Modelling the effects of dust on galaxy spectra

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1 Dust: Why is it important?

- Dust makes up roughly 1 percent of the interstellar medium
- Scatters and absorbs UV/optical photons and re-emits them in the IR
- Dust emission seen in the local universe (spirals, ellipticals, ULIRGs)
- And in the high-z universe (SCUBA, Lyman-break galaxies, EROs)
- IR/submm background implies much of the energy radiated by stars throughout the history of the universe has been reprocessed by dust!
- The magnitude of reprocessing of starlight grows with star formation rate

It is clear that in order to accurately reproduce the UV-submm SEDs of galaxies we must take dust into account in any model for galaxy formation and evolution.

2 The interstellar medium of the Milky Way

The interstellar medium is mostly comprised of atomic and molecular hydrogen, with some helium present also. Only one percent of the ISM is in the form of dust. The dust component is made up of:

- Graphite grains
- Silicate grains
- Water ice
- CO, PAH and over a hundred other different molecules
- Small quantities of heavy metals (Fe etc.)

3 GRASIL

GRASIL allows treatment of the overall SED of a galaxy with realistic geometry, taking the effects of dust on starlight into account. For a complete description of the code and applications, see Silva (1998) and Silva (PhD thesis, 1999).

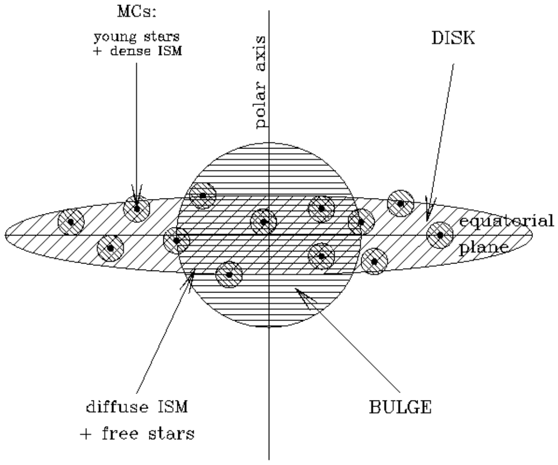

Figure 1: The components of a model galaxy

3.1 The galaxy model

A model galaxy can have both a disk and bulge component. The gas is distributed in discrete molecular clouds and a diffuse cirrus component, as depicted in figure 1.

The disk takes the form of a double exponential:

$$\rho_d = \rho_0 \exp\left(-R/R_d\right) \exp\left(-|z|/z_d\right) \qquad \text{where} \qquad z = r\cos\theta$$

The bulge is given by a King profile:

$$\rho_b = \rho_0 \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-\gamma} \qquad \text{where} \qquad \gamma = 3/2 \text{ (stars) and} \qquad \gamma = 3/4 \text{(dust)}$$

Disks are truncated at $R_{gal} = 6R_d$, and bulges are truncated at the tidal radius $R_t = 10^{2.2} r_c$.

Stars, molecular clouds and diffuse gas all have the same scalelengths: $R_d^{stars} = R_d^{mc} = R_d^{cirrus}$ and $z_d^{stars} = z_d^{mc} = z_d^{cirrus}$.

Any star formation takes place in molecular clouds, $N_{mc} = \delta M_g f_{mc}$ where δ is the dust to gas ratio and f_{mc} is the fraction of the gas in molecular clouds.

The initial stages of star formation are virtually invisible in the UV and optical. Stellar winds, outflows and UV flux from massive stars will destroy a molecular cloud on a timescale $\sim 10^6 - 10^7$ years (the typical lifetime of O/B stars).

Let f be the fraction of starlight reradiated in a cloud. This parameter will decrease with time according to the age of the stars t and the parameter t_0 which sets the escape time for stars:

$$f = \begin{cases} 1 & : \quad t \leq t_0 \\ 2 - t/t_0 & : \quad t_0 < t \leq 2t_0 \\ 0 & : \quad t > 2t_0 \end{cases}$$

The starlight locked inside a molecular cloud is modelled as a single central source.

3.2 The dust model

For the dust component of the interstellar medium, GRASIL takes into account the following:

- graphite grains
- silicate grains
- (optionally) PAH molecules

3.2.1 Graphite and silicate grains

The 2175 $\AA$ feature in the extinction curve of the Milky Way indicates the prescence of graphite grains in the ISM. Features seen in the near-IR imply the prescence of silicate grains.
The size distribution of graphite grains is given by:

$$\frac{dn_g}{da} = \begin{cases} A_g n_H a^{\beta_1} & : \quad a_b < a < a_{max} \\ A_g n_H a^{\beta_1 - \beta_2} a^{\beta_2} & : \quad a_{min} < a < a_b \end{cases}$$

where a_{min} = 8 Å,a_b = 50 Å, a_{max} = 0.25μm, β_1 = -3.5, β_2 = -4.0 and $A_g = 10^{-25.22} cm^{2.5} H^{-1}$

The size distribution of silicate grains is given by:

$$\frac{dn_s}{da} = A_S n_H a^{\beta}$$

where a_{min} = 50 Å, a_{max} = 0.25μm, β = -3.5 and $A_s = 10^{-25.11} cm^{2.5} H^{-1}$

In the code the size distribution of grains is discretized into twenty logarithmic bins, where $b_o = a_{min}, b_N = a_{max} and b_i = \sqrt{a_i a_{i+1}}$ for N = 1,2,3...N-1
The cross section for absorption or scattering Q is given by:

$$Q = \frac{C}{\pi a^2}$$

where C is the extinction cross section, units m^2. Values of Q are obtained from laboratory measurements.

The dust mixture cross section is given by a sum over grain radii:

$$\sigma_\lambda = \sum_{i=1}^{N} w_i Q(a_i), \lambda \qquad cm^2 \tag{1}$$

where the weights w_i are given by:

$$w_i = \pi a_i^{-1} \int_{b_{i-1}}^{b_i} a^3 \frac{1}{n_H} \frac{dn}{da} da \sim \frac{\pi a_i^2 n_i}{n_H}$$

The absorption coefficent α_λ is defined by $dI_\lambda = -\alpha_\lambda I_\lambda ds$ where ds is the path through the absorbing medium, and α_λ is given by:

$$\alpha_\lambda = n_H \sigma_\lambda \qquad cm^{-1} \tag{2}$$

The optical depth τ_λ is gien by $\tau_\lambda = \int_s n_H ds$, where again ds is the path through the absorbing medium.

The emission coefficent j_λ again is given by a sum over discrete grain radii. It is assumed that large grains reach thermal equilbrium dur to the radiation field. All grains of a given size above a particular size and composition (silicate or graphite) emit as a grey body at temperature T:

$$j_\lambda^{(big)} = \sum_{i=1, a_i > a_{flu}}^{N} w_i Q_{abs}(a_i, \lambda) B_\lambda(T(a_i)) n_H \qquad erg s^{-1} cm^{-2} sr^{-1} \AA^{-1} \tag{3}$$

where $B_\lambda(T)$ is the Planck function.

The source function S_λ is defined as $S_\lambda \equiv j_\lambda/\alpha_\lambda$. Hence, using equations (2) and (3):

$$S_\lambda = \frac{\sum w_i Q_{abs}(a_i,\lambda) B_\lambda(T(a_i) n_H}{\sum w_i Q_{abs}(a_i,\lambda) n_H} \qquad ergs^{-1}cm^{-2}sr^{-1}\mathring{A}^{-1} \tag{4}$$

Very small grains in a radiation field with $a < a_{flu} = 100\mathring{A}$ are subject to temperature fluctuations, due to the energy of incident radiation being larger than the typical energy content of the small grain. There can be significant deviations from the stable temperature the larger grains settle down to. Each grain size and composition will have a temperature distribution P(T) which I will not derive here. For those who are interested, the details can be found in Silva (1999) and references therein. The emission coefficent of small grains is given by:

$$j_\lambda^{(small)} = \sum_{i=1,a_i<a_{flu}}^{N} w_i Q_{abs}(a_i,\lambda) n_H \int_{T_{min}}^{T_{max}} B_\lambda(T(a_i)) P(T(a_i)) dT \qquad ergs^{-1}cm^{-2}sr^{-1}\mathring{A}^{-1} \tag{5}$$

3.2.2 PAH molecules (optional! Not covered today.)

In order to accurately model the Milky Way emission features seen at 3.3, 6.2, 7.7, 8.6 and 11.3 μm, a population of polycyclic aromatic hydrocarbons can be included in the dust model. Astrophysical PAH are both *dehydrogenated* (not every hydrogen site is occupied) and *catacondensed* (the most compact and stable structure).

The following details will not be covered in the lecture, but are included here for completeness.

The model PAH distribution as derived in Leger et. al (1989) is used. The size of a molecule is given by $a = 0.9\sqrt{N_C}$ $\mathring{A}$ where N_C is the number of carbon atoms in the molecule. The number of hydrogen atoms $N_H = x_H N_{s,H}$ where n_H is the hydrogen coverage and $N_{s,H}$ is the number of hydrogen sites, where $N_{s,H} = \sqrt{6N_C}$.

From this, we see that:

$$N_C = n_C a^2 \qquad n_C = 1.23 \qquad \mathring{A}^{-2}$$

$$N_H = n_H x_H a \qquad n_H = 2.72 \qquad \mathring{A}^{-1}$$

The size distribution is given by:

$$\frac{dn}{dN_C} \propto N_C^{-2.25}$$

or, using the above relations:

$$\frac{dn}{da} = X a^{-3.5}$$

Values of x_H and X are set to fit observational data.

The emissivity of a PAH molecule in frequency band ν_i is:

$$4\pi\epsilon_{PAH}(\nu_i) = 2\int_0^\infty \int_{a_{min}}^{a_{max}} S(\nu,\nu_i,a) \frac{4\pi J_\mu}{h\nu} 2\sigma_{PAH} \frac{dn}{da} d\nu da \qquad ergs^{-1}Hz^{-1}H^{-1}$$

where the energy inside band i from PAH a is:

$$S(\nu,\nu_i,a) = \sigma_i(a) \int_{T_{min}}^{T_{max}(\nu,a)} \frac{B_{\nu_i}(T) C(T,a)}{F(T,a)} \qquad ergHz^{-1}$$

where σ_i is the band absorption cross section of feature i, and C(T,a) is the heat capacity of the molecule,given by:

$$\frac{C(T,a)}{C_{max}} = \begin{cases} 9.25\text{x}10^{-4}T & : \quad T < 800K \\ 2\text{x}10^{-4}T + 0.58 & : \quad 800 \leq T < 2100K \\ 1 & : \quad T > 2100K \end{cases}$$

where $C_{max} = 3(N_C + N_H + 2)k$.

The power inside molecule a is:

$$F(T,a) = \sum_j B_{\nu_j} \triangle\nu_j \sigma_j(a) \qquad ergs^{-1}sr^{-1}$$

where $\triangle\nu_j$ is the width of feature j.

The maximum temperature T_{max} is derived from:

$$h\nu = \int_{T_{min}=5K}^{T_{max}(\nu,a)} C(T,a)dT$$

Values of $X = 3.03\text{x}10^{-25}cm^{2.5}H^{-1}$ and $x_H = 0.2$ give a good fit to the Milky Way cirrus emission data. From this, the number of carbon atoms in the form of PAH molecules is $\int N_C(a)\frac{dn}{da}da = 18$ ppM.

The emission coefficent of PAH molecules $j_{\nu,PAH}$ is then given by:

$$j_{\nu,PAH} = \frac{\epsilon_{PAH}\rho_{PAH}}{4\pi}$$

3.3 The equation of radiative transfer in molecular clouds

To take into account the effects of dust in optically thick molecular clouds on starlight, we must solve the equation of radiative transfer:

$$\frac{dI_\lambda}{ds} = -\alpha_\lambda I_\lambda + j_\lambda \tag{6}$$

This can be split into 2 components: $I_{\lambda,1}$, the specific intensity of radiation emitted by the central source, and $I_{\lambda,2}$, the specific intensity of radiation emitted by the dust.

For $I_{\lambda,1}$ the transfer equation is satisfied with j_λ=0:

$$I_{\lambda,1}(r,\theta) = I_{\lambda,1}(0)\exp[-\tau_\lambda(r,\theta)] \qquad ergs^{-1}cm^{-2}sr^{-1}\mathring{A}^{-1} \tag{7}$$

Where $\tau_\lambda(r,\theta)$ is the optical depth from the origin to the point (r,θ). The mean specific intensity is given by:

$$J_\lambda = \frac{1}{4\pi}\int_{4\pi} I_\lambda d\Omega$$

For $I_{\lambda,2}$, the full transfer equation must be solved, and is given by:

$$I_{\lambda,2}(r,\theta,\beta,\phi) = \int_{(r,\theta)}^{(r_b,\theta_b)} S_\lambda(r',\theta')\exp[-\tau_\lambda(r',\theta')]d\tau_\lambda \tag{8}$$

The integration extends from point (r,θ) to the outer boundary of the cloud (r_b,θ_b) along the direction $(\pi-\beta,\pi+\phi)$, and τ_λ is the optical depth from (r,θ) to (r_b,θ_b). The source function S_λ is given by equation (4) above.

It is assumed that the net energy transfer is dominated by radiative flux, so thermal equlibrium holds (energy input=energy output). This can be written as:

$$\int_0^\infty Q_{abs}(a,\lambda)J_\lambda d\lambda = \int_0^\infty Q_{abs}(a,\lambda)B_\lambda(T(a))d\lambda \tag{9}$$

So for each grainsize a, T is determined by numerically integrating equation (9).

3.3.1 The lambda iteration method

There are no analytic solutions to the equation of radiative transfer above, and so it must be solved numerically. The most common way of doing this is known as the **lambda iteration method**. This involves selecting a trial value of the mean specific intensity at each point, from which a value for T and the source function S_λ are obtained. Using this an updated value of the mean specific intensity is found using the equation of radiative transfer, and the process is repeated in an iterative manner. The method proposed by Collison and Fix (1991) is used in GRASIL.

It is assumed that the local integrated flux of radiation is proportional to the change in the source function weighted by the absorption coefficent. The region of interest has N radial grid points i = 1, 2, 3...N.

$$F(r,\theta) \sim \frac{S(r_i,\theta) - S(r_{i+1},\theta)}{r_i - r_{i+1}} \tag{10}$$

where

$$S(r,\theta) = \int_0^\infty \alpha_\lambda S_\lambda(r,\theta)d\lambda = \int_0^\infty \alpha_\lambda J_\lambda(r,\theta)d\lambda$$

due to radiative equilibrium.

Hence, for the n^{th} iteration:

$$S^{(n)}(r,\theta) = \int_0^\infty \alpha_\lambda J_\lambda^{(n)}(r,\theta)d\lambda \tag{11}$$

and the calculated luminosity for the n^{th} iteration at r_i is:

$$L_i^{(n)} = \int_{4\pi} F(r_i,\theta) r_i^2 d\Omega$$

where $F = \int\int I_\lambda \cos\theta d\lambda d\Omega$. Using equation (10) we have:

$$\frac{L_i^{(n)}}{L_i^{(n+1)}} = \frac{\int_{4\pi} S^{(n+1)}(r_i,\theta) - S^{(n+1)}(r_{i+1},\theta)d\Omega}{\int_{4\pi} S^{(n)}(r_i,\theta) - S^{(n)}(r_{i+1},\theta)d\Omega} \tag{12}$$

To speed up the convergence to a solution, set:

$$J^{(n+1)}(r_i,\theta) = f_i J'_\lambda(r_i,\theta) \tag{13}$$

$$J^{(n)}(r_i,\theta) = J'_\lambda(r_i,\theta) \tag{14}$$

where f_i is a multipicative factor only dependent on the radius, J'_λ is the mean specific intensity calculated using the current source function. There is a recursion relation for the values of f_i, and this can be obtained using equations (13), (14) and (15):

$$f_i = f_{i+1}\left(\frac{Y_{i+1}}{Y_i}\right) + \frac{L_{central}}{L_i^{(n+1)}}\left(1 - \frac{Y_{i+1}}{Y_i}\right) \tag{15}$$

where

$$Y_i = \int_{4\pi}\int_0^\infty \alpha_\lambda J'_\lambda(r,\theta_i)d\lambda d\Omega \qquad , L_i^{(n+1)} = L_{central}/L_i^{(n)} \qquad , f_{N-1} = L_{central}/L_{N-1}^{(n)}$$

A summary of this is shown in flowchart 1.

3.4 Radiative transfer through a diffuse dusty medium

We need not solve the full equation of radiative transfer for light passing through the diffuse dust component. The 'cirrus' component is optically thin in the IR and is transparent to its own photons.
Consider the cirrus component to be an infinite homogenous medium. A photon travelling through this medium will undergo a random walk due to scattering events before ending in an absorption event. The probability of a random walk ending in an absorption event is $\epsilon_\lambda = \frac{\alpha_\lambda}{\alpha_\lambda+\sigma_\lambda}$. The mean number of free paths is $N = \epsilon_\lambda^{-1}$.
The mean effective path (rms displacement of photon after random walk) is:

$$l_* = \sqrt{N} l \qquad \Rightarrow \qquad l_* = \epsilon^{-1/2} l$$

where l is the mean free path, $l = (\alpha_\lambda + \sigma_\lambda)^{-1}$ Thus:

$$l_* = [\alpha_\lambda(\alpha_\lambda + \sigma_\lambda)]^{-1/2}$$

We can define an effective optical depth τ_{eff}:

$$\tau_{eff} = \sqrt{\tau_{abs}(\tau_{abs} + \tau_{scat})}$$

where $\tau_{abs} = \alpha_\lambda L$, $\tau_{scat} = \sigma_\lambda L$ and L is the size of the region.

The galaxy is divided into small volume elements V_i. Due to azimuthal symmetry the local radiation field in $V_i(r, \theta, \phi = 0)$ is calculated from the contributions from all other elements $V_k(r, \theta, \phi > 0)$. There will be contributions from both molecular cloud emission and unobscured starlight.

The specific intensity in element V_i is given by:

$$J_{\lambda,i} = \sum_k \frac{V_k(j^{mc}_{\lambda,k} + j^{stars}_{\lambda,k}) \exp -\tau_{eff,\lambda}(i,k)}{r^2_{i,k}} \qquad ergs^{-1}cm^{-2}sr^{-1}\mathring{A}^{-1}$$

where $\tau_{eff}(i,k)$ is the effective optical depth between i and k and $r^2(i,k)$ is the volume averaged square distance from i to all points in k:

$$r^2(i,k) = \frac{\int_{\triangle\phi}\int_{\triangle\theta}\int_{\triangle r} V_k r^2 \sin\theta d^2(i,k) dr d\theta d\phi}{V_k}$$

where

$$V_k = \int_{\triangle\phi}\int_{\triangle\theta}\int_{\triangle} r^2 \sin\theta dr d\theta d\phi$$

and

$$d^2(i,k) = (r\sin\theta_i - r\sin\theta\cos\phi)^2 + (-r\sin\theta\sin\phi)^2 + (r\cos\theta_i - r\cos\theta)^2$$

Then the emission from the cirrus dust is calculated, using equations (3) and (5). This gives a total emission coefficent: $j_{\lambda,k} = j^{mc}_{\lambda,k} + j^{stars}_{\lambda,k} + j^{cirrus}_{\lambda,k}$

Finally, the specific flux as seen by a distant observer in direction β is given by:

$$F_\lambda(\beta) = 4\pi \sum_k V_k j_{\lambda,k} \exp -\tau_{eff,\lambda}(k,\beta) \qquad ergs^{-1}\mathring{A}^{-1}$$

where $\tau_{eff,\lambda}$ is the optical depth from k to the outskirts of the galaxy along the direction β.

3.5 Inputs for the model

From the galaxy formation model, the following inputs are required:

$\Psi(t, Z)$	SF history as a function of time and metallicity
M_d	disk mass
M_b	bulge mass
$R_{d,half}$	disk half mass radius ($R_d = 0.595 R_{d,half}$)
$R_{b,half}$	bulge half mass radius ($R_b = 0.068 R_{b,half}$)
M_{gas}	mass of cold gas present
Z_{gas}	metallicity of cold gas

Parameters related to the dust model:

δ	dust to gas mass ratio ($\delta \propto Z$ where $\delta = 1/110$ for $Z = Z_\odot$)
m_{mc}	typical mass of molecular cloud ($10^6 M_\odot$)
r_{mc}	typical radius of molecular cloud (15 pc)
t_0	typical escape time for stars in clouds (0.007 Gyr)
T_S	sublimation temperature of grains inside clouds (400 K)

Also require an initial mass function: Salpeter IMF $\phi(m) \propto m^{-1.35}$, where the lower and upper limits are $0.1 M_\odot$ and $120 M_\odot$ respectively.

Stars are represented by simple stellar populations, where a population of given age and metallicity has a specific spectrum. The total flux is an integral over the appropriate SSPs, weighted by the star formation rate at each population's birth:

$$F_\lambda(t_{gal}) = \int_0^{t_{gal}} SSP_\lambda(t_{gal} - t, Z(t))\Psi(t)dt$$

We can now evaluate the SED of a dusty galaxy. An example output SED is shown in figure 2. This particular galaxy contained $\sim 10^{10} M_\odot$ of cold gas at the output time, and was forming stars at a rate of $40 M_\odot yr^{-1}$. The contributions from stalight, molecular cloud emission and diffuse cirrus emission are all shown. The strong extinction in the UV/visible region is clearly visible. PAH emission lines in the mid-IR are visible, and molecular cloud and cirrus emission contribute strongly in the mid to far-IR.

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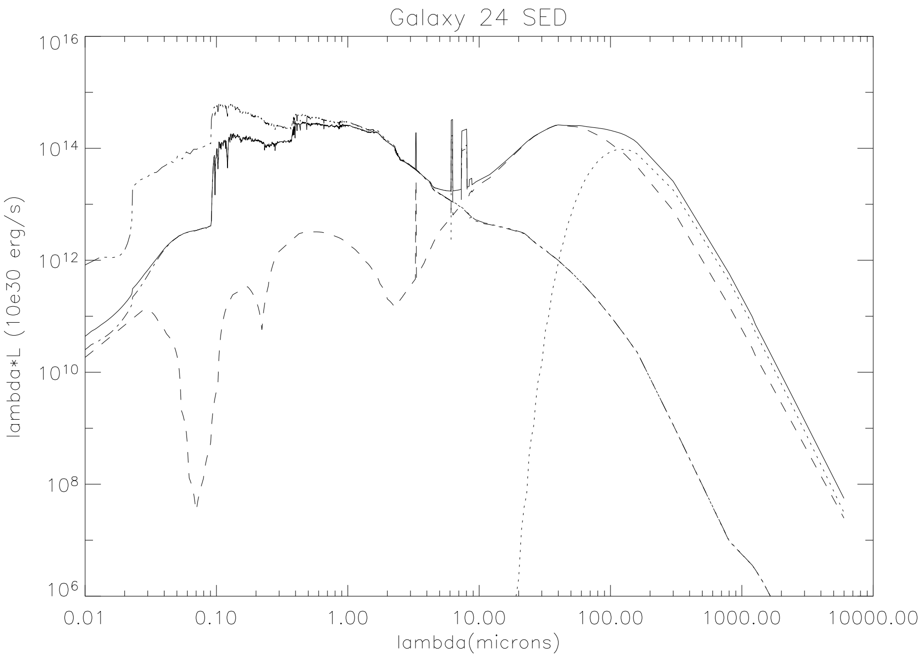

Figure 2: Example galaxy SED. Solid:total, dotted:cirrus emission, dashed:molecular cloud emission, dash-dot:extinguished stalight, dash dot dot: unextinguished starlight

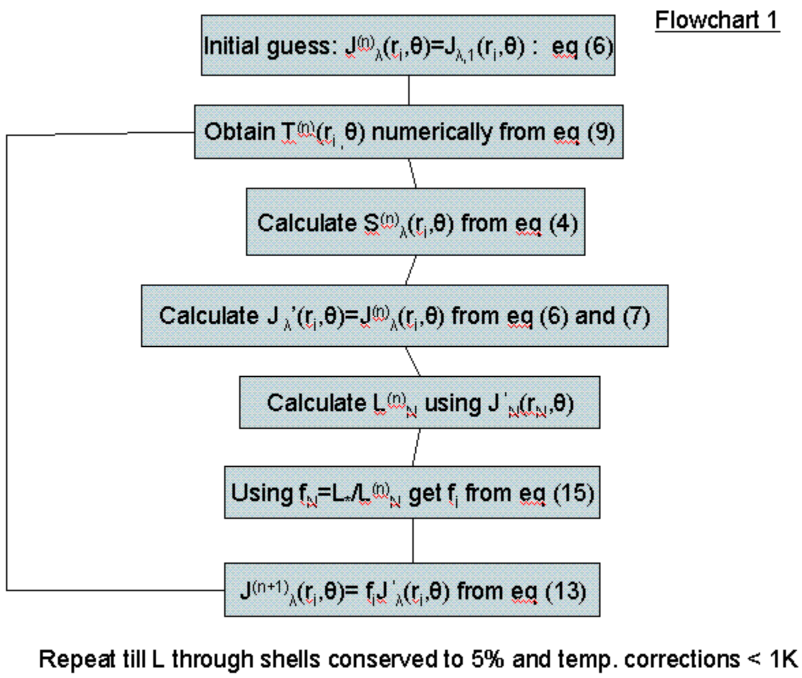

Figure 3: flowchart 1

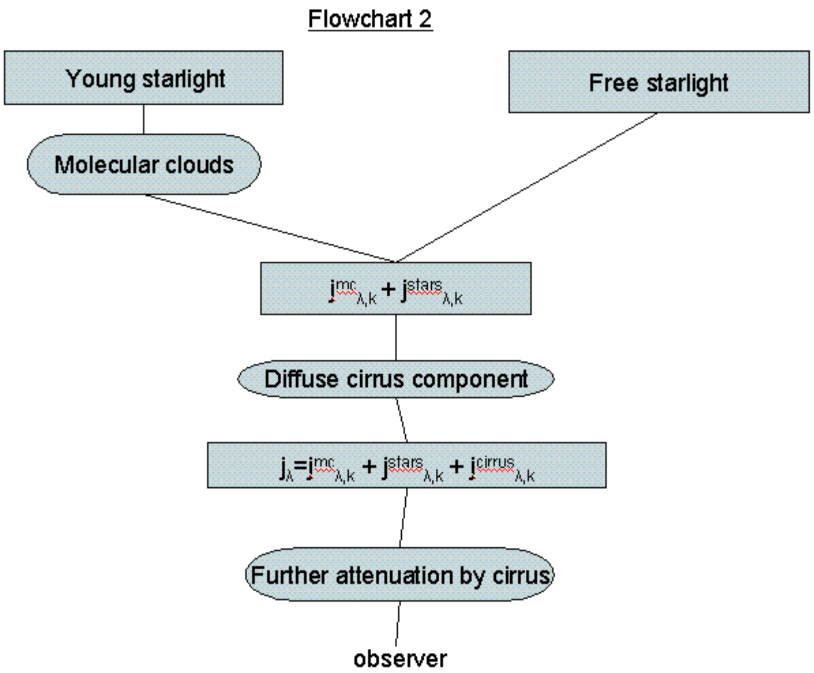

Figure 4: flowchart 2