ie_2$. The intrinsic source ellipticity e_A of a galaxy in original image A is thus related to the intrinsic source ellipticity e_B of the same galaxy in the counterpart rotated image B,

$$e_A = -e_B\,. \tag{2}$$

In the absence of PSF smearing and shear measurement errors, the observed ellipticity $e^{\rm obs}$ is related to the intrinsic source ellipticity by

$$e^{\rm obs} = \frac{e+g}{1+g^*e}, \tag{3}$$

[Seitz and Schneider, 1997] where g is the complex reduced shear that is applied to each image. As in the first STEP project $\kappa = 0$ in all simulations, and hence $g = \gamma$.

In the first STEP project we measured shear γ from averaging over the measured ellipticities of many galaxies $\gamma = < e^{\rm obs} >$. Our results were however subject to an intrinsic shot noise error SN

$$\mathrm{SN} \simeq \langle e \rangle = 0 \pm \frac{0.1}{\sqrt{N}} \tag{4}$$

from the N observed galaxies. Note that in the STEP2 simulations $\sigma_e = \sqrt{\langle e_i^2 \rangle} \sim 0.1$.
In this STEP project we therefore use combined ellipticity measurements from the original and rotated images to determine a measure of the shear γ where the shot noise error is significantly reduced. This works as follows;

$$\begin{aligned} e_A^{\rm obs} + e_B^{\rm obs} &= \frac{e_A+\gamma}{1+\gamma^* e_A} + \frac{e_B+\gamma}{1+\gamma^* e_B} \\ &= 2\left(\frac{\gamma - \gamma^* e_A^2}{1-(\gamma^* e_A)^2}\right) \end{aligned} \tag{5}$$

where we have used equation 2. When averaging over N galaxy pairs we find

$$\gamma = < e_A^{\rm obs} + e_B^{\rm obs} > /2 \tag{6}$$

with a shot noise error SN that is now significantly reduced to

$$\mathrm{SN} \simeq \gamma\langle e^2\rangle = 0 \pm \gamma\frac{0.05}{\sqrt{2N}} \tag{7}$$

where, in the STEP2 simulations $\sqrt{\langle e^4\rangle} \sim 0.05$.
We first match galaxies in each pair of simulations images A and B producing a catalogue of N galaxy pairs. In the cases of methods that use weights w_i for each galaxy i, we determine normalised weights w_i' where

$$w_i' = \frac{N w_i}{\sum_{j=1}^{N} w_j} \tag{8}$$

and then calculate the following three shear estimates;

$$\begin{aligned} \gamma_A &= \frac{1}{N}\sum(w_A' e_A^{\mathrm{obs}}) \\ \gamma_B &= \frac{1}{N}\sum(w_B' e_B^{\mathrm{obs}}) \\ \gamma &= \frac{1}{2N}\sum(w_A' e_A^{\mathrm{obs}} + w_B' e_B^{\mathrm{obs}}) \end{aligned} \tag{9}$$

Errors on the shear measurements are estimated using a bootstrap technique.

For each author and PSF type, following the first STEP project, we determine, from the range of applied sheared images, the best-fit parameters to

$$\begin{aligned} \gamma_1 - \gamma_1^{\mathrm{true}} &= m_1\gamma_1^{\mathrm{true}} + c_1 \\ \gamma_2 - \gamma_2^{\mathrm{true}} &= m_2\gamma_2^{\mathrm{true}} + c_2 \end{aligned} \tag{10}$$

where γ_i^{true} is the external shear applied to each image. We find that $m \simeq (m_A + m_B)/2$ and $c \simeq c_A - c_B$, with errors on the calibration error m and PSF error c from the combined analysis significantly reduced. Typically $m_A \simeq m_B$ and $c_A \simeq -c_B$ which we would expect if the shear measurement errors were stable to changes in image rotation. This is however not always the case which needs to be investigated further.

References

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C. Seitz and P. Schneider. Steps towards nonlinear cluster inversion through gravitational distortions. III. Including a redshift distribution of the sources. *A&A*, 318:687–699, February 1997.