Estimating Black Hole Masses in Active Galactic Nuclei

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1 Introduction

This seminar will outline a number of popular methods for the determination of black hole masses in active galactic nuclei (AGN). The origins of the physical models behind the techniques will be discussed, and the mathematical techniques for implementation of these methods will be outlined where relevant.

The four techniques discussed today are:

- Correlations between velocity dispersion and black hole mass: the $M_{bh} - \sigma$ relation
- Reverberation mapping
- Photoionisation modelling
- Correlations between bulge mass and black hole mass: the $M_{bh} - M_{bulge}$ relation

2 Velocity Dispersion Analysis: the $M_{bh} - \sigma$ Relation

The supermassive black hole paradigm for AGN was launched when it was realised that the gravitational potential energy in a masses of $10^6 - 10^9 M_{\odot}$ was more than sufficient to produce the observed quasar and Seyfert luminosities. The success of the black hole model in bringing together all of the observed properties of AGN into a coherent framework meant that acceptance of the paradigm came before there was any concrete evidence for the existence of black holes.

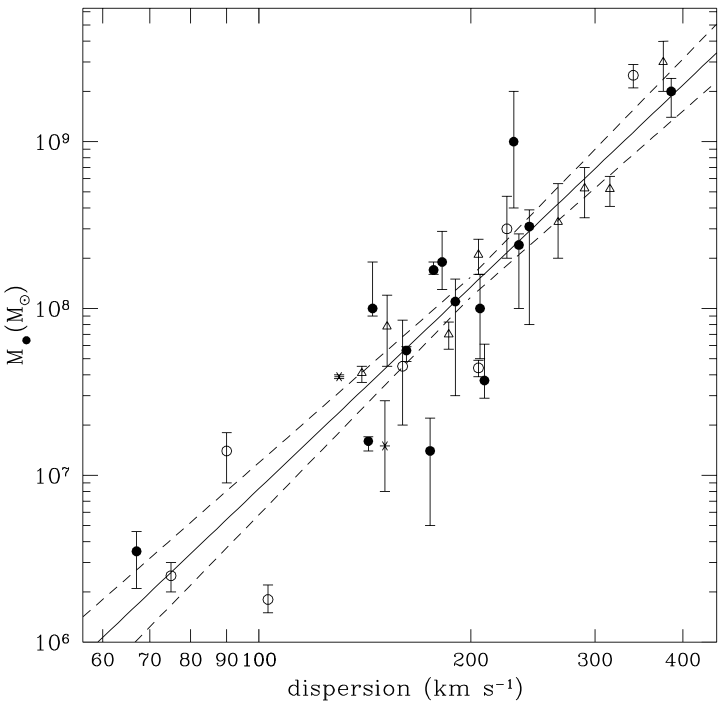

Figure 1: The correlation between black hole mass and stellar velocity dispersion. From Tremaine et al. (2002).

Evidence for high mass-to-light ratios in the centres of a number of nearby non-active galaxies mounted as better spatial resolution arrived with the advent of the Hubble Space Telescope. Observations were consistent with the existence of a central massive dark object in all galaxies that possessed a spheroidal component, i.e. ellipticals and spiral galaxies with bulges. Observations of the Galactic centre and also the galaxy NGC 4258 were able to rule out astrophysical alternatives to black holes in these two cases, and so the focus turned from whether the massive dark object was a supermassive dark hole to what relationship the putative black hole had with the host galaxy.

Analysis of the line-of-sight velocity dispersion for a number of nearby galaxies revealed a correlation between the measured velocity dispersion of the stars in the bulge component and the mass of the central black hole. The best fitting linear relation to the masses shown in Figure 1 is given by

$$\frac{M_{bh}}{M_\odot} = (8.13 \pm 0.06) + (4.02 \pm 0.32) log \frac{\sigma}{\sigma_0} \tag{1}$$

The measured scatter in this relationship is 0.3dex in black hole mass for a fixed dispersion σ, although likely measurement errors mean that the intrinsic scatter is more like half that.

This relationship can be used to estimate the black hole masses in AGN as well as quiescent galaxies.

2.1 Measuring the velocity dispersion from stellar absorption lines

The basic assumption of this method is that the stellar absorption lines are the convolution of a suitable template spectrum (adjusted for instrumental broadening) and a broadening function corresponding to the distribution of stars over line-of-sight velocities (the *line profile*). Generally, the line profile is assumed to be Gaussian and the best fitting Gaussian can be determined in either Fourier or pixel space. I shall be discussing a method implemented in pixel space today.

Since models of galaxies are not uniquely constrained by knowledge of rotation velocities and velocity dispersion alone, it is difficult to estimate any errors on black hole masses determined by Gaussian fitting alone. Different models *can* be distinguished by fitting the full line profile, however, allowing effects like velocity anisotropy in the bulge to be quantified. The line fitting technique discussed here tests whether the line profile deviates significantly from Gaussian by expanding the model profile as a Gauss-Hermite series, exploiting the fact that Gaussian functions are in general a good low-order approximation to the actual line profile.

The routine is implemented as follows:

- The galaxy spectrum $G(\lambda)$ and stellar template spectrum $S(\lambda)$ are rebinned linearly so that a relative velocity ΔV implies a Doppler shift $\Delta x = \Delta V/c$.
- A polynomial fit is made to the continuum of each spectrum. The continuum fit is subtracted and the spectra are divided by the mean continuum level.
- We assume that our logarithmically rebinned, continuum subtracted spectrum $G(x)$ is a convolution of the stellar template spectum $S(x)$ and a broadening function $B(x)$. The broadening function $B(x)$ is related to the line profile $L(v)$ by

$$L(v) = B(v/c) \tag{2}$$

- If we assume that $B(x)$ depends on a finite number of free parameters, the parameters can then be found by minimising the quantity

$$\chi^2 = \int [G(x) - B(x) * S(x)]^2 dx \tag{3}$$

Here we consider fitting an idealised solution where we know the required template fit exactly, and have information corresponding to all velocities. In this case the quantity that we want to minimise is proportional to

$$\chi^2_{id} = \int [\gamma_0 L_0(v) - L(v)]^2 dv \tag{4}$$

where $L_0(v)$ is the true normalised distribution of stars over line of sight velocities and L(v) is our model distribution.

The usual approach is to assume that the line profile $L(v)$ is a Gaussian with free parameters line strength γ, mean radial velocity V and velocity dispersion σ. So

$$L(v) = \frac{\gamma}{\sigma} a(w) \tag{5}$$

where $a(y)$ is a standard Gaussian and $w = (v - V)/\sigma$.

To determine deviations from a Gaussian we expand the line profile as a sum of orthogonal functions. We consider the Gauss-Hermite series:

$$L(v) = \frac{\gamma}{\sigma} a(w) \sum_{j=0}^{N} h_j H_j(w) \tag{6}$$

where the coefficients h_j are fit as free parameters and the functions $H_j(w)$ are the Hermite polynomials. These satisfy the orthonormal relation

$$\int_{\infty}^{\infty} H_m(y) H_n(y) a^2(y) dy = \frac{1}{2\sqrt{\pi}} \delta_{mn} \tag{7}$$

The first 5 Hermite polynomials as used in this method are:

$$H_0(y) = 1, \qquad H_1(y) = \sqrt{2}y, \qquad H_2(y) = \frac{1}{\sqrt{2}}(2y^2 - 1) \tag{8}$$

$$H_3(y) = \frac{1}{\sqrt{6}}(2\sqrt{2}y^3 - 3\sqrt{2}y), \qquad H_4(y) = \frac{1}{\sqrt{24}}(4y^4 - 12Y^2 + 3) \tag{9}$$

Hermite polynomials of odd degree are odd functions and therefore tell us about asymmetric deviations of the line profile from a Gaussian. The even degree polynomials are even functions and so tell us about symmetric deviations.

3 Reverberation Mapping and Photoionisation Modelling

3.1 Reverberation Mapping

Reverberation mapping utilises variability in both the continuum and line emission to constrain the size of the emitting region in the active nucleus. Using the line width to obtain an estimate of the velocity of the line emitting gas, the mass of the central engine is obtained by assuming that the gas is gravitationally bound.

The size of the region is measured by observing the response of the emission line to the driving continuum variations. This lag is assumed to arise entirely as a result of the light travel time between the accretion disk and the line-emitting clouds.

Since we quantify the time lag using the line *variability*, we also want to quantify the cloud velocities using the variable part of the line to try to compare simlar size scales. The variability of the emission lines can be quantified by looking at the r.m.s. of the combined spectrum. The width of the r.m.s line then gives an estimate for the varying cloud velocities.

3.2 Photoionisation modelling

An alternative method of estimating the virial mass is based on estimating the size of the broad-line region using photoionisation theory, and using the emission-line width as an indicator of the lin-of-sight velocity dispersion. Thie method only assumes that the gas is gravitationally bound and does not rely on the specific geometry of the BLR as does reverberation mapping.

The simplest method is to used line ratios to determine the density of the line-emitting gas and its ionisation parameter. This constrains the size of the BLR. The cloud velocity is then obtained from the wide of the broad line under scrutiny. The fact that the specific geometry of the BLR and its inclination are unknown is parameterised in the mass determination.

The principle problem with this method is estimating the broad-line luminosity. As a zeroth-order estimate we can assume that it is proportional to the optical luminosity, and try and determine the proportionality constant using a sample with a mass-estimate derived from another method. Kaspi and collaborators found a correlation between the monochromatic luminosity at

5100Å and the size fo the broad-line region. This is given by:

$$R_{BLR} \propto L^{0.7} \tag{10}$$

Naively we might expect that $R_{BLR} \propto L^{0.5}$ from our simple photoionisation model, indicating that other factors (such as inclination) need to be taken into account here.

4 The Bulge-Mass - Black Hole Relation

As mentioned previously during the discussion of the $M_{bh} - \sigma$ relation, the high spatial resolution fo the HST allowed detailed modelling of stellar kinematics in galaxy bulges, yielding comparatively accurate values for the masses of black holes at the centre of quiescent galaxies. Prior to the correlation between the black hole mass and the stellar velocity dispersion being determined, a similar correlation between the mass of the black hole and the mass of the bulge component of the host galaxy was discovered.

Early analysis of this correlation by John Magorrian utilised HST photometry to determine mass-to-light ratios, but ground-based spectroscopic measurements to determine the stellar kinematics. Consequently, due to the inaccuracies in the kinematic modelling the scatter in this relationship was thought to be quite large (0.5 dex).

However, later measurements that used photoionisation modelling to measure the black hole masses in AGN, together with accurate host galaxy morphologies and luminosities, showed that the scatter in this relationship to be considerably smaller - comparable to the 0.3dex scatter in the $M_{bh} - \sigma$ relationship. This meant that black hole masses in the brightest quasars could be determined (velocity dispersion analysis for these objects is difficult or impossible).

4.1 Modelling the bulge mass in AGN

This is a tricky proposition in AGN, because the low surface-brightness of the host galaxy is often completely masked by the high luminosity nucleus. Therefore a modelling technique is used that allows the successful decomposition of the nuclear an host galaxy light. This requires an accurate model of the PSF for the optical system.

The next steop is to model the host galaxy and nuclear component and fit the model to the data. This is done as follows:

- A model galaxy and nuclear component is constructed with 6 free parameters
  - The luminosity profile parameter β
  - Host galaxy position angle
  - Host galaxy axial ratio
  - Host galaxy scalelength
  - Host galaxy luminosity
  - Nuclear component luminosity
- The 6 parameters ar ethen allowed to vary continuously until the modle, when convolved with the constructed PSF, best fits the available data. The χ^2-fitting is done using a downhill-simplex minimisation technique such as *amoeba*
- The 6 free parameters define 6 vertices of a 5-dimensional hypersurface. The parameters are varied until the global minimum corresponding to the minimum-χ^2 is found.
- Once the model has been found that best fits the observational data, the mass of the bulge component is determined by considering the host galaxy luminosity and morphological type.

References

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