

# 11 The puzzle of dark energy

## 11.1 Cosmological effects of the vacuum

One of the most radical conclusions of recent cosmological research has been the necessity for a non-zero vacuum density. This was detected on the assumption that Einstein's **cosmological constant**,  $\Lambda$ , might contribute to the energy budget of the universe. But if this ingredient is a reality, it raises many questions about the physical origin of the vacuum energy; as we will see, a variety of models may lead to something similar in effect to  $\Lambda$ , and the general term **dark energy** is used to describe these.

The properties of dark energy can be probed by the same means that we used to deduce its existence in the first place: via its effect on the expansion history of the universe. The vacuum density is included in the Friedmann equation, independent of the equation of state

$$\dot{R}^2 - \frac{8\pi G}{3} \rho R^2 = -kc^2. \quad (318)$$

At the outset, then we should be very clear that the deduced existence of dark energy depends on the correctness of the Friedmann equation, and this is not guaranteed. Possibly we have the wrong theory of gravity, and we have to replace the Friedmann equation by something else. Alternative models do exist, particularly in the context of extra dimensions, and these must be borne in mind. Nevertheless, as a practical framework, it makes sense to stick with the Friedmann equation and see if we can get consistent results. If this programme fails, we may be led in the direction of more radical change.

To insert vacuum energy into the Friedmann equation, we need the equation of state

$$w \equiv p/\rho c^2 \quad (319)$$

If this is constant, adiabatic expansion of the vacuum gives

$$\frac{8\pi G\rho}{3H_0^2} = \Omega_v a^{-3(w+1)}. \quad (320)$$

More generally, we can allow  $w$  to vary; in this case, we should regard  $-3(w+1)$  as  $d \ln \rho / d \ln a$ , so that

$$\frac{8\pi G\rho}{3H_0^2} = \Omega_v \exp \left( \int -3(w(a) + 1) d \ln a \right). \quad (321)$$

In general, we therefore need

$$H^2(a) = H_0^2 \left[ \Omega_v e^{\int -3(w(a)+1) d \ln a} + \Omega_m a^{-3} + \Omega_r a^{-4} - (\Omega - 1)a^{-2} \right]. \quad (322)$$

Some complete dynamical model is needed to calculate  $w(a)$ . Given the lack of a unique model, a common empirical parameterization is

$$w(a) = w_0 + w_a(1 - a). \quad (323)$$

Frequently it is sufficient to stick with constant  $w$ ; most experiments are sensitive to  $w$  at a particular redshift of order unity, and  $w$  at this redshift can be estimated with little dependence on whether we allow  $dw/dz$  to be non-zero.

If  $w$  is negative at all, this leads to models that become progressively more vacuum-dominated as time goes by. When this process is complete, the scale factor should vary as a power of time. The case  $w < -1$  is particularly interesting, sometimes known as **phantom dark energy**. Here the vacuum energy density will eventually diverge, which has two consequences: this singularity happens in a finite time, rather than asymptotically; as it does so, vacuum repulsion will overcome the normal electromagnetic binding force of matter, so that all objects will be torn apart in the **big rip**. Integrating the Friedmann equation forward, ignoring the current matter density, the time to this event is

$$t_{\text{rip}} - t_0 \simeq \frac{2}{3} H_0^{-1} |1 + w|^{-1} (1 - \Omega_m)^{-1/2}. \quad (324)$$

**OBSERVABLE EFFECTS OF THE VACUUM** The comoving distance-redshift relation is one of the chief diagnostics of  $w$ . The general definition is

$$D \equiv R_0 r = \int_0^z \frac{c}{H(z)} dz. \quad (325)$$

Perturbing this about a fiducial  $\Omega_m = 0.25$   $w = -1$  model shows a **sensitivity multiplier** of about 5 – i.e. a measurement of  $w$  to 10% requires  $D$  to 2%. Also, there is a near-perfect degeneracy with  $\Omega_m$ , so this parameter must be known very well before the effect of varying  $w$  becomes detectable.

The other main diagnostic of  $w$  is its effect on the growth of density perturbations. These are also sensitive to the vacuum, as may be seen from the growth equation:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_0\delta. \quad (326)$$

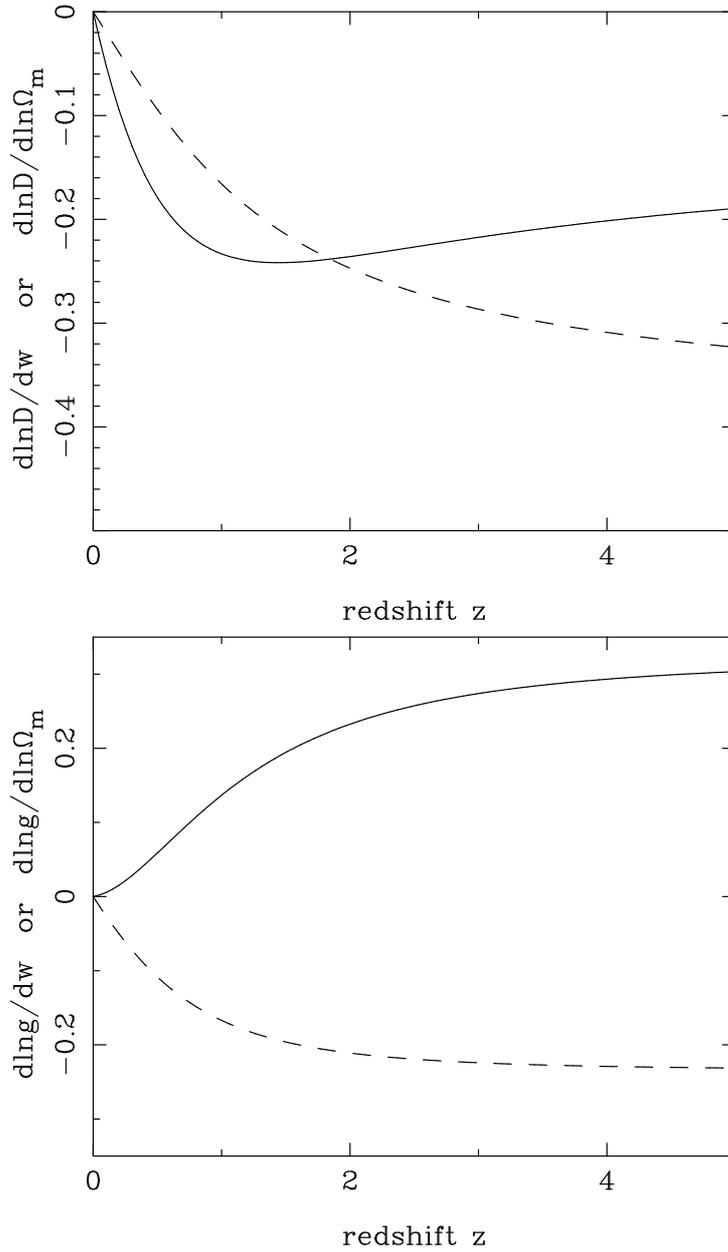
The vacuum energy manifests itself in the factor of  $H$  in the ‘Hubble drag’ term  $2(\dot{a}/a)\dot{\delta}$ . For flat models with  $w = -1$ , we have seen that the growing mode for density perturbations is approximately as  $g(a) \propto a\Omega(a)^{0.23}$ . If  $w$  is made more negative, this makes the growth law closer to the Einstein–de Sitter  $g(a) \propto a$  (for very large negative  $w$ , the vacuum was unimportant until very recently). Therefore, increasing  $w$  (making it less negative) has an effect in the same sense as *decreasing*  $\Omega_m$ . As shown in figure 28, the degeneracy between variations in  $\Omega_m$  and  $w$  thus has the opposite sign to the degeneracy in  $D(z)$ . Ideally, one would therefore try to observe both effects.

## 11.2 Observing the properties of dark energy

What are the best ways to measure  $w$ ? We have seen that the two main signatures are alterations to the distance-redshift relation and the perturbation growth rate. It is possible to use both of these effects in the framework we have been discussing: observing the perturbed universe in both the CMB and large-scale structure.

In the CMB, the main observable is the angle subtended by the horizon at last scattering

$$\theta_{\text{H}} = D(z_{\text{LS}})/D(z = 0). \quad (327)$$



**Figure 28.** Perturbation around  $\Omega_m = 0.25$  of distance-redshift and growth-redshift relations. Solid line shows the effect of increase in  $w$ ; dashed line the effect of increase in  $\Omega_m$

This has the approximate scaling with cosmological parameters (for a flat universe)

$$\theta_H \propto (\Omega_m h^{3.3})^{0.15} \Omega_m^{\alpha-0.4}; \quad \alpha(w) = -2w/(1 - 3.8w). \quad (328)$$

The latter term comes from a convenient approximation for the current horizon size:

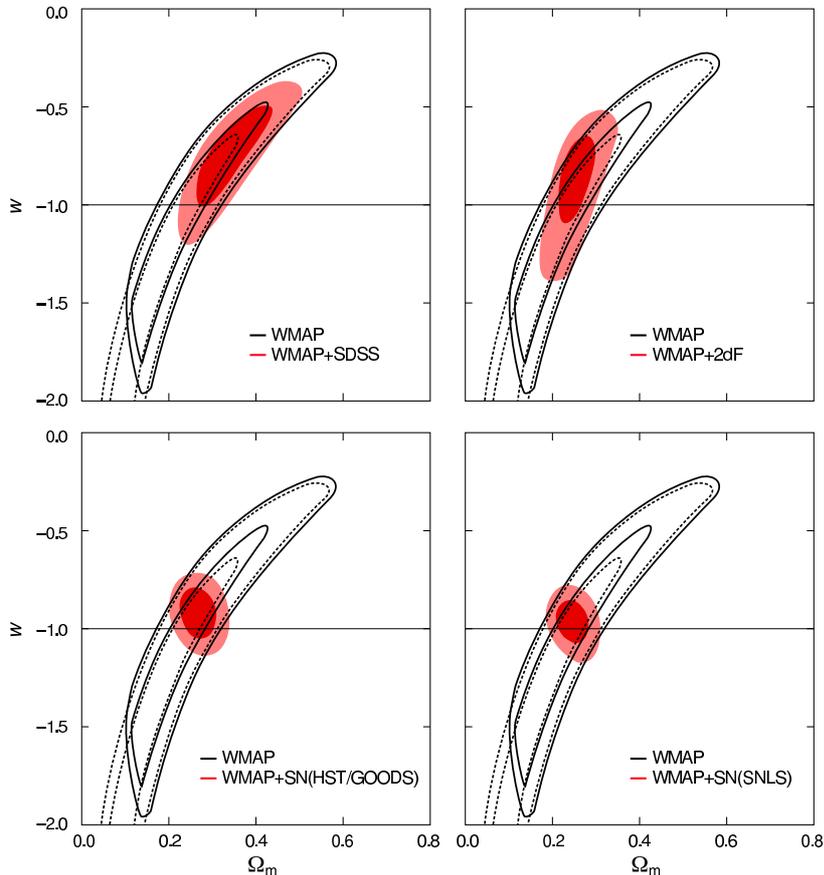
$$D_0 = 2 \frac{c}{H_0} \Omega_m^{-\alpha(w)}. \quad (329)$$

At first sight, this looks bad: the single observable of the horizon angle depends on three parameters (four, if we permit curvature). Thus, even in a flat model, we can only pin down  $w$  if we know both  $\Omega_m$  and  $h$ .

However, if we have more detail on the CMB than just the main peak location, then we have seen that the  $\Omega_m - h$  degeneracy is weakly broken, and that this situation improves with information from large-scale structure, which yields an estimate of  $\Omega_m h$ . In effect, we have two constraints on the  $\Omega_m - h$  plane that are consistent if  $w = -1$ , but this is not the case for other values of  $w$ . In this way, the current combined constraints from CMB plus alternative probes (LSS and the Supernova Hubble diagram) yield an impressive accuracy:

$$w = -0.926^{+0.054}_{-0.053}, \quad (330)$$

for a spatially flat model – see Spergel et al. (2006). The confidence contours are plotted in detail in figure 29, and it is clear that so far there is very good consistency with a simple cosmological constant. But as we will see, plenty of models exist in which some deviation is predicted. The next goal of the global cosmology community is therefore to push the errors on  $w$  down substantially – to about 1%. There is no guarantee that this will yield any signal, but certainly it will cut down the range of viable models for dark energy.



**Figure 29.** The marginalized WMAP3 confidence contours on the plane of dark-energy equation of state ( $w$ ) vs  $\Omega_m$  (from Spergel et al. 2006). A flat universe is assumed, although this is not critical to the conclusions.

One of the future tools for improving the accuracy in  $w$  will be large-scale structure. We have seen how this helps pin down the parameter degeneracies inherent in a CMB-only analysis, but it also contains unique information from the acoustic horizon. Earlier, we approximated this without considering how the speed of sound would depend on the baryon density; a good approximation to the exact result is

$$D_a \simeq 60 (\Omega_m h^2)^{-0.25} (\Omega_b h^2)^{-0.08} \text{ Mpc}. \quad (331)$$

This forms a standard measuring rod, as seen in the ‘baryon wiggles’ in the galaxy power spectrum. In future galaxy surveys, the measurement of this signature as a function of redshift will be a further useful geometrical probe.

The amplitude constraint from the CMB has been harder to implement. Although WMAP provides an accurately determined normalization, it involves the uncertain optical depth due to reionization:

$$\sigma_8(\text{CMB}) = 0.75(\Omega_m/0.3)^{+0.4} \exp(\tau) \pm 4\%. \quad (332)$$

The value of  $\tau$  is constrained by large-angle polarization data, and lies close to 0.1 according to WMAP. The current accuracy would be useful if we had an accurate independent estimate of  $\sigma_8$ . This can be attempted using the abundance of clusters of galaxies and also gravitational lensing, although the test is not really properly mature as yet.

### 11.3 Quintessence

The simplest physical model for dynamical vacuum energy is a scalar field. We know from inflationary models that this can yield something close in properties to a cosmological constant, and so we can immediately borrow the whole apparatus for modelling vacuum energy at late times. This idea of scalar fields as a dynamical substitute for  $\Lambda$  was first explored by Ratra & Peebles (1988). Of course, this means yet another scalar field that is introduced without much or any motivation from fundamental physics. This hypothetical field is given the fanciful name ‘quintessence’, implying a new addition to the ancient Greek list of elements (fire, air, earth, water).

The Lagrangian density for a scalar field is as usual of the form of a kinetic minus a potential term:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi). \quad (333)$$

In familiar examples of quantum fields, the potential would be

$$V(\phi) = \frac{1}{2} m^2 \phi^2, \quad (334)$$

where  $m$  is the mass of the field. However, as before we keep the potential function general at this stage.

Suppose the Lagrangian has no explicit dependence on space-time (*i.e.* it depends on  $x^\mu$  only implicitly through the fields and their 4-derivatives). Noether’s theorem then gives the energy–momentum tensor for the field as

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \mathcal{L}. \quad (335)$$

From this, we can read off the energy density and pressure:

$$\begin{aligned}\rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}(\nabla\phi)^2 \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{1}{6}(\nabla\phi)^2.\end{aligned}\tag{336}$$

If the field is constant both spatially and temporally, the equation of state is then  $p = -\rho$ , as required if the scalar field is to act as a cosmological constant; note that derivatives of the field spoil this identification.

For a homogeneous field, we have the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0,\tag{337}$$

which is most easily derived via energy conservation:

$$\frac{d\ln\rho}{d\ln a} = -3(1+w) = -3\dot{\phi}^2/(\dot{\phi}^2/2 + V),\tag{338}$$

following which the relations  $H = d\ln a/dt$  and  $\dot{V} = \dot{\phi}V'$  can be used to change variables to  $t$ , and the damped oscillator equation for  $\phi$  follows. The solution of the equation of motion becomes tractable if we make the **slow-rolling approximation** that  $|\ddot{\phi}|$  is negligible in comparison with  $|3H\dot{\phi}|$  and  $|dV/d\phi|$ , so that

$$3H\dot{\phi} = -dV/d\phi.\tag{339}$$

From this, we know that a sufficiently flat potential can provide a dynamical vacuum that is arbitrarily close to a cosmological constant in its equation of state. However, there are good reasons why we might want to imagine the slow-roll conditions being violated in the case of dark energy.

**COSMIC COINCIDENCE AND QUINTESSENCE** Accepting the reality of vacuum energy raises a difficult question. If the universe contains a constant vacuum density and normal matter with  $\rho \propto a^{-3}$ , there is a unique epoch at which these two contributions cross over, and we seem to be living near to that time. This coincidence calls for some explanation.

We already have one coincidence, in that we live relatively close in time to the era of matter-radiation equality ( $z \sim 10^3$ , as opposed to  $z \sim 10^{28}$  for the GUT era). This is relatively simple to understand: structure formation cannot begin until after  $z_{\text{eq}}$ , and so we would expect observers to appear before the universe has expanded much beyond this point. The vacuum coincidence problem could therefore be solved if the vacuum density was some dynamical entity that was triggered to become  $\Lambda$ -like by the change in expansion history at  $z_{\text{eq}}$ . Zlatev, Wang & Steinhardt (1999) suggested how this might happen. We have seen that the density and pressure for a quintessence field will be

$$\begin{aligned}\rho_\phi &= \dot{\phi}^2/2 + V \\ p_\phi &= \dot{\phi}^2/2 - V.\end{aligned}\tag{340}$$

This gives us two extreme equations of state: (i) vacuum-dominated, with  $V \gg \dot{\phi}^2/2$ , so that  $p = -\rho$ ; (ii) kinetic-dominated, with  $V \ll \dot{\phi}^2/2$ , so that  $p = \rho$ . In the first case, we know that  $\rho$  does

not alter as the universe expands, so the vacuum rapidly tends to dominate over normal matter. In the second case, the equation of state is the unusual  $\Gamma = 2$ , so we get the rapid behaviour  $\rho \propto a^{-6}$ . If a quintessence-dominated universe starts off with a large kinetic term relative to the potential, it may seem that things should always evolve in the direction of being potential-dominated. However, this ignores the detailed dynamics of the situation: for a suitable choice of potential, it is possible to have a **tracker field**, in which the kinetic and potential terms remain in a constant proportion, so that we can have  $\rho \propto a^{-\alpha}$ , where  $\alpha$  can be anything we choose.

Putting this condition in the equation of motion shows that the potential is required to be exponential in form. The Friedmann equation with  $\rho \propto a^{-\alpha}$  requires  $a \propto t^{2/\alpha}$ , so we have  $\rho \propto t^{-2}$  as usual. But now both  $V$  and  $\dot{\phi}^2$  must scale in the same way as  $\rho$ , so that  $\dot{\phi} \propto 1/t$ . Both the  $\ddot{\phi}$  and  $3H\dot{\phi}$  terms are therefore proportional to  $V$ , so an exponential potential solves the equation of motion. More importantly, we can generalize to the case where the universe contains scalar field and ordinary matter. Suppose the latter obeys  $\rho_m \propto a^{-\alpha}$ ; it is then possible to have the scalar-field density obeying the same  $\rho \propto a^{-\alpha}$  law, provided

$$V(\phi) \propto \exp[-\lambda\phi/M], \quad (341)$$

where  $M = m_{\text{P}}/\sqrt{8\pi}$ . The scalar-field density is  $\rho_{\phi} = (\alpha/\lambda^2)\rho_{\text{total}}$ . The impressive thing about this solution is that the quintessence density stays a fixed fraction of the total, whatever the overall equation of state: it automatically scales as  $a^{-4}$  at early times, switching to  $a^{-3}$  after matter-radiation equality.

This is not quite what we need, but it shows how the effect of the overall equation of state can affect the rolling field. Because of the  $3H\dot{\phi}$  term in the equation of motion,  $\phi$  ‘knows’ whether or not the universe is matter dominated. This suggests that a more complicated potential than the exponential may allow the arrival of matter domination to trigger the desired  $\Lambda$ -like behaviour. Zlatev, Wang & Steinhardt suggested two potentials which might achieve this:

$$V(\phi) = M^{4+\beta}\phi^{-\beta} \quad \text{or} \quad V(\phi) = M^4[\exp(m_{\text{P}}/\phi) - 1]. \quad (342)$$

They show that these can yield an evolution in  $w(t)$  so that it switches from  $w \simeq 1/3$  in the radiation era to  $w \simeq -1$  today.

However, a degree of fine-tuning is still required, in that the trick only works for  $M \sim 1$  meV, so there is no natural reason for tracking to cease at matter-radiation equality. The idea of tracker fields thus does not remove completely the puzzle concerning the level of present-day vacuum energy. But such models are at least testable: because the  $\Lambda$ -like behaviour only switches on quite recently, it is hard to complete the transition, and the prediction is of something around  $w \simeq -0.8$  today. As we have seen, this can be firmly ruled out with current data. These ideas about the dynamical vacuum are therefore already interesting testable science.

**K-ESSENCE** In a sense, quintessence is only half the story. We started with the usual Lagrangian for a simple massive scalar field,  $\mathcal{L} = \dot{\phi}^2/2 - m^2\phi^2/2$  and generalized the quadratic mass term to an arbitrary potential,  $V(\phi)$ . Why not take the same liberties with the kinetic term? Even though such **k-essence** models lack the intuitive analogies of quintessence, a Lagrangian can be anything we like. The

simplest models try to express things in terms of the normal kinetic expression

$$X \equiv \partial^\mu \phi \partial_\mu \phi / 2, \quad (343)$$

and one assumes that

$$\mathcal{L} = K(\phi)f(X) \quad (344)$$

In the homogeneous case,  $X = \dot{\phi}^2/2$ .

The pressure and density are

$$\begin{aligned} \rho &= 2X\mathcal{L}_{,X} - \mathcal{L} \\ P &= \mathcal{L} \end{aligned} \quad (345)$$

so that the equation of state is

$$w = \frac{f}{2Xdf/dX - f}. \quad (346)$$

For a normal kinetic term, this gives  $w = +1$  if there is no potential. The equation of motion is derived just by writing conservation of energy as for quintessence:

$$\frac{d \ln \rho}{d \ln a} = -3(1 + w). \quad (347)$$

What sort of  $k$ -essence Lagrangian will yield tracking? We want to fix  $w$  at the value of the dominant component, which requires

$$\frac{d \ln f}{d \ln X} = (1 + 1/w)/2 \quad \Rightarrow \quad f(X) \propto X^{(1+1/w)/2}. \quad (348)$$

Thus, a Lagrangian proportional to the square of the usual kinetic term will produce tracking during the radiation era, but tracking in the matter era requires a step to  $f(X) = 0$  to be encountered just as the universe becomes matter dominated. This is the opposite to the case of quintessence: now fine-tuning would be required in order for tracking to be maintained. The real question is whether a simple model can achieve sufficiently strong departure from tracking to get somewhere close to  $w = -1$  in the matter era in an inevitable way. This seems to be controversial: Armendariz-Picon, Mukhanov & Steinhardt (0006373) claimed that it could be done, but Malquarti, Copeland & Liddle (0304277) disagreed. The issue, as with quintessence, is the extent to which a tracking solution arises inevitably independent of initial conditions – i.e. whether it is an **attractor**. This has certainly not been demonstrated.

**PERTURBATIONS IN THE VACUUM** In dynamical models for the vacuum, we have a peculiar kind of fluid, so it is able to respond to gravity and grow inhomogeneities. The key parameter here is the vacuum sound speed, which obeys the usual relation

$$c_s^2 = \frac{\partial p}{\partial \rho}. \quad (349)$$

In practice, this is evaluated as

$$c_s^2 = \frac{\partial p / \partial X}{\partial \rho / \partial X} \quad (350)$$

i.e. ignoring perturbations in the field. The justification for this is that a gauge freedom exists, and that  $\delta\phi = 0$  corresponds to the rest-frame of the vacuum fluid.

This means that, for quintessence, the sound speed is always  $c_s = c$ . Even a completely flat potential with initial condition  $\dot{\phi} = 0$  does not mimic a cosmological constant. This only happens if the Lagrangian is set up completely lacking any kinetic term. The low sound speeds in some  $k$ -essence models can have quite large effects on the CMB anisotropies, and so can be probed observationally beyond just  $w$  and its evolution.

#### 11.4 The anthropic approach

Whether or not one finds the ‘essence’ approach compelling, there remains one big problem. All the models are constructed using Lagrangians with a particular zero level. All quintessence potentials tend to zero for large fields, and  $k$ -essence models lack a potential altogether. They are therefore subject to the classical dilemma of the cosmological constant: adding a pure constant to the Lagrangian has no affect on field dynamics, but mimics a cosmological constant. With so many possible contributions to this vacuum energy from the zero-point energies of different fields (if nothing else), it seems contrived to force  $V(\phi)$  to asymptote to zero without a reason.

This leads us in the direction of anthropic arguments, which are able to limit  $\Lambda$  to some extent: if the universe had become vacuum-dominated at  $z > 1000$ , gravitational instability would have been impossible – so that galaxies, stars and observers would not have been possible (Weinberg 1989). Indeed, Weinberg made the astonishingly prescient prediction on this basis that a non-zero vacuum density would be detected at  $\Omega_v$  of order unity, since there was no reason for it to be much smaller.

**MANY UNIVERSES** At first sight, this argument seems quite appealing, but it rapidly leads us into deep waters. How can we talk about changing  $\Lambda$ ? It has the value that it has. We are implicitly invoking an **ensemble picture** in which there are many universes with differing properties. This is a big step (although exciting, if this turns out to be the only way to explain the vacuum level we see). In fact, the idea of an ensemble emerges inevitably from the framework of inflationary cosmology, since the fluctuations in the scalar field can affect the progress of inflation itself. We have used this idea to look at the changes in when inflation ends – but fluctuations can affect the field at all stages of its evolution. They can be thought of as adding a random-walk element to the classical rolling of the scalar field down the trough defined by  $V(\phi)$ . In cases where  $\phi$  is too close to the origin for inflation to persist for sufficiently long, it is possible for the quantum fluctuations to push  $\phi$  further out – creating further inflation in a self-sustaining process. This is the concept of **stochastic eternal inflation** due to Linde. Sufficiently far from the origin, the random walk effect of fluctuations becomes more marked and can overwhelm the classical downhill rolling. This means that some regions of space can inflate for an indefinite time, and a single inflating universe

automatically breaks up into different bubbles with their own histories. Some random subset of these eventually random-walk close enough to the origin that the classical end of inflation can occur, thus creating a set of ‘universes’ each of which can potentially host observers.

With this as a starting point, the question now becomes whether we can arrange for the different members of this ensemble to have different values of  $\Lambda$ . This is easily achieved. Let there be some quintessence field with a very flat potential, so that it is capable of simulating  $\Lambda$  effectively. Quantum fluctuations during inflation can also displace this field, so that each member of the **multiverse** would have a different  $\Lambda$ .

**THE DISTRIBUTION OF  $\Lambda$**  We are now almost in a position to calculate a probability distribution for  $\Lambda$ . First, we have to set some ground rules: what will vary and what will be held fixed? We should try to change as little as possible, so we assume that all universes have the same values for

- (1) The Baryon fraction  $f_b = \rho_b/\rho_m$ .
- (2) The entropy per particle  $S = (T/2.73)^3/\Omega_m h^2$
- (3) The horizon-scale inhomogeneity  $\delta_H \simeq 10^{-5}$ .

It is far from clear that these minimal assumptions are correct. For example, string theorists have evolved the notion of the **landscape**, in which there is no unique form for low-energy particle physics, but instead a large number of possibilities in which numbers such as the fine-structure constant, neutrino masses etc. are different. From the point of view of understanding  $\Lambda$ , we need there to be at least  $10^{100}$  possible states so that at least some have  $\Lambda$  smaller than the natural  $m_p^4$  density by a sufficient factor. The landscape hypothesis really took off in 2001, when this number was first shown to be about  $10^{500}$ . But to start with, the simplest approach makes sense: if the simplest forms of anthropic variation can be ruled out, this might be taken as evidence in favour of the landscape picture.

We then take a Bayesian viewpoint to the distribution of  $\Lambda$  given the existence of observers:

$$P(\Lambda \mid \text{Observer}) \propto P_{\text{prior}}(\Lambda)P(\text{Observer} \mid \Lambda), \quad (351)$$

where we need both the prior distribution of  $\Lambda$  between different members of the ensemble and how the chance of getting an observer is modified by  $\Lambda$ . The latter factor should be proportional to the number of stars, which is generally take to be proportional to the fraction of the baryons that are incorporated into nonlinear structures. We can estimate this using the Press-Schechter apparatus to get the collapse fraction into systems of a galaxy-scale mass. The exact definition of this is not very important, since the CDM power spectrum is very flat on small scales: any mass at all close to  $10^{12} M_\odot$  gives similar answers.

The more difficult part is the prior distribution of  $\Lambda$ , and a common argument is to say that it has a uniform distribution – which seems reasonable enough if we are to allow it to have either sign, but know that we will be interested in a very small range near zero. This is the startling proposition of the anthropic model: the vacuum density takes large ranges, and in almost all realizations, the values are comparable in magnitude to the natural scale  $m_p^4$ ; such models are stupedously inimical to life.

We therefore have the simple model

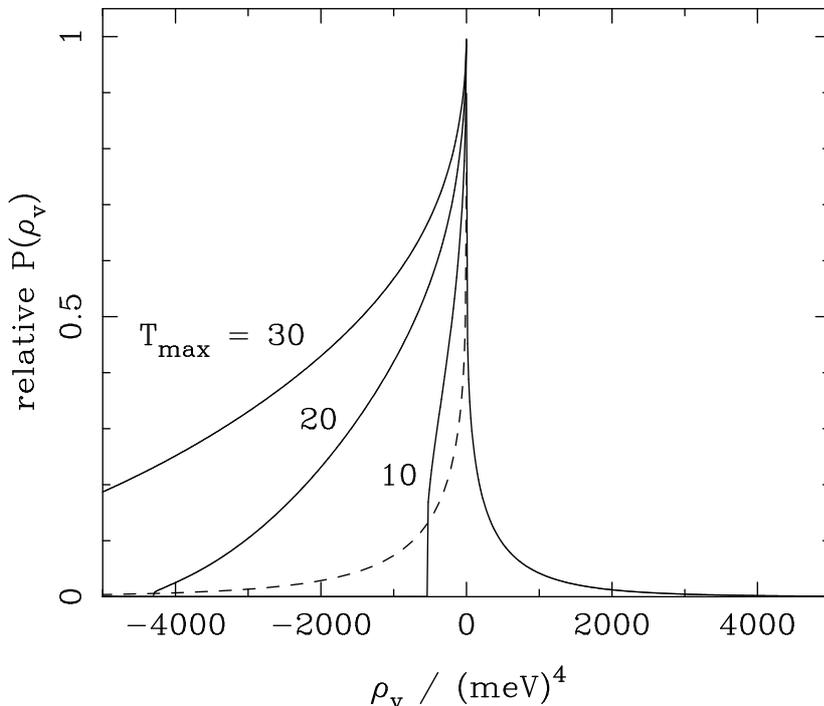
$$dP(\rho_v) \propto f_c d\rho_v, \quad (352)$$

where  $f_c$  is the collapse fraction into galaxy-scale objects. For large values of  $\Lambda$ , growth ceases at high redshift, and  $f_c$  is exponentially suppressed. But things are less clear-cut if  $\Lambda < 0$ . Here the universe eventually recollapses, and the high density means that the collapse fraction always tends to unity. So why do we not observe  $\Lambda < 0$ ? The answer is that we have to cut off the calculation at late stages of recollapse: once the universe becomes too hot, star-formation may be affected and in any case there is little time for life to form.

With this proviso, figure 30 shows the posterior distribution of  $\Lambda$  conditional on the existence of observers in the multiverse. We express things in natural units: if we adopt the values  $\Omega_v = 0.75$  and  $h = 0.73$  for the key cosmological parameters, then

$$\rho_v = 7.51 \times 10^{-27} \text{ kg m}^{-3} = \frac{\hbar}{c} \left( \frac{E_v}{\hbar c} \right)^4, \quad (353)$$

where  $E_v = 2.39$  meV is known to a tolerance of about 1 %. Provided we consider recollapse only to a maximum temperature of about 10 K, the observed figure is matched well by the anthropic prediction: with this cutoff, most observers will see a positive  $\Lambda$ , and something of order 10% of observers will see  $\Lambda$  as big as we do, or smaller.



**Figure 30.** The collapse fraction as a function of the vacuum density, which is assumed to give the relative weighting of different models. The dashed line for negative density corresponds to the expanding phase only, whereas the solid lines for negative density include the recollapse phase, up to maximum temperatures of 10 K, 20 K, 30 K.

So is the anthropic explanation the correct one? Many people find the hypothesis too radical: why postulate an infinity of universes in order to explain a detail of one of them? Certainly, if an alternative explanation for the ‘why now’ problem existed in the form of e.g. a naturally successful quintessence model, one might tend to prefer that. But so far, there is no such alternative. The longer this situation persists, the more we will be forced to accept that the universe we see can only be understood by making proper allowance for our role as observers.