Numerical simulation of a new tomographic reconstruction method for wide FoR MOAO.

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Introduction

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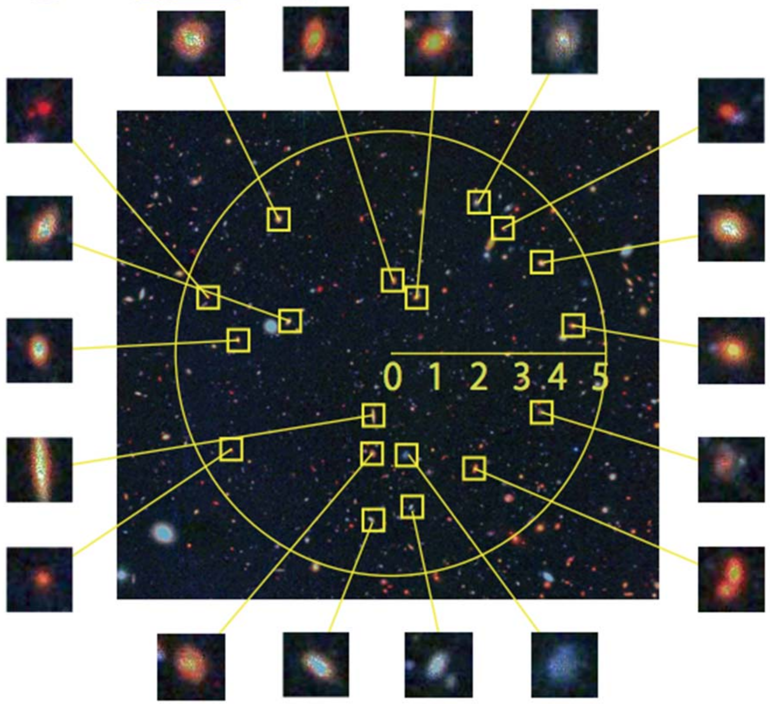
Introduction : Wide-field Multi Object Adaptive Optics

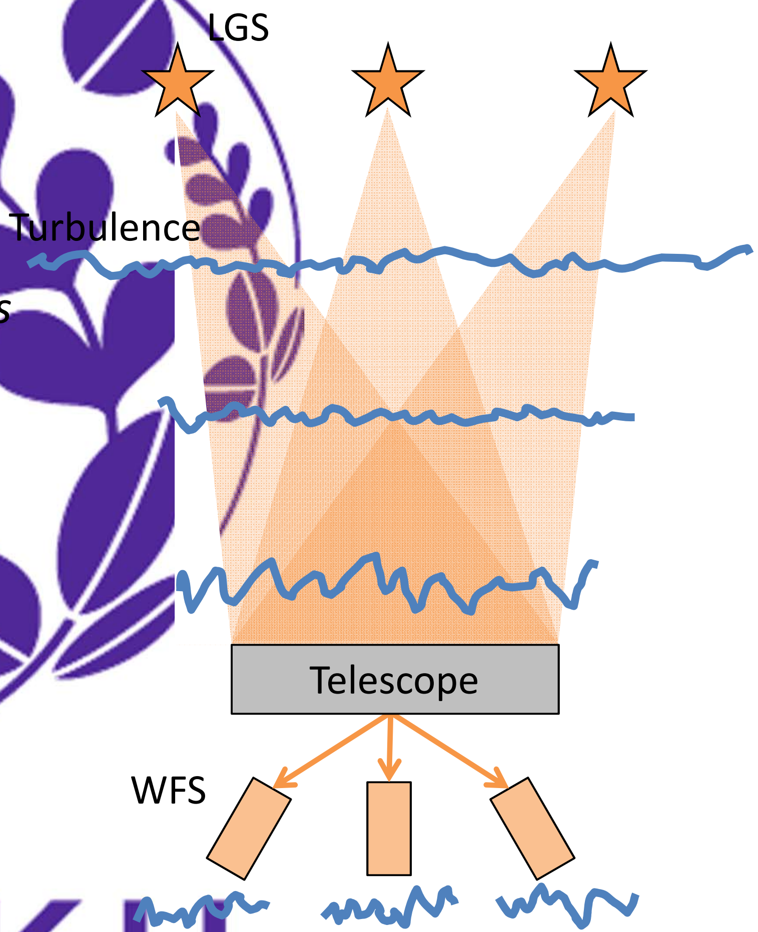
- *MOAO is a system which realize high spatial-resolution observations of multiple objects in a Field of Regard (FoR).*
- *MOAO will be a powerful tool for statistical studies of high-redshift galaxies.*
- *Extreme large telescopes can reveal internal structure of relatively bright galaxies which redshifts are larger than 4.*
- *Since these objects have a small number density, the FoR as wide as 10'φ is required to detect multiple high-redshift galaxies in the FoR.*
- *We are trying to enlarge FoR of MOAO system with new tomographic reconstruction method without increasing the number of LGS.*



Introduction : Limitation of AO tomography

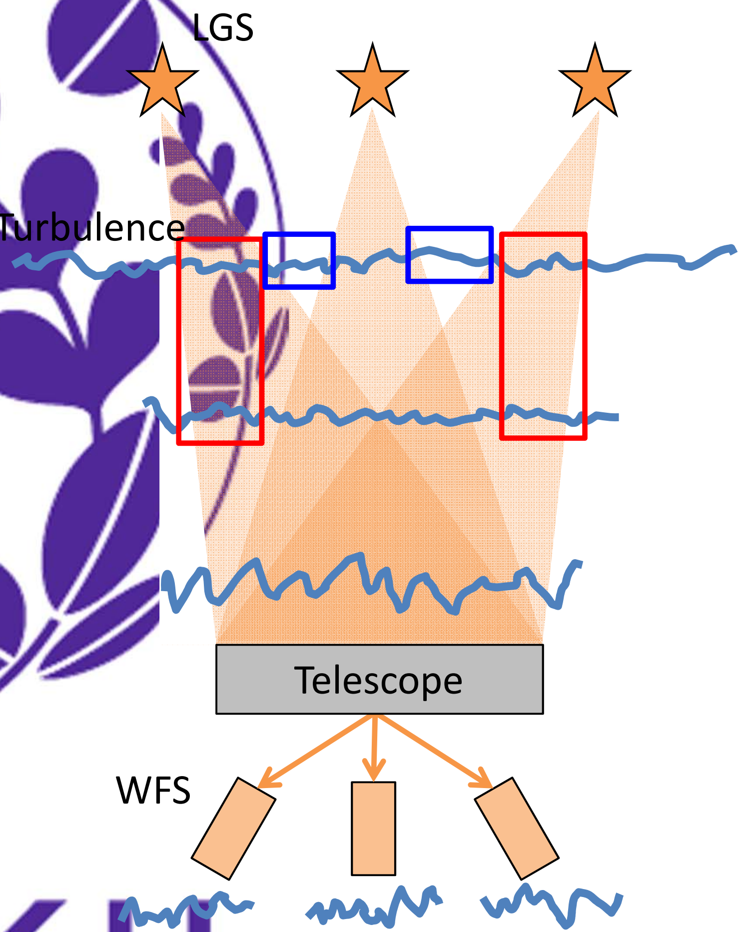
- *Enlarging the FoR is possible by increasing the size of LGS configuration.*

- *Larger LGS configuration make uncertainty of tomographic reconstruction larger, because we can't get sufficient information of atmospheric turbulences from limited number of guide stars.*



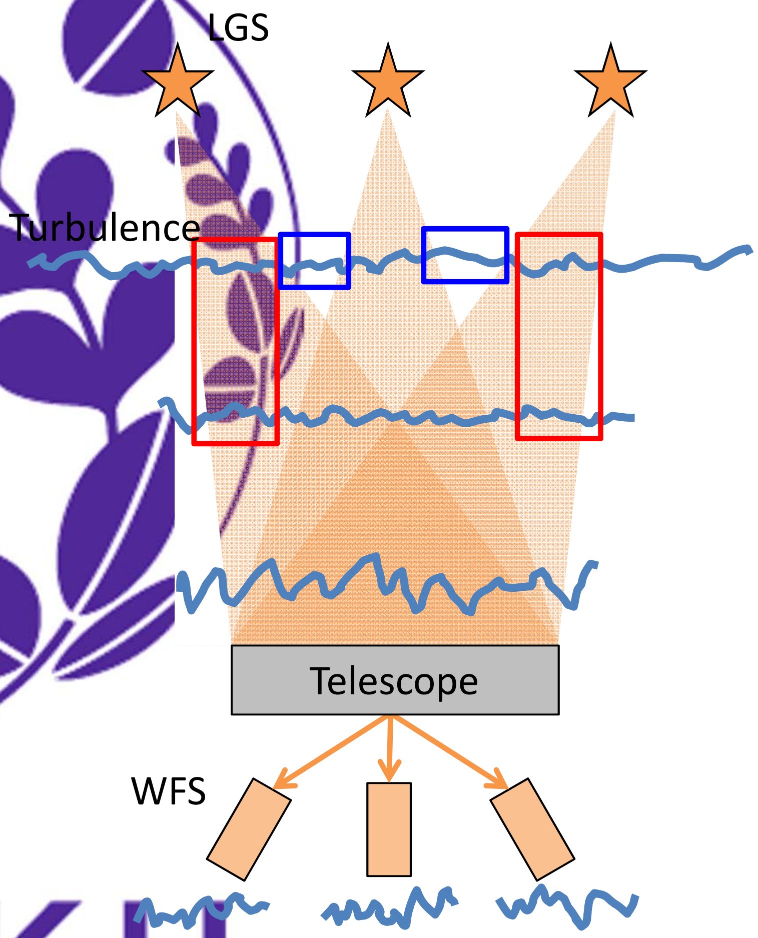
Introduction : Limitation of AO tomography

- Enlarging the FoR is possible by increasing the size of LGS configuration.
- Larger LGS configuration make uncertainty of tomographic reconstruction larger, because we can't get sufficient information of atmospheric turbulences from limited number of guide stars.
- In the case of the right figure, some regions are un-sensed or less-sensed, because they are covered with no LGS or only one LGS.
- These regions cause a significant tomographic error even if using noise-free WFSs and perfect knowledge of the C_n^2 profile.
- This error limits the FoR since these un-sensed and less-sensed regions increase as LGSs separation increases.



Introduction : Idea of a new tomography method

- *In order to overcome this limitation, my idea is using measurements at previous time steps and knowledge of wind speeds and directions at each turbulence layer (wind profile).*
- *It is assumed that an atmospheric layer shifts by wind with keeping their turbulence structure within a short timescale. It's called "frozen flow".*
- *If wind profiles are given, we can fill in the un-sensed and less-sensed regions with measurements at previous time steps and reduce a tomographic error.*



Method

- New reconstruction method
- estimation of wind speeds and directions at each altitude

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Method : New tomographic reconstruction [1/2]

- *The equation between WFS measurements and turbulence phases at current time step (t) is written as (1).*

$$\boldsymbol{s}(t) = \boldsymbol{G}\boldsymbol{\phi}(t) + \boldsymbol{\eta}(t) \quad \cdots\cdot(1)$$

↑ *phase-to-slope influence Matrix*

WFS measurements

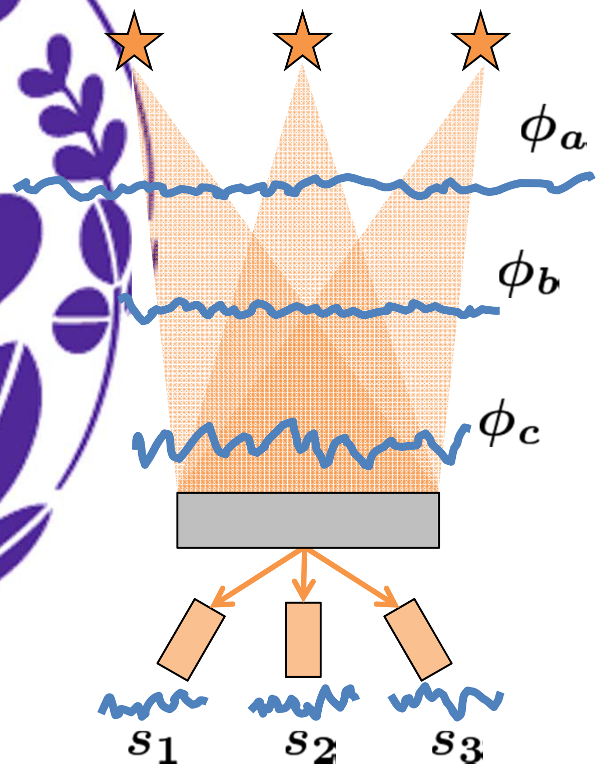
$$\boldsymbol{s}(t) = \begin{bmatrix} \boldsymbol{s_1}(t) \\ \boldsymbol{s_2}(t) \\ \boldsymbol{s_3}(t) \end{bmatrix}$$

turbulence phases

$$\boldsymbol{\phi}(t) = \begin{bmatrix} \boldsymbol{\phi_a}(t) \\ \boldsymbol{\phi_b}(t) \\ \boldsymbol{\phi_c}(t) \end{bmatrix}$$

measurement noises

$$\boldsymbol{\eta}(t) = \begin{bmatrix} \boldsymbol{\eta_1}(t) \\ \boldsymbol{\eta_2}(t) \\ \boldsymbol{\eta_3}(t) \end{bmatrix}$$



- *In my method, we also consider the equation between WFS measurements at previous time step (t-Δt) and current turbulence phases using information of winds.*

$$\boldsymbol{s}(t-\Delta t) = \boldsymbol{G'}\boldsymbol{\phi}(t) + \boldsymbol{\eta}(t-\Delta t) \quad \cdots\cdot(2)$$

↑ *influence Matrix between current turbulence phases and previous measurements*

- *Wind profiles are necessary to calculate G'.*

Method : New tomographic reconstruction [2/2]

- *Considering equation (1) and (2), the equation which should be solved is (3).*

$$\begin{bmatrix} \boldsymbol{s}(t) \\ \boldsymbol{s}(t-\Delta t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{G} \\ \boldsymbol{G}' \end{bmatrix} \boldsymbol{\phi}(t) + \begin{bmatrix} \boldsymbol{\eta}(t) \\ \boldsymbol{\eta}(t-\Delta t) \end{bmatrix} \quad \cdots\cdots(3)$$

- *Derive φ(t) with Minimum-Variance Method (Ellerbroek 2002) (4) and solve the equation (4) with Conjugate Gradient Method (CGM).*

measurements noise → σ; *atmospheric turbulence covariance matrix* → $\boldsymbol{C}_{\phi}^{-1}$

$$\left([\boldsymbol{G}]^T[\boldsymbol{G}] + \sigma \boldsymbol{C}_{\phi}^{-1}\right)\hat{\boldsymbol{\phi}}(t) = [\boldsymbol{G}]^T[\boldsymbol{s}] \quad \cdots\cdots(4)$$

$$\boldsymbol{A} \quad \boldsymbol{x} \quad = \quad \boldsymbol{b}$$

$$[\boldsymbol{s}] = \begin{bmatrix} \boldsymbol{s}(t) \\ \boldsymbol{s}(t-\Delta t) \end{bmatrix}$$

$$[\boldsymbol{G}] = \begin{bmatrix} \boldsymbol{G} \\ \boldsymbol{G}' \end{bmatrix}$$

Method : Wind Estimation [1/3]

- Wind speeds and directions at each altitude are necessary for the new reconstruction method.

- We construct a estimation method of wind profiles which uses the result of a classical tomographic reconstruction.

Method : Wind Estimation [2/3]

- *We extract turbulence structure of each layer from classical tomographic estimation. We consider regions within LGS light paths in order to escape from uncertainty associated with the tomographic reconstruction outside of LGS light path.*
- *P is a matrix which extracts a wavefront within a LGS light path from reconstructed turbulence phases and calculate slope maps from extracted wavefront.*

Method : Wind Estimation [3/3]

- We calculate auto-correlations and temporal-correlations of slope maps and average the correlation maps over a few seconds time steps.

Auto Correlation

$$\boldsymbol{A}_{\mathbf{1}\boldsymbol{a}} = \frac{1}{N_{\mathrm{frame}}} \sum_{t}^{N_{\mathrm{frame}}} \overline{\mathcal{F}[\hat{\boldsymbol{s}}_{\mathbf{1}\boldsymbol{a}}(t)]}\, \mathcal{F}[\hat{\boldsymbol{s}}_{\mathbf{1}\boldsymbol{a}}(t)]$$

Temporal Correlation

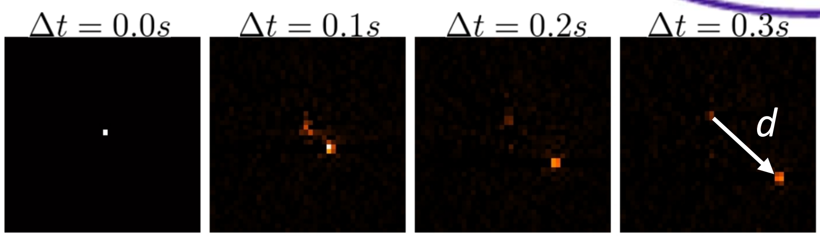
$$\boldsymbol{T}_{\mathbf{1}\boldsymbol{a}}(\Delta t) = \frac{1}{N_{\mathrm{valid}}} \sum_{t}^{N_{\mathrm{valid}}} \overline{\mathcal{F}[\hat{\boldsymbol{s}}_{\mathbf{1}\boldsymbol{a}}(t+\Delta t)]}\, \mathcal{F}[\hat{\boldsymbol{s}}_{\mathbf{1}\boldsymbol{a}}(t)]$$

- We deconvolve the temporal-correlation with the auto-correlation.

2-Dimensional Deconvolution

$$\boldsymbol{D}_{\mathbf{1}\boldsymbol{a}}(\Delta t) = \mathcal{F}^{-1}\left[\frac{\mathcal{F}[\boldsymbol{T}_{\mathbf{1}\boldsymbol{a}}(\Delta t)]}{\mathcal{F}[\boldsymbol{A}_{\mathbf{1}\boldsymbol{a}}]}\right]$$

- We calculate the peak position on the deconvolved maps with a center of gravity, and estimate wind speeds and directions from shifts of correlation peaks and Δt.



temporal-correlation map after deconvolution

$$\frac{d[m]}{\Delta t[s]} = v[m/s]$$

Simulation

Simulation Setup : Basic Parameters

- *30m circular aperture*
- *r_0 = 0.156m, L_0 = 30m, 7 layers (Fig.1 red)*
- *wind model from Hardy 1998. (Fig.1 blue)*
- *8 Na LGS, 2 configuration*
  - *5'φ configuration (TMT, IRMOS) (Fig.2 red)*
  - *10'φ configuration (Fig.2 blue)*
- *Shack Hartmann WFS*
  - *60×60 subapertures, subaperture size = 0.5m*
  - *pixel scale 0.75", 11×11 pixels / subaperture*
  - *700 counts / subaperture*
  - *read out noise = 5 counts / pixel / read*
- *DM segment size = 0.5m*

Fig.1 Turbulence and wind model

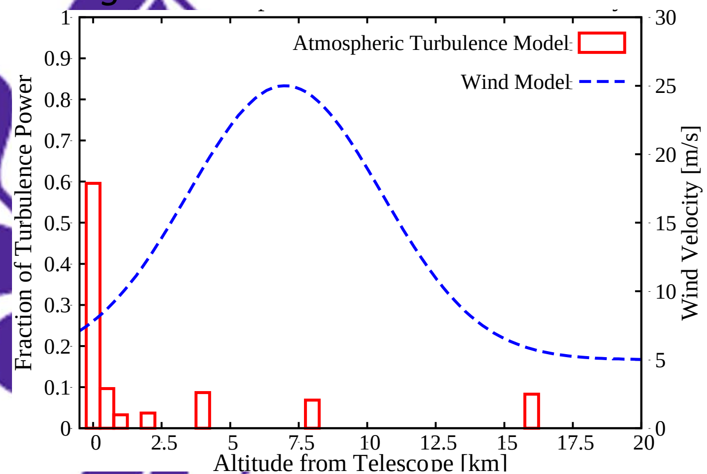
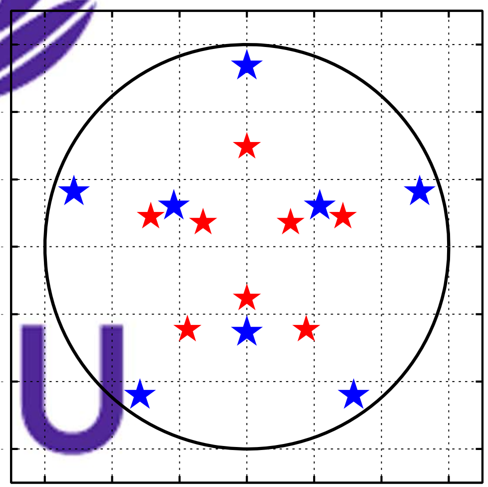


Fig.2 Guide stars configuration



Simulation Setup : Assumptions

There are some assumptions in my simulation.

- *Turbulence profiles, heights and C_n^2 of turbulence layers, are given.*
- *Wind speeds and directions don't change with time.*
- *100% frozen flow turbulences.*
- *There is no time delay due to calculation time.*

I will mention the effects of these assumptions later.

Results

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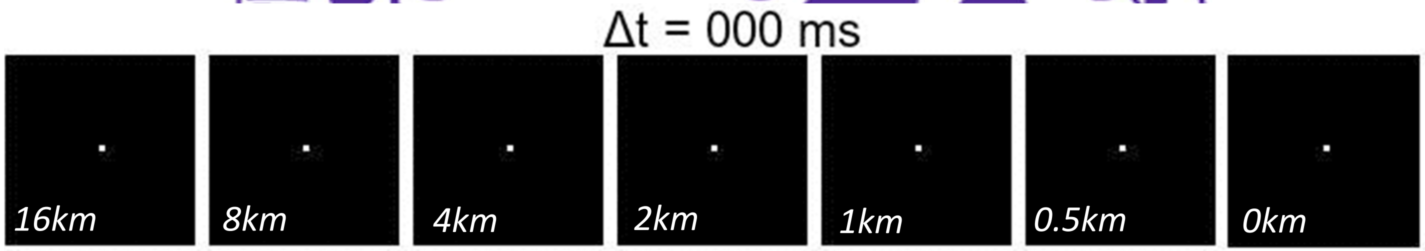
Simulation Results : Wind profile estimation

- *Wind estimation is performed using 200 frames (100Hz, 2s) simulated WFS's data.*

- *Correlation map movie at each altitude after a deconvolution.*

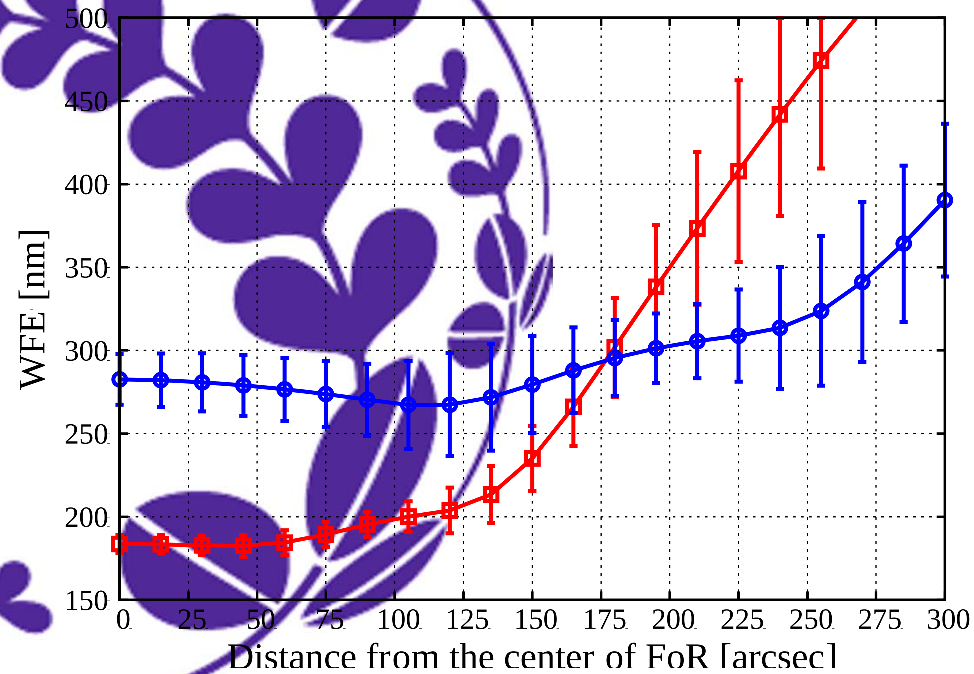


- *The error of the estimated shift of turbulence layers for 0.1s due to wind estimation errors are less than 0.25m at all altitude.*
  (※I note that this is ideal case that heights of atmospheric layers are given.)

Height (km)	*0*	*0.5*	*1*	*2*	*4*	*8*	*16*
C_n^2 fraction (%)	*59.6*	*9.63*	*3.25*	*3.72*	*8.69*	*6.84*	*8.26*
Shifts of turbulence layers for 0.1s due to given wind (m)	*0.78*	*0.87*	*0.97*	*1.24*	*1.9*	*2.42*	*0.58*
Error of the estimated shifts of turbulence layers for 0.1s due to wind estimate error (m)	*0.02*	*0.02*	*0.03*	*0.02*	*0.09*	*0.23*	*0.13*

Simulation Results : Wavefront Error [1/3]

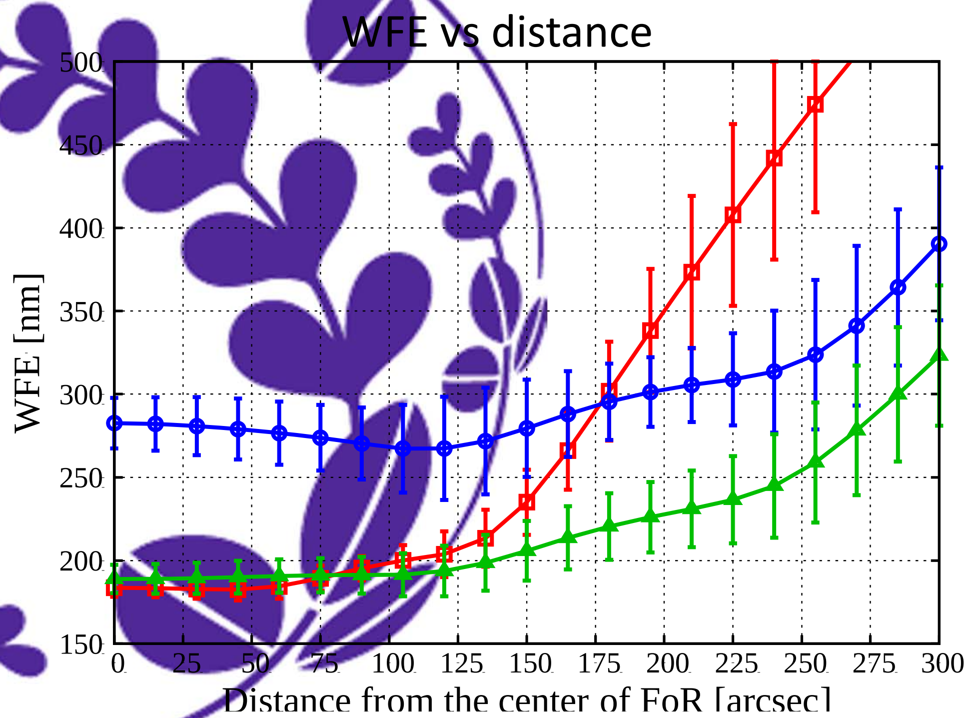
- Right figure shows residual WFE profiles in a FoR after MOAO correction.
- The red and blue line in the right figure indicate the result of a classical reconstruction method for 5'φ configuration and 10'φ configuration.
- As the LGS configuration extends, the WFE at outside of the FoR get smaller, but inside WFE becomes larger because un- and less-sensed regions increase.



Red : Classical method for 5'φ LGS configuration
Blue : Classical method for 10'φ configuration

Simulation Results : Wavefront Error [2/3]

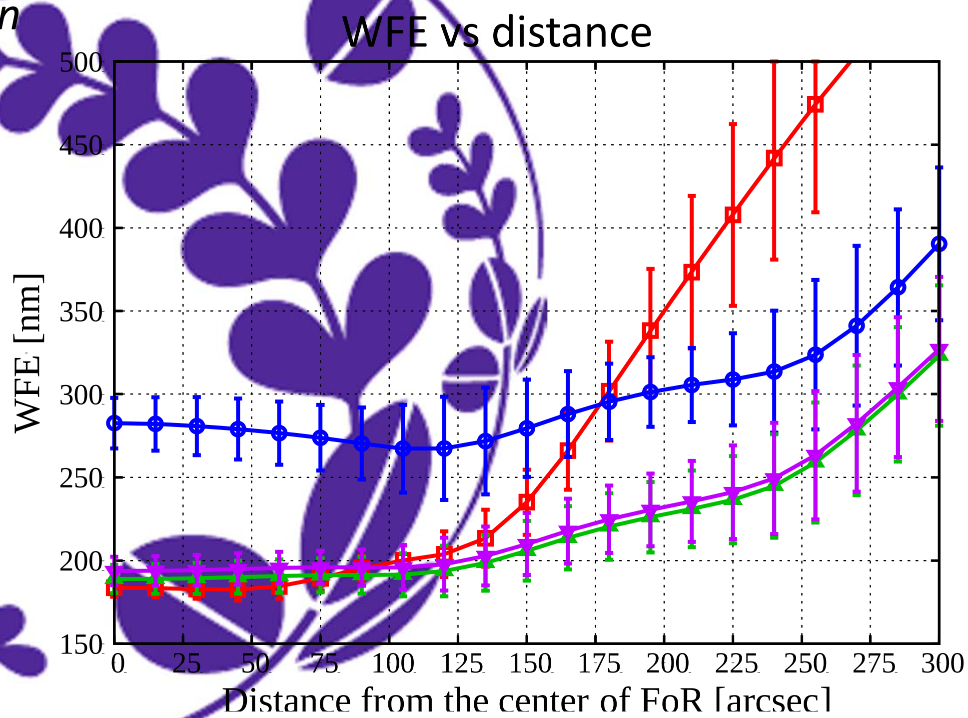
- *This green line shows the new reconstruction result for 10'φ configurations with perfect knowledge of wind profiles.*
- *We use measurements 0.1s before for the new tomographic reconstruction.*
- *Outside WFE decrease with keeping inside WFE since un- and less- sensed regions are filled with previous measurements.*



Red : Classical method for 5'φ LGS configuration
Blue : Classical method for 10'φ configuration
Green : New method with perfect knowledge of wind profiles for 10'φ configuration

Simulation Results : Wavefront Error [3/3]

- *Magenta line shows the new reconstruction result for 10'φ configurations with estimated wind profiles.*
- *Even if I use estimated wind profiles, the WFE profile is same as the result with perfect information of wind profiles.*
- *In this case, the effect of wind estimation error for the reconstruction result is small.*

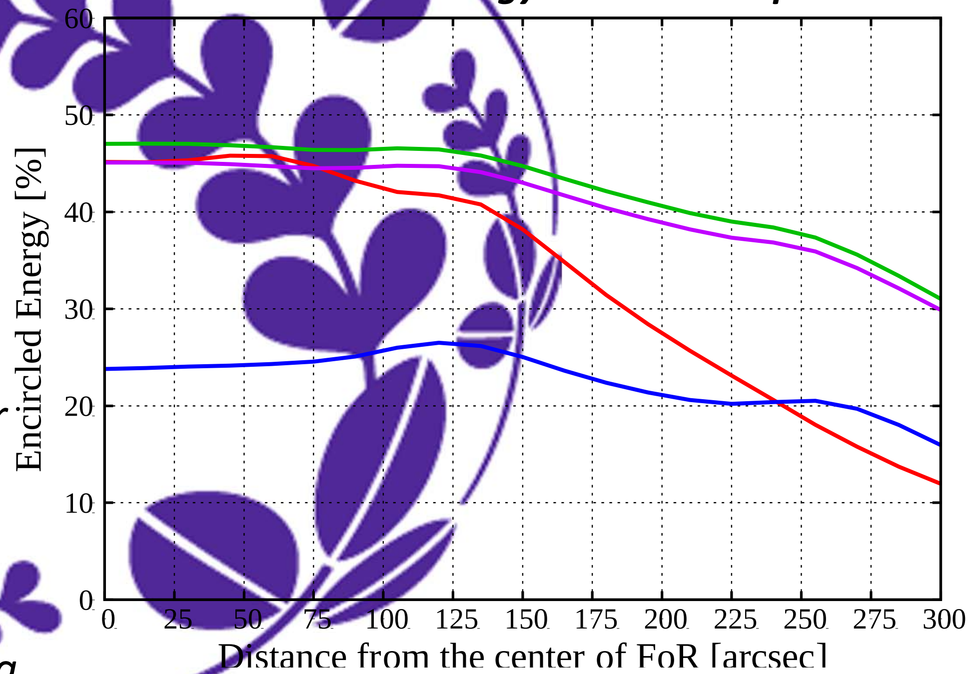


Red : Classical method for 5'φ LGS configuration
Blue : Classical method for 10'φ configuration
Green : New method with perfect knowledge of wind profiles for 10'φ configuration
Magenta : New method with estimated wind information for 10'φ configuration

Simulation Results : Encircled Energy in a 50mas aperture

- The right figure shows encircled energy profiles in a 50mas aperture at J-band.
- The profiles are same as the WFE profiles in the previous slide.
- At outside of a FoR, encircled energies with the new method get 2-3 times larger than the encircled energy with classical method.
- From this result, I think that I can extend a FoR of MOAO with the new reconstruction method.

J-band Encircled Energy in a 50mas aperture



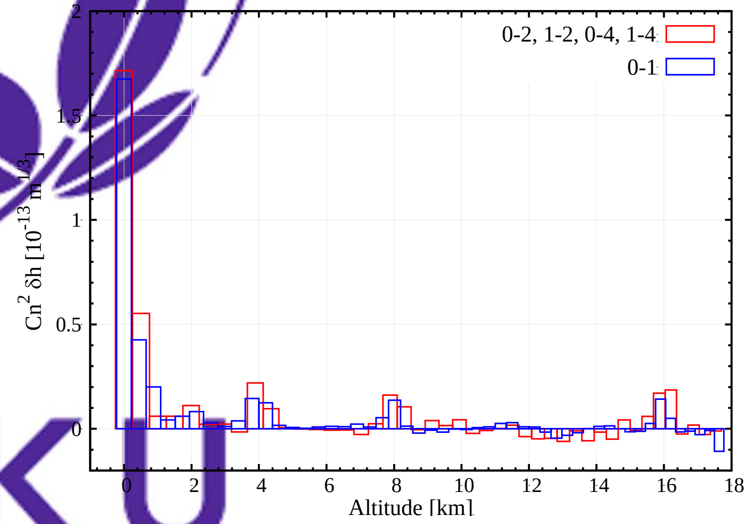
Red : Classical method for 5'φ LGS configuration
Blue : Classical method for 10'φ configuration
Green : New method with perfect knowledge of wind profiles for 10'φ configuration
Magenta : New method with estimated wind information for 10'φ configuration

Discussion

Discussion : Effect of Estimation Error of Turbulence profiles

- It is assumed that turbulence profiles, atmospheric layer heights and C_n^2, are given in this simulation.
- In fact, we should estimate turbulence profiles before tomographic reconstructions (for example, SLODAR).
- We are trying to simulate estimation of turbulence profiles with SLODAR.
- The error of turbulence profile estimation may affect wind estimations and tomographic reconstructions.
- We will evaluate the new reconstruction method including the estimation error of turbulence profiles.

SLODAR simulation



Discussion : Effect of Wind Estimation Error

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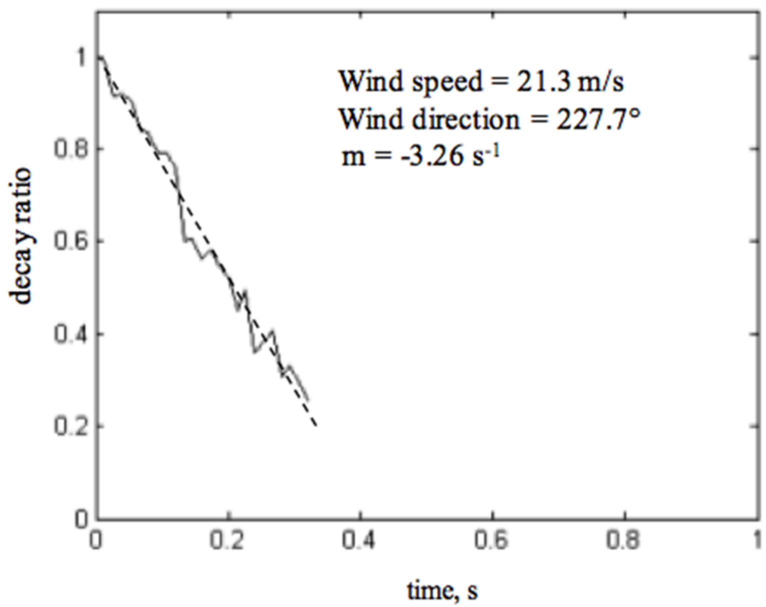
Shifts of turbulence layers for 0.1s due to given wind (m)	*0.78*	*0.87*	*0.97*	*1.24*	*1.9*	*2.42*	*0.58*
Error of the estimated shifts of turbulence layers for 0.1s due to wind estimate error (m)	*0.02*	*0.02*	*0.03*	*0.02*	*0.09*	*0.23*	*0.13*

- *In this simulation, the errors of estimated shift of turbulence layers for 0.1s due to wind estimation error are smaller than 0.25m which is a half of DM segment size.*

- *Since the WFE caused by this error is correspond to a fitting error which is smaller than tomographic error, the wind estimation errors don't affect the result of reconstruction.*

Discussion : Effects of time variation of turbulence

- Guesalaga et al. 2014 shows the peak of cross-correlation of WFS measurements become weaker up to 70-80% for 0.1s. This means the turbulence structure vary with time.
- This decay of correlation peak makes wind estimation error larger.
- Time variation of turbulence structure breaks frozen flow assumption and make residual WFE of the new reconstruction method larger.



Guesalaga et al. 2014

Discussion : Effects of time variation of wind profiles

- *Poyneer et al. 2009 shows that the RMS of change of wind speeds for 10s is around 0.5m/s at Maunakea.*

- *The error of the estimated shifts of turbulence layers due to the time variation is much smaller than 0.1m, and this error doesn't affect the result of this simulation.*

Summary & Future Works

Summary :

- *I construct a new tomographic reconstruction method which use wind information and measurements at previous time steps for extending a FoR of MOAO.*
- *I also construct the method of wind estimation.*
- *I performed numerical simulation of wind estimations and the new reconstruction method.*
- *Encircled energies at 5’ from the center of the FoR become more than 30% with the new method when turbulence profiles are perfectly known.*
- *Increasing a FoR of MOAO is possible by the new method.*

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Future works :

Simulation

- *I will also simulate a estimation of turbulence profiles and evaluate the new reconstruction method including the estimation error of turbulence profiles.*

On-sky test

- *We have the first engineering observation with RAVEN/Subaru in May 2014.*
- *Using the on-sky WFSs data from RAVEN, I will try to*
  - *estimate turbulence profiles,*
  - *estimate wind profiles,*
  - *investigate time variation of turbulence profiles and wind profiles,*
  - *evaluate the effectiveness of new tomographic method.*

- *Apply the new method for real-time system of RAVEN.*

Thank you very much !!

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