Calculating Luminosity Functions

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1 Preliminaries

1.1 The luminosity function in cosmology

The galaxy luminosity function is defined as the number density of galaxies per unit magnitude per unit volume:

$$\phi(M) = \frac{\Delta N}{\Delta M \Delta V} \text{ or } \phi(L)\frac{\Delta N}{\Delta V \Delta L} \tag{1}$$

The luminosity function is of great importance to observational cosmology, since it is essential for:

- the estimation of the total content of luminous matter in the universe in the form of galaxies
- to provide a reference for testing models of galaxy formation and evolution
- the deprojection of the angular correlation function
- the estimation of the selection function of redshift surveys

to name but a few.

1.2 Parameterization of the LF

The most common parameterization of the luminosity function is the Schechter function, proposed by Schecter (1979):

$$\phi(L) = \phi^*(\frac{L}{L^*})^a e^{-\frac{L}{L^*}} \frac{1}{L^*}$$

or, if we want a function of absolute magnitude:

$$\phi(M) = 0.4 \ln 10 \phi^* [10^{0.4(M^*-M)}]^{(1+a)} e^{-10^{0.4(M^*-M)}}$$

where $L^*(M^*)$ is a characteristic luminosity(magnitude) where the LF exhibits a rapid change in its slope in the $(\log \phi, \log L)$ plane, and a is the slope of the LF for values $L \leq L^*$ (faint end). ϕ^* is a constant with units of number density and is used in the normalization of the LF.

2 Estimation of the LF

Consider a survey of N galaxies, covering a solid angle of Ω in the sky, down to magnitude m_{lim} and redshift range from z_{min} to z_{max}.

2.1 Classical Estimator

If we bin the galaxies according to absolute magnitude M, in bins with width ΔM, then the number of objects in a given bin is from the definition

$$N_{bin} = \int_{M1}^{M2} \phi(M) dM V_{lim} = \Phi(M) V_{lim}$$

where V_lim is defined by the maximum distance a galaxy with absolute magnitude M can have and still be included in the survey. For small ΔM we get $\phi(M) = \Phi(M)/(\Delta M V_{lim})$

This estimator is seriously flawed. The assumption this method makes is that the galaxies populate the sky uniformly without taking in to account an inhomogeneities in the sample. Additionally, information on the exact position of the galaxies in the absolute magnitude bins is lost.

2.2 V/V_{max} method

To correct for the loss of information due to the binning one may sum over all galaxies in an absolute magnitude bin :

$$\Delta M \phi(M) = \sum^{N} \frac{1}{V_{max}}$$

where $Vmax$ is the maximum volume for each galaxy.

This estimator is not a great improvement from the classical one, but it has the small advantage of giving the slope and the normalization of the luminosity function at the same time. It is a good first approximation of the LF if the magnitude binning is kept small.

2.3 STY estimator

Te probability that a galaxy at redshift z would have absolute magnitude M:

$$p = \frac{\phi(m)}{\int_{-\infty}^{M} \phi(M) dM}$$

If we assume that the number of objects in a given magnitude range and a given volume element is

$$\nu(x, y, z, M) = \phi(M) D(x, y, z)$$

, then the density parameter $D(x, y, z)$ cancels out in the probability relation. What we have implied here is that the luminosity function is independent of the local density, a fact that it is not true. For an approximation though it is a safe assumption.

The likelihood for all the galaxies to belong to the same parent distribution would then be:

$$L = \prod_i p_i$$

Assuming a Schecter function for the LF, we can maximize

$$\ln L = \sum_i \ln pi + const$$

and retrieve the parameters to our model.

This estimator gives good results, sine it uses all the information available and (in general) is not sensitive to inhomogeneities in the sample.

2.4 SWML estimator

The STY estimator relies on a functional form of the LF. One non parametrical estimator is the SWML estimator of Efstathiou, Ellis and Peterson. If we assume that the luminosity function is a function of N steps:

$$\phi(L) = \phi_k, \quad L \in (L_k - \Delta L/2, L_k + \Delta L/2), \quad k = 1, \ldots, N$$

The likelihood, as in the previous method, then is:

$$\ln L = \sum_{i=1}^{N} W(L_i - L_k) \ln \phi_k - \sum_{i=1}^{N} \ln\{\sum_{j=1}^{N} \phi_j \Delta L H[L_j - L_{min}(z_i)]\} + C$$

where C is a constant and we have defined the functions

$$W(x) = \left\{ \begin{array}{ll} 1 & x - \frac{\Delta x}{2} \leq x \leq x + \frac{\Delta x}{2} \\ 0 & \text{everywhere else} \end{array} \right.$$

and

$$H(x) = \left\{ \begin{array}{ll} 0 & x \leq -\frac{\Delta L}{2} \\ \frac{x}{\Delta l} + \frac{1}{2} & x - \frac{\Delta x}{2} \leq x \leq x + \frac{\Delta x}{2} \\ 1 & x + \frac{\Delta l}{2} \end{array} \right.$$

The normalization constant resulting from this method is arbitrary, therefore it is important to impose some constrains in order ot be able to directly compare results from different samples. The constrain adopted is of the form

$$g = \sum_k \phi_k (\frac{L_k}{L_f})^{\beta} \Delta L - 1 = 0$$

where L_f is a fiducial luminosity and β is a constant. introducing the constraint to our estimator we have to maximize the quantity $\ln L' = \ln L + \lambda g(\phi_k)$ with respect to λ and ϕ_k

Finally, we have:

$$\phi_k \Delta L = \frac{\sum_i W(L_i - L_k)}{\sum_i \{ \frac{H[L_k - L_m in(z_i)]}{\sum_{j=1}^{N} \phi_j \Delta L H[L_j - L_{min}(z_i)]} \}} \tag{2}$$

from which we can calculate the ϕ_k parameters with iteration.

2.5 Choloniewski method

Another method that does not rely on any assumption of a functional form of the luminosity function is the Choloniewski method. This method gives along with the shape of the luminosity function, the normalization constant of the LF as well.

The method bins the galaxies of the sample according to distance modulus μ and absolute magnitude M, in bins of equal length $\Delta\mu = \Delta M = \Delta$.

The average number density of the sample is

$$n = \frac{N}{V}$$

and for the integrated luminosity function

$$\int_{M_{min}}^{M_{max}} \Phi(M)\delta M = n$$

. If $\rho(x, y, z)$ is the number density then it follows that

$$\int\int\int_V \rho(x, y, z)\delta x \delta y \delta z = N$$

.

The method assumes that the number of galaxies k in the interval $\delta M \delta x \delta y \delta z$ are randomly distributed in a poissonian process, with probability:

$$P_k = e^{-\lambda} \frac{\lambda^k}{k!}$$

with $\lambda = \frac{1}{n}\Phi(M)\rho(x, y, z)\delta M \delta x \delta y \delta z$ (notice again that M and x,y,z are statistically independent)

If we bin the galaxies according to their distance modulus μ_j and their absolute magnitude M_i then the number of galaxies in the (i,j) bin would be N_{ij} and described by the probability $P_{N_{ij}} = \exp(-\lambda_{ij}) \frac{\lambda_{ij}^{N_{ij}}}{N_{ij}!}$

where $\lambda_{ij} = \frac{1}{n}\Phi_i(M)\Delta M \rho_j V_{ij}$

The values of N_{ij} are ONLY known for bins that satisfy the relation $i + j \leq (m_{lim} - M_{min} - \mu_0)/\Delta M$ The likelihood for the sample is then:

$$L = \prod_i^A \prod_j^B P_{ij} \quad , \; i + j \leq S$$

Maximazing the likelihood gives us the best estimates for ϕ_i and ρ_j.

We can attempt to "fill in" the unused pixels (decreasing the length Δ doesn't help very much, because we end up with pixels with too few galaxies in them), but to do so, we need a model for the galaxy distribution, which in effect cancels our need for a non-parametrical estimator.

2.6 C method (as revised by Choloniewski)

Consider again galaxies in the (M, μ) plane. The observed distribution of galaxies in the region of the plane can be expressed as follows:

$$N(M, \mu) = \phi(M) D(\mu) \Theta(m_{lim} - (M + \mu))$$

where $\phi(M)$ and $D(\mu)$ are distribution functions and $\Theta(m_{lim} - M - \mu)$ is the unit step function to account for the magnitude limit cutoff.

The function $N(M, \mu)$ can be expressed as a sum of delta functions over the galaxies of the sample

$$N(M, \mu) = \sum_{k=1}^{N} \delta(M - M_k, \mu - \mu_k)$$

we can express the $\phi(M)$ and $D(\mu)$ in a similar way:

$$\phi(M) = \sum_{i=1}^{N} \psi_i \delta(M - M_i)$$

$$D(\mu) = \sum_{j=1}^{N} d_j \delta(\mu - \mu_j)$$

thus we have

$$\sum_{k=1}^{N} \delta(M - M_k, \mu - \mu_k) = \overbrace{\sum_i \sum_j}^{M_i + \mu_j \leq m_{lim}} \phi_i d_j \delta(M - M_i) \delta(\mu - \mu_j)$$

Integrating this equation over M and μ gives , respectively:

$$1 = d_j \sum_i^{M_i + \mu_j \leq m_{lim}} \psi_i \quad j = 1, \ldots, N$$

$$1 = \psi_i \sum_j^{M_i + \mu_j \leq m_{lim}} d_j \quad i = 1, \ldots, N$$

providing us with a system of equations that can be solved to yield the ψ_i and d_j parameters.

Knowledge of the $\phi(M)$ and $D(\mu)$ functions allows us to calculate the distribution of galaxies above the selection line.

$$N_t(M,\mu) = \phi(M)D(\mu)$$

then we have

$$\phi(M) = \int_{\mu_{min}}^{\mu_{max}} N_t(M,\mu)d\mu\frac{1}{V_{total}}$$

$$N_t = \int_{M_{min}}^{M_{max}}\int_{\mu_{min}}^{\mu_{max}} N_t(M,\mu)dMd\mu$$

which, using the previous formalism, become:

$$\phi(M) = \sum_{i=1}^{N}\phi_i\delta(M-M_i)\sum_{j=1}^{N}d_j\frac{1}{V_{total}}$$

$$N_t = \sum_{i=1}^{N}\psi_i\sum_{j=1}^{N}d_j$$

with the total number density given by

$$n = \frac{N_t}{V_t}$$

.

This method returns a normalized LF and uses all the data, although again makes the assumption that the luminosity function is independent of the local density.

3 Normalization of the LF

Most methods recover the shape but not the normalization of the luminosity function ϕ^*, with a few notable exceptions. There are again quite a few methods of calculating the normalization. A simple estimator was one proposed by EEP that relies on the survey selection function.

The probability that a galaxy at distance x is included in the catalogue is

$$S(x) = \frac{\int_{min(L_{min}(x),L_{min})}^{L_{max}}\phi(L)dL}{\int_{L_{min}}^{L_{max}}\phi(L)dL}$$

and for a Shechter function this gives for the estimate of ϕ:

$$\phi^* = \frac{\frac{1}{V}\sum_i\frac{1}{S(x_i)}}{\Gamma(a+1,L_{min}/L^*)-\Gamma(a+1,Ł_{max}/L^*)}$$

summing over all galaxies in the volume V.

4 Which method is the best?

There is not a definitive answer to that question. The general consensus is that the classical estimation and the V/V_{max} estimator give the worse results but this was to be expected, since they are quite limited. Probably the best solution is to use more than one method and compare results. Generally, using the STY and SWML method together provides good results. It is noted however that all the estimators fail at the faint end although that is understandable given the limited number of objects in that range.

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