Testing the cosmic evolution of radio galaxies

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March 18, 2005

1 Introduction

The population evolution of radio galaxies and quasars can be inferred from large surveys which provide a measure of the redshift distribution of these objects. Evolution can be said to occur if a difference is found between their co–moving space density locally and at high redshift. In this seminar I will outline two methods which can be applied to a survey to test for evolution:

- the $\log N - \log S$ test and
- the V/V_{max} test

These large surveys were first carried out at radio wavelengths since local, stellar, radio sources tend to be very dim so any bright objects detected are likely to lie at large, cosmological, distances. These large distances however mean that redshift effects need to be taken into account as outlined below.

1.1 Luminosity distance, d_L

If a photon is emitted by a distant source at some frequency ν_1 then it would be detected by an observer at a frequency $\nu_0 = \nu_1/(1+z)$. If the energy of the photon is $h\nu$ then the measured photon energy flux density is

$$F'_\nu(\nu_0) = h\ \nu_0\ n_\nu\ (\nu_0) = h\ \frac{\nu_1}{1+z}\ n_\nu(\nu_1) = \frac{F_\nu(\nu_1)}{1+z}, \tag{1}$$

where $'$ denotes measurements in the observers frame and n is the total number of photons. The total energy flux received over all frequencies is therefore

$$F' = \int \frac{F_\nu(\nu_1)}{(1+z)} \frac{d\nu_1}{(1+z)} = \frac{F}{(1+z)^2}, \tag{2}$$

For an object at co–moving co–ordinate r_1 and with luminosity L the flux measured in the emitted reference frame would be $F_1 = L/(4\pi\ (a_0 r_1)^2)$ where a_0 is the cosmic scale factor. Using the previous result, the flux measured by the observer would be

$$F_0 = \frac{F_1}{(1+z)^2} = \frac{L}{4\pi\ (1+z)^2\ (a_0 r_1)^2} = \frac{L}{4\pi d_L^2} \tag{3}$$

where $d_L = (1+z)\ a_0 r_1$ is the luminosity distance.

1.2 K and Evolutionary corrections

A K correction is applied to an object's magnitude in a particular band to correct for the effects of redshift. It is normally applied along with the evolutionary correction (EC) which corrects for any intrinsic evolution which may have occurred in the object. For more information on these corrections see Poggianti (1997).

Consider a galaxy at a redshift z. The light emitted by this object at a time t_1 is then observed at a time t_0 with flux

$$F_{\lambda_0} = \frac{1}{4\pi\ d_L^2(1+z)} \int_0^\infty E(\frac{\lambda}{1+z}, t_1)\ T(\lambda)\ d\lambda, \tag{4}$$

where $E(\frac{\lambda}{1+z}, t_1)$ is the spectral energy distribution (SED) of the object observed at t_1 and $T(\lambda)$ is the transmission function of the filter used.

The above equation can be re–written as:

$$F_{\lambda_0} = \frac{\int_0^\infty E(\lambda, t_0)\ T(\lambda)\ d\lambda}{4\pi\ d_L^2(1+z)} \frac{\int_0^\infty E(\frac{\lambda}{1+z}, t_0)\ T(\lambda)\ d\lambda}{\int_0^\infty E(\lambda, t_0)\ T(\lambda)\ d\lambda} \frac{\int_0^\infty E(\frac{\lambda}{1+z}, t_1)\ T(\lambda)\ d\lambda}{\int_0^\infty E(\frac{\lambda}{1+z}, t_0)\ T(\lambda)\ d\lambda}, \tag{5}$$

which then becomes

$$\begin{aligned} m_{\lambda_0} &= M(\lambda_0, t_0) + 5\ \log d_L + const \\ &+ \left[2.5\ \log\left(1+z\right) + 2.5\ \log \frac{\int_0^\infty E(\lambda, t_0)\ T(\lambda)\ d\lambda}{\int_0^\infty E(\frac{\lambda}{1+z}, t_0)\ T(\lambda)\ d\lambda}\right] + 2.5\ \log \frac{\int_0^\infty E(\frac{\lambda}{1+z}, t_0)\ T(\lambda)\ d\lambda}{\int_0^\infty E(\frac{\lambda}{1+z}, t_1)\ T(\lambda)\ d\lambda}. \end{aligned} \tag{6}$$

The term in the square brackets is the K correction and the last term is the evolutionary correction. Once the SED of an object is known these two corrections can be calculated and the observed magnitude, m_{λ_0} can be corrected.

2 The $\log N - \log S$ test

The cosmic evolution of the population of radio galaxies in general can be inferred from basic number count statistics via the log N-log S test. Firstly the simple Euclidean version of the test is outlined followed by the more useful non-Euclidean treatment. See Peterson (1997) for more details.

2.1 Euclidean case

Consider a Euclidean volume element of space dV containing a certain number of radio sources each of luminosity L (for simplicity). If the space density of these sources is $n(r)$ as a function of distance r then the total number in the volume will be

$$d\mathcal{N}(r) = n(r)dV = n(r)r^2 dr d\Omega. \tag{7}$$

Therefore the number of sources per unit solid angle is

$$\frac{d\mathcal{N}}{d\Omega} = n(r)r^2 dr, \tag{8}$$

Next assume that there is some limiting flux, S, below which a source would not have been detected. The total number of sources is now

$$N(S) = \int \frac{d\mathcal{N}}{d\Omega}(F \geqslant S) = \int \frac{d\mathcal{N}}{d\Omega}(r \leqslant r_{max}) = \int_0^{r_{max}} n(r) r^2 dr, \tag{9}$$

If $n(r) = n_0$, i.e. there is a uniform density of radio sources, (9) integrates to

$$N(S) = n_0 \frac{r_{max}^3}{3}, \tag{10}$$

Finally, substituting for r_{max} and logging both sides, (10) becomes

$$\log N(S) = \log \left[\frac{n_0 L^{3/2}}{3(4\pi)^{3/2}} \right] - \frac{3}{2} \log S. \tag{11}$$

This equation shows that for a Euclidean universe with a uniform density of radio sources, the number observed (above some limiting flux S) should be proportional to $S^{-3/2}$ and a plot of log N against log S should be a straight–line with a gradient of -3/2 if the assumptions are correct.

2.2 Non–Euclidean case

The observed flux density, $S_\nu(\nu_0)$ of a radio source is related to the source's emitted flux density $S_\nu(\nu_1)$ by

$$S_\nu(\nu_0) = S_\nu(\nu_1)/(1+z). \tag{12}$$

Therefore the observed flux density is given by

$$S_\nu(\nu_0) = \frac{L_\nu(\nu_1)(1+z)}{4\pi d_L^2}, \tag{13}$$

where $L_\nu(\nu_1)$ is the luminosity at ν_1 and d_L is the luminosity distance.

Next, suppose that the spectrum of a radio galaxy can be modelled as a power law, $L_\nu \propto \nu^{-\alpha}$, where $< \alpha > \approx 0.7$. Now

$$S_\nu(\nu_0) = \frac{L_\nu(\nu_0)}{4\pi d_L^2 (1+z)^{\alpha-1}}. \tag{14}$$

In this case the total number of sources in dV is a function of z:

$$d\mathcal{N} = n(z) dV(z), \tag{15}$$

and the volume element $dV(z)$ is given by

$$dV(z) = (a_0 r)^2 \; dr \; d\Omega = \frac{c \; d_L^2 \; d\Omega \; dz}{H_0 \; (1+2q_0 z)^{1/2} \; (1+z)^3}. \tag{16}$$

Here, instead of assuming a constant density of radio sources we are assuming that the co–moving density of radio sources is constant with redshift, i.e. $n(z) = n_0(1+z)^3$. The surface density of sources is now $dN(z) = d\mathcal{N}(z)/d\Omega$ giving

$$dN(z) = \frac{n(z) \; d_L^2 c \; dz \; d\Omega}{H_0 \; (1+z)^3 \; (1+2q_0 \; z)^{1/2} \; d\Omega} \Rightarrow \frac{dN(z)}{dz} = \frac{n_0 \; d_L^2 \; c}{H_0 (1+2q_0 \; z)^{1/2}}, \tag{17}$$

where H_0 is the Hubble constant and q_0 is the deceleration parameter at t_0 assuming the cosmological constant, Λ, is zero.

The differential source counts per unit flux, dN/dS can now be calculated by combining (17) with the differential of (14). The resulting expression, normalised by the differential source counts in the Euclidean case, dN_E, is:

$$\frac{dN/dS}{dN_E/dS} = \frac{dN}{dN_E} = \frac{2\ c\ (1+z)^{-3(\alpha-1)/2}}{H_0\ (1+2\ q_0\ z)^{1/2}} \left[\frac{2}{d_L}\frac{d\ (d_L)}{dz} + \frac{(\alpha-1)}{(1+z)}\right]^{-1}. \tag{18}$$

If $q_0 = 1/2$ (Einstein de Sitter universe) and $< \alpha > \approx\ 0.7$ then dN/dN_E is 1 at $z = 0$ and is < 0 for higher z values. Therefore, the $\log N - \log S$ test in a non-Euclidean universe is predicted to have a shallower slope than the Euclidean case if the co-moving density of sources is constant.

The major drawback to this test is that it is very dependent on the completeness of the sample. If sources are missed then $\log N$ will not be accurate.

3 The V/V_{max} test

The V/V_{max} test is a useful counterpart to the previous test because it is not limited by the completeness of the sample. Again the test needs to be derived for both Euclidean and non-Euclidean cases and further details can be found in Peterson (1997).

3.1 Euclidean case

Consider a group of randomly distributed sources all with the same luminosity, L. The maximum volume beyond which these sources could not be detected is

$$V_{max} = \frac{4}{3}\pi r_{max}^3. \tag{19}$$

If the sources are uniformly distributed throughout this volume then half the sources would be expected to fall in the inner half of this space, and half in the outer. Therefore if a survey extends out to a volume V,

$$\left\langle \frac{V}{V_{max}} \right\rangle = 0.5. \tag{20}$$

To be useful the error in this result is also needed. A simple calculation gives $\sigma_{V/V_{max}} = (12N)^{-1/2}$ so

$$\left\langle \frac{V}{V_{max}} \right\rangle = 0.5 \pm (12N)^{-1/2}. \tag{21}$$

If this statistic is found to be > 0.5 then the number density of sources is higher at larger distances, suggesting evolution. Alternatively, if it is < 0.5 then the present number density is higher again implying evolution.

3.2 Non–Euclidean case

Here, the luminosity distance d_L must be used instead of r and, as before, the constant density assumption becomes one of constant density per unit co–moving volume. Also, as V_{max} is now a function of z, the maximum redshift beyond which a source could not be detected is needed. This is usually found by calculating the absolute magnitude M (with K-corrections) of a source of apparent magnitude m at a distance d_L. The final step is to calculate the value of z_{max} at which a source of this absolute magnitude would be at the survey detection limit, m_{lim}.

The co–moving volume element is identical to that used in the $\log N - \log S$ test, therefore, for each source in the sample the integral

$$V_i(z') = \int_0^{z'} dV(z) = \int_0^{z'} \frac{d_L^2(z)\ c}{H_0\ (1+z)\ (1+2q_0\ z)^{1/2}} dz, \tag{22}$$

must be evaluated for both $z' = z$ and $z' = z_{max}$. (The integral over $d\Omega$ is ignored as it cancels in the V/V_{max} ratio.) The statistic is then

$$\left\langle \frac{V}{V_{max}} \right\rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{V_i(z)}{V_i(z_{max})}. \tag{23}$$

It should also be noted that in some cases there is a lower limit to the V_{max} as well. Here the test is modified such that only the accessible volume is considered:

$$\frac{V}{V_a} = \frac{V - V_{min}}{V_{max} - V_{min}} \tag{24}$$

References

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