Advanced Cosmology: Summer 2007

Section A Answer two Questions

A.1.

(a) Consider the universe at a GUT-scale temperature of $kT = 10^{15}$ GeV. Assuming a standard radiation-dominated model, estimate the comoving size of the particle horizon at the GUT era (currently, $kT = 10^{-3.6}$ eV). **[5]**

(b) Write down the integral for the current size of the particle horizon. If the universe begins with a phase of inflation, during which the Hubble parameter, H, is constant, how long must inflation continue in order to reconcile the GUT-scale horizon with the present value of $\sim c/H_0$? **[5]**

(c) Write down the equation of motion obeyed by a homogeneous scalar field, ϕ, commenting on the physical significance of the terms that appear. Explain how the equation can be solved in the slow-roll approximation. **[5]**

(d) For a potential $V(\phi) \propto \phi^\alpha$, show that the solution to the slow-roll equation is

$$\phi/\phi_{\rm initial} = (1 - t/t_{\rm final})^{1/\beta} ,$$

where $\beta = 2 - \alpha/2$. Explain why this equation does not hold near $\phi = 0$. **[10]**

A.2. The equation describing the growth of density fluctuations in a matter-dominated expanding universe is

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \delta(4\pi G\bar{\rho}_m - c_s^2 k^2/a^2),$$

where δ is the fractional density fluctuation, $\rho = \bar{\rho}_m(1+\delta)$, $\bar{\rho}_m$ is the mean matter density, c_s is the speed of sound, and k is comoving wavenumber.

(a) Show that, for a static model, density fluctuations grow exponentially as long as the wavelength is sufficiently large, and explain physically why this is so. **[5]**

(b) Derive the solutions to the perturbation equation for a universe of critical density, in the limit of infinitely long wavelength. **[5]**

(c) At time $t = t_c$, a homogeneous critical-density universe is given a velocity perturbation, such that $\dot{\delta} = A$. Evaluate the density perturbation as a function of time following this event. **[7]**

(d) If the universe contains a homogeneous component in addition to matter that can clump, the perturbation equation still applies – but $\bar{\rho}_m$ does not include the homogeneous component. Consider a flat universe that contains a mixture of hot and cold dark matter: for sufficiently small wavelengths, the hot component can be assumed to be uniform, with density parameter Ω_h. Show that fluctuations in the cold component grow as $\delta \propto t^\alpha$, where $\alpha = (\sqrt{25 - 24\Omega_h} - 1)/6$. **[8]**

A.3.

(a) Write brief notes on the mechanisms that contribute to the anisotropies of the cosmic microwave background. Include a sketch of the temperature power spectrum and indicate the angular scales where each effect is important. **[5]**

(b) The properties of the spatially-varying temperature fluctuations must be determined by the physical densities of dark matter and baryons, and thus depend on the parameter combinations $\omega_{dm} = \Omega_{dm}h^2$ and $\omega_b = \Omega_b h^2$. Write down the integral for the comoving distance to the last scattering surface, and hence explain why the CMB alone cannot be used to measure all cosmological parameters. **[5]**

(c) If the universe is reionized by UV radiation from galaxies and quasars, photons from the CMB can be scattered by foreground matter. If the transition from neutral to ionized is assumed to happen suddenly at redshift z_c ($\gg 1$), calculate the optical depth to last scattering as a function of z_c (the Thomson cross-section is $\sigma_t = 6.652 \times 10^{-29}$ m^2 and the proton mass is $m_p = 1.673 \times 10^{-27}$ kg). **[10]**

(d) Explain briefly why the effects of reionization on the CMB power spectrum tend to be degenerate with tilt. **[5]**