

Galaxy Structure through Bayesian Hierarchical modelling

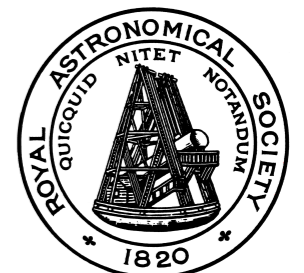
Josh Argyle

Supervisors: Jairo Méndez-Abreu & Vivienne Wild

DEX, Edinburgh, 9th January 2017.

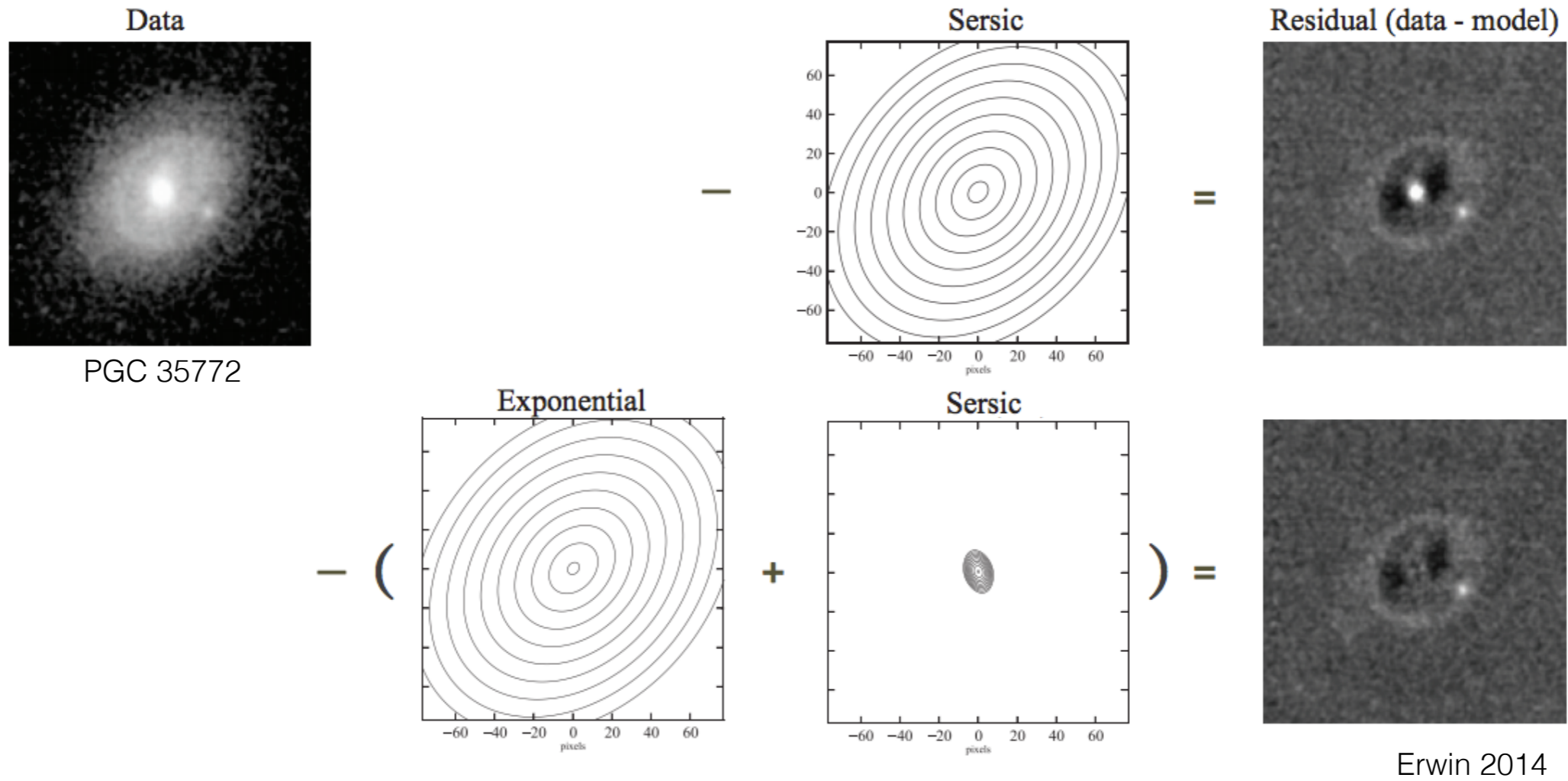


University of
St Andrews



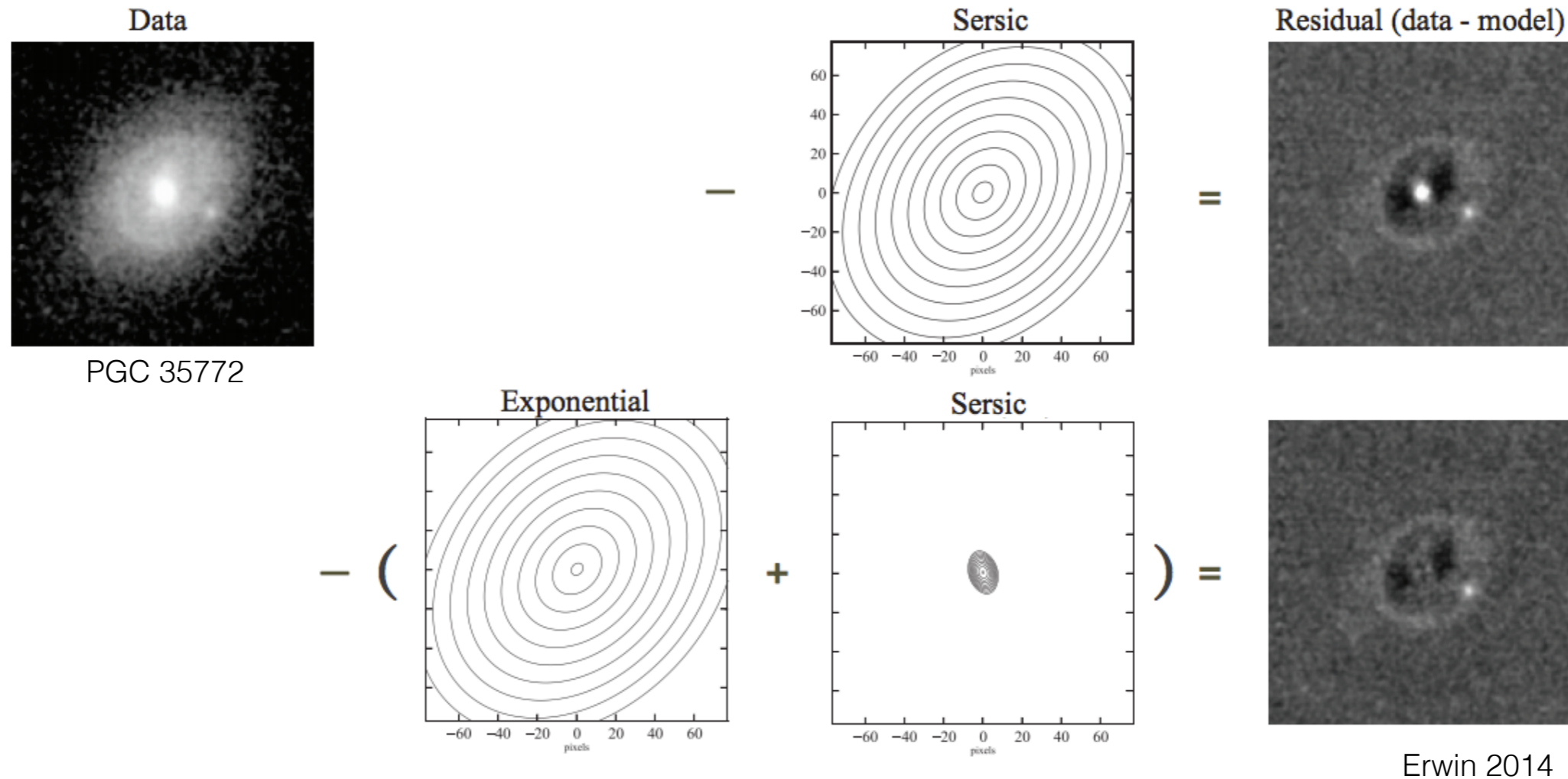
ja66@st-andrews.ac.uk

2D Photometric Decompositions



*see Lange *et al.* 2016 for more details.

2D Photometric Decompositions



Erwin 2014

Problems with conventional methods:

- 1) Local minima trapping.
- 2) Unrealistic solutions.
- 3) Which model?
- 4) Representation of errors.

*see Lange *et al.* 2016 for more details.

Bayesian Inference

Baye's Rule:

$$P(\theta|Data) = \frac{P(Data|\theta)P(\theta)}{\sum P(Data|\theta_i)P(\theta_i)}$$

Posterior Joint distribution \propto Likelihood function \times Prior distribution

The diagram illustrates Bayes' Rule with the following components and their relationships:

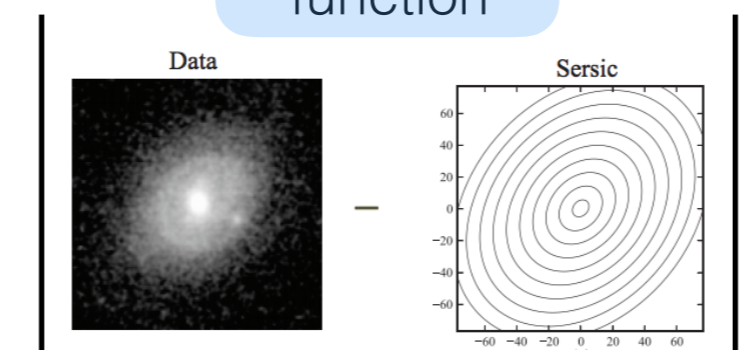
- Posterior Joint distribution** (grey box) is proportional to the product of the **Likelihood function** (blue box) and the **Prior distribution** (red box).
- The **Likelihood function** is represented by $P(Data|\theta)$ in the numerator of the fraction.
- The **Prior distribution** is represented by $P(\theta)$ in the numerator of the fraction.
- The denominator of the fraction is $\sum P(Data|\theta_i)P(\theta_i)$, representing the sum of the products of likelihood and prior for all possible parameter values.

Bayesian Inference

Baye's Rule:

$$P(\theta | Data) = \frac{P(Data | \theta) P(\theta)}{\sum P(Data | \theta_i) P(\theta_i)}$$

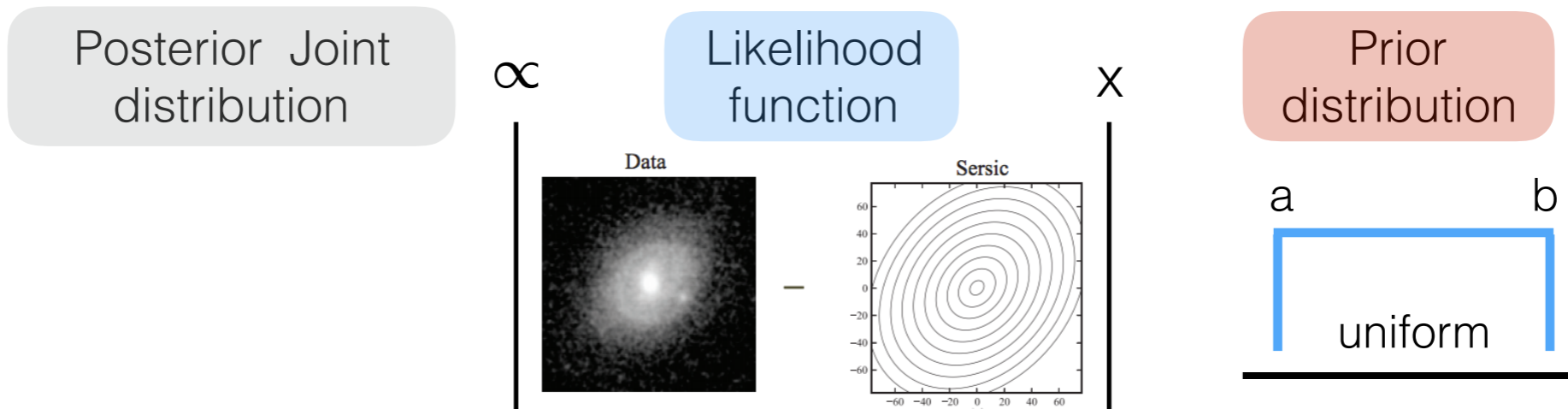
Posterior Joint distribution \propto Likelihood function \times Prior distribution



Bayesian Inference

Baye's Rule:

$$P(\theta|Data) = \frac{P(Data|\theta)P(\theta)}{\sum P(Data|\theta_i)P(\theta_i)}$$



Bayesian Inference

Baye's Rule:

$$P(\theta|Data) = \frac{P(Data|\theta)P(\theta)}{\sum P(Data|\theta_i)P(\theta_i)}$$

Posterior Joint distribution \propto Likelihood function \times Prior distribution

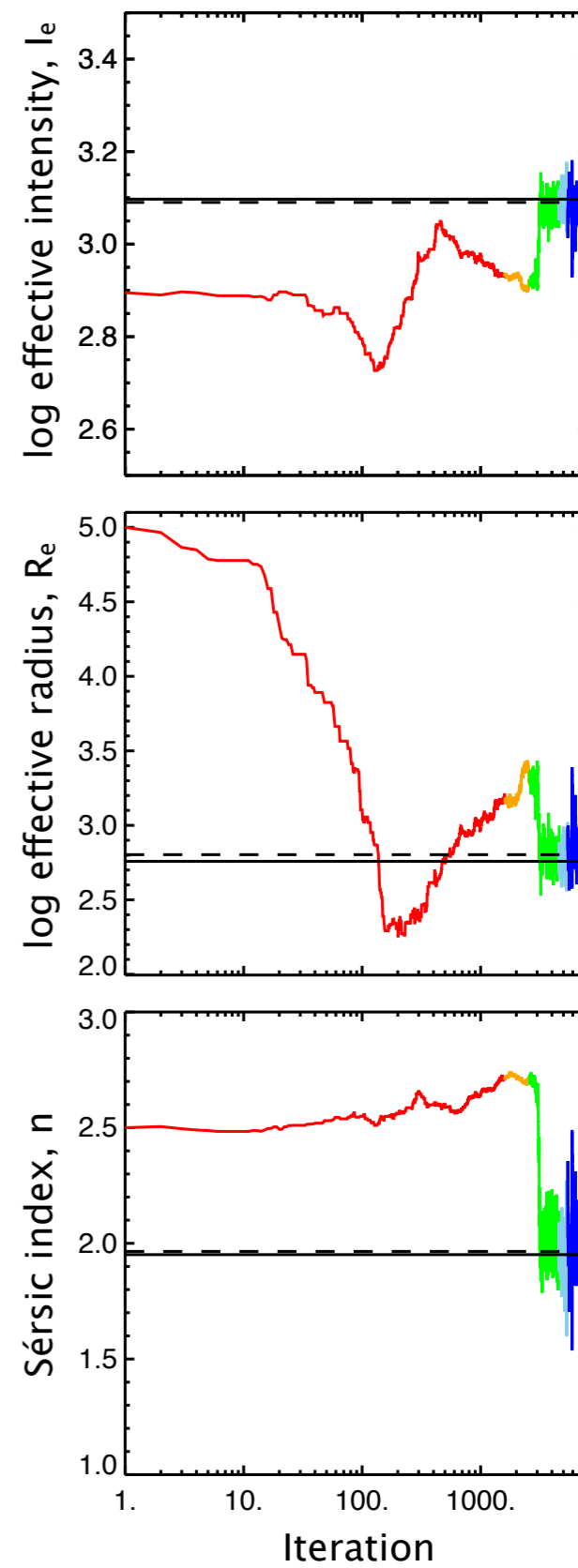
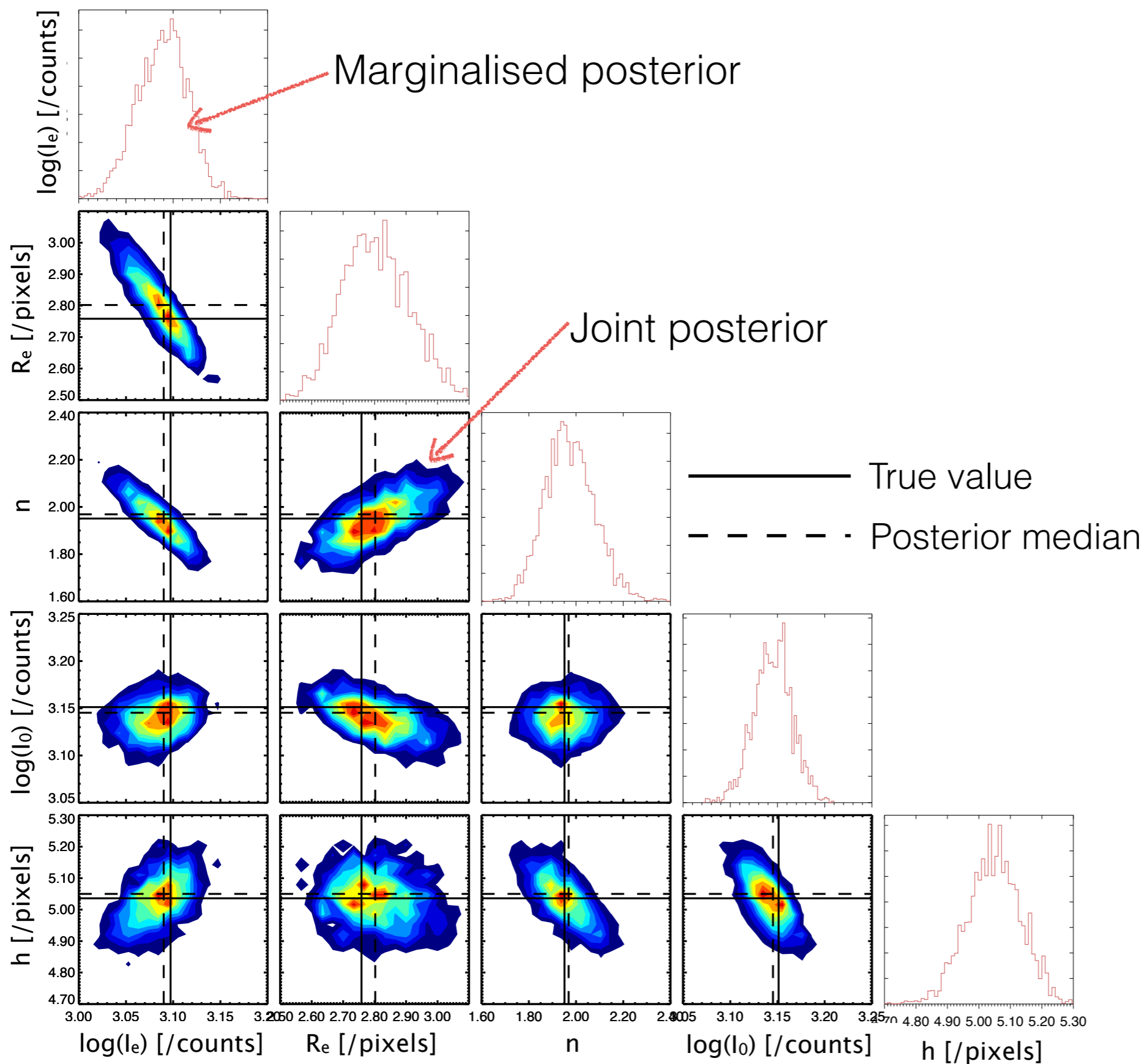
Previous Problems:

- 1) Local minima trapping.
- 2) Unrealistic solutions.
- 3) Which model?
- 4) Representation of errors.

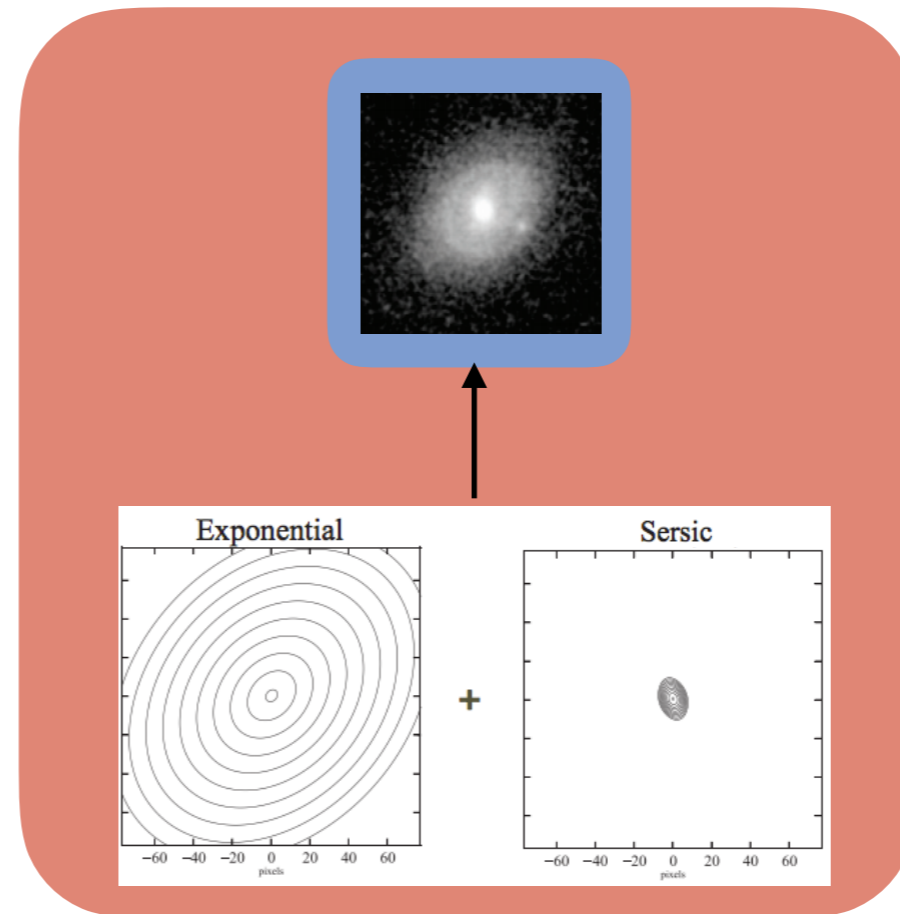
Markov Chain Monte Carlo Solutions:

- 1) Exploration of parameter space.
- 2) Priors.
- 3) Bayesian model selection.
- 4) Posterior probabilities.

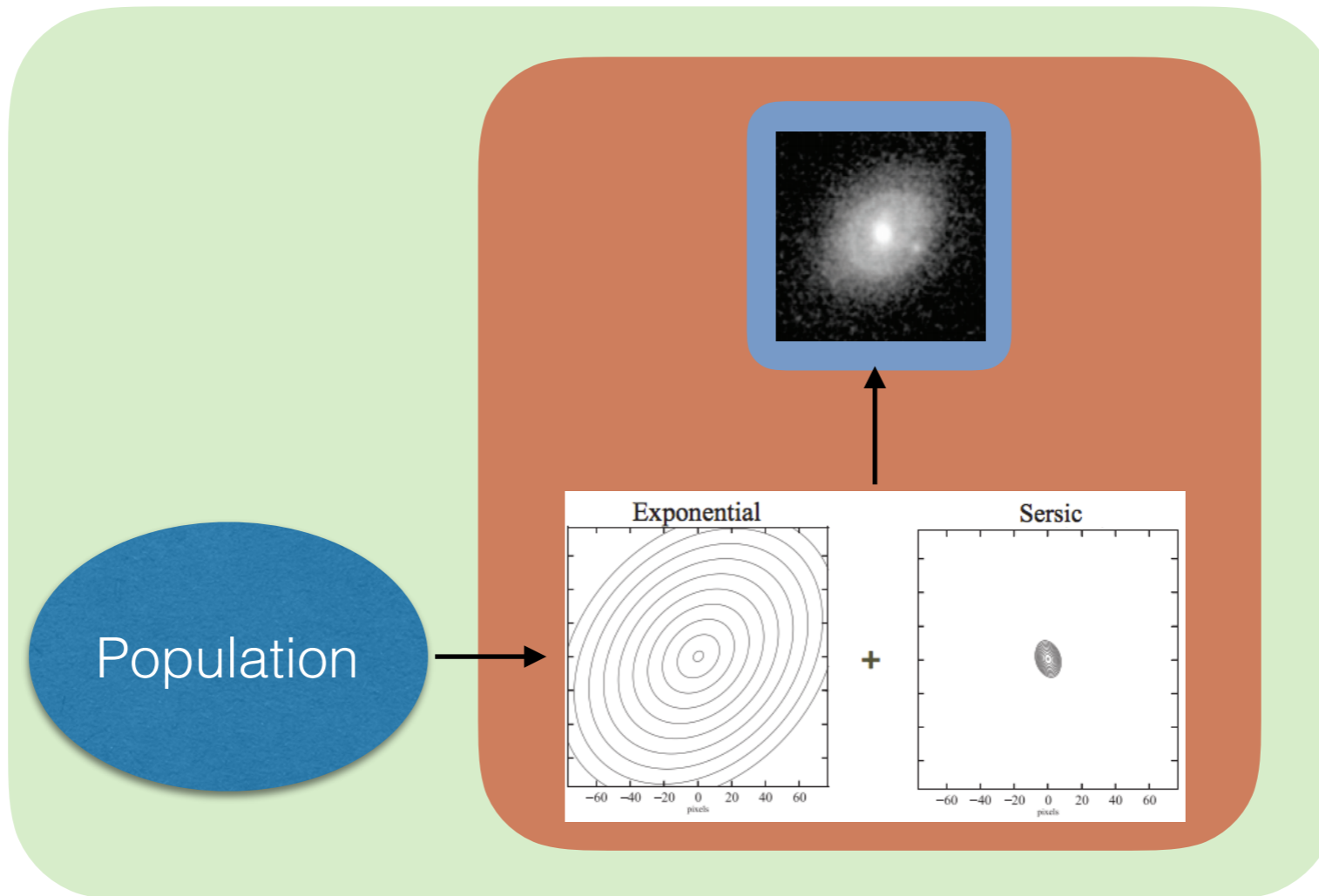
Bayesian Inference



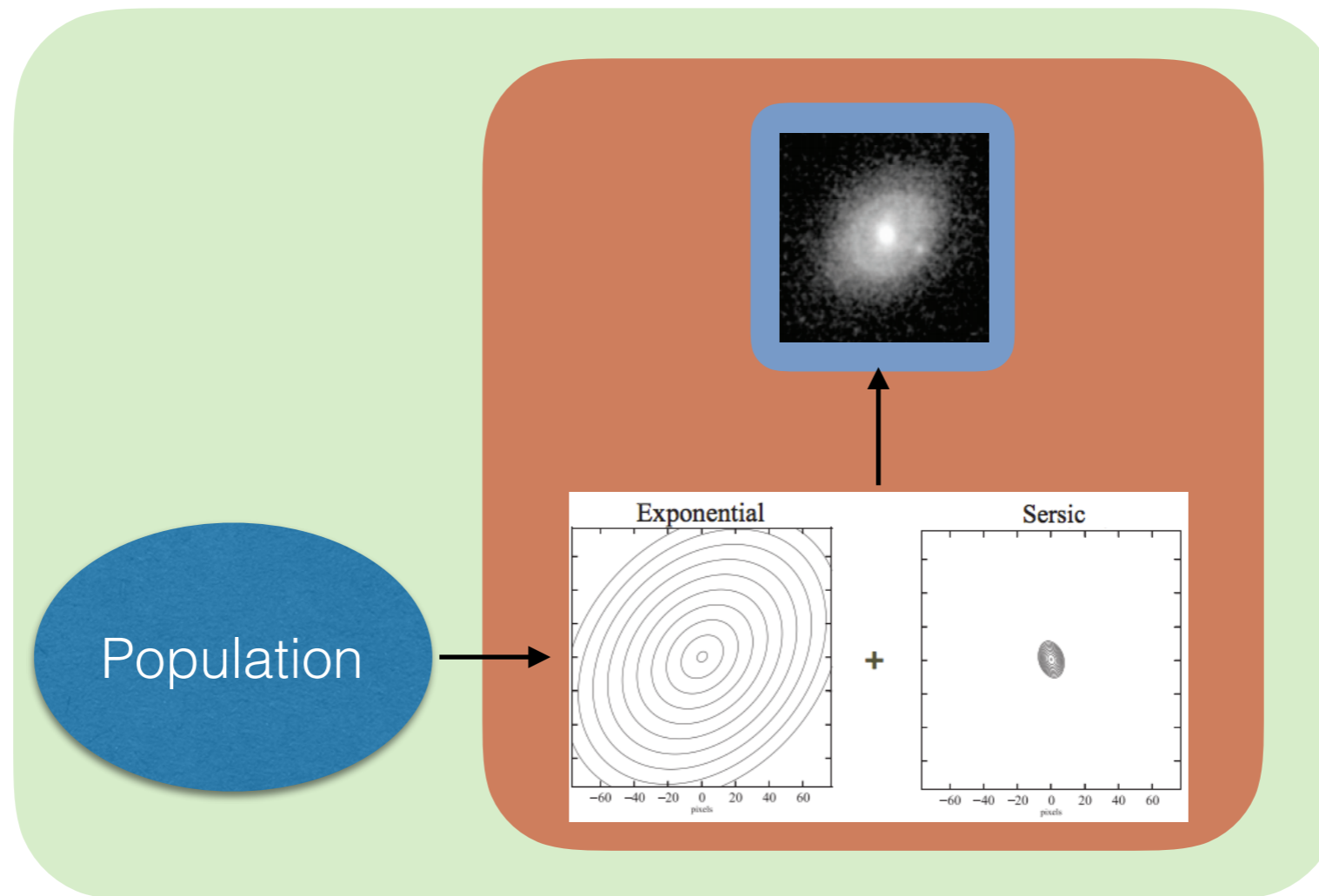
Hierarchical Bayesian Inference



Hierarchical Bayesian Inference



Hierarchical Bayesian Inference



Piece-wise constant representation*:

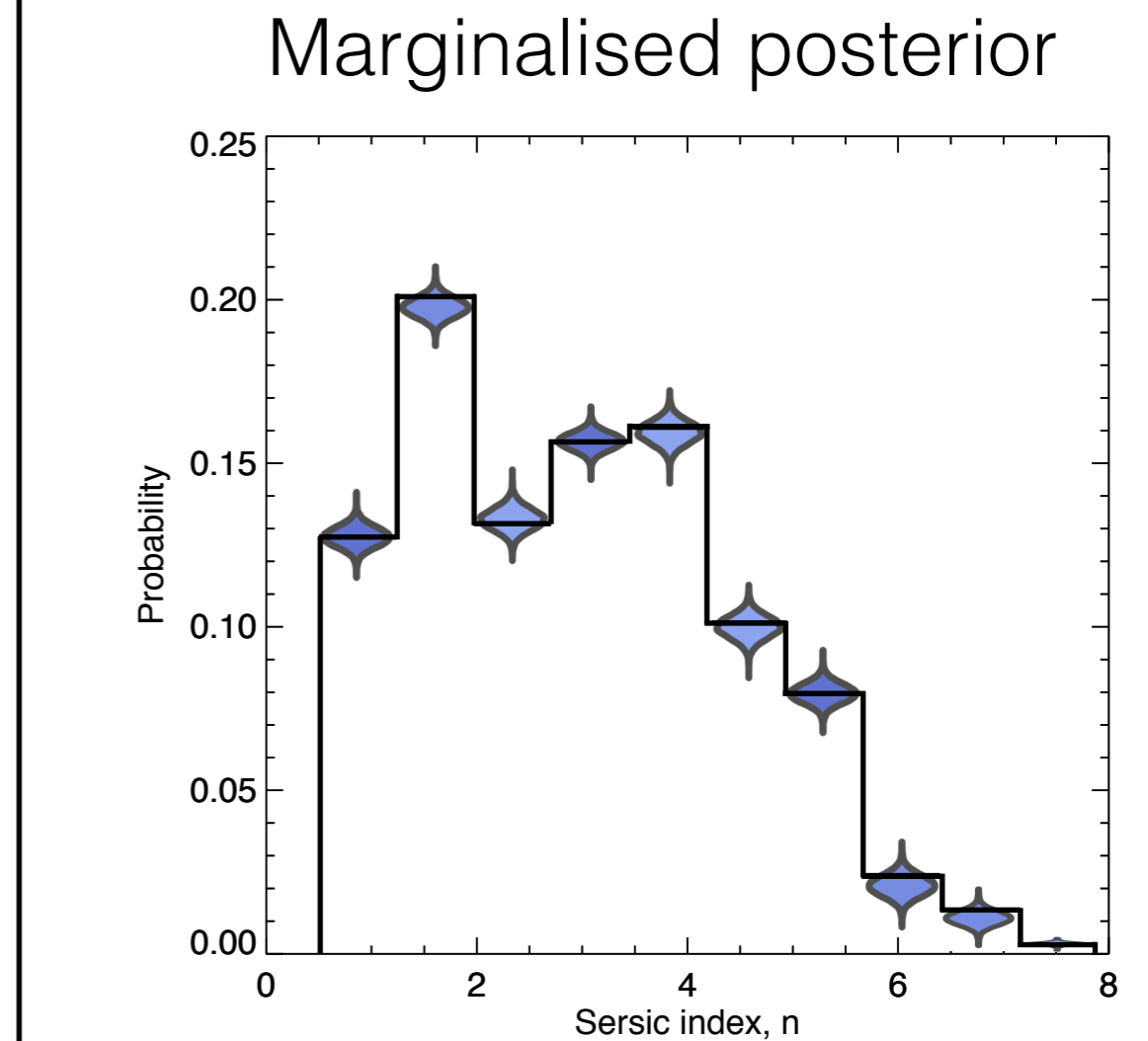
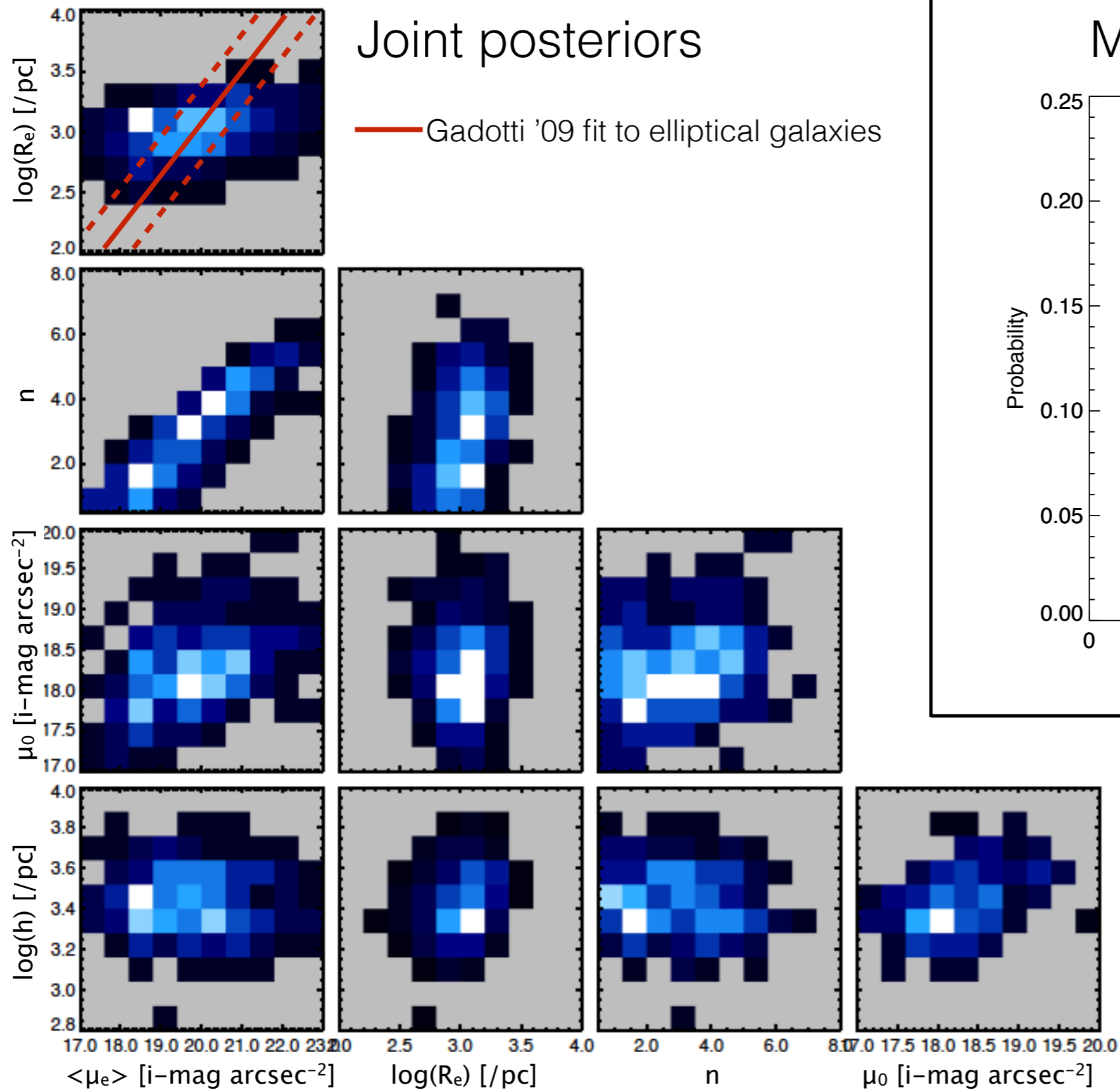
$$P(\theta | \{f_d\}) = \sum_d \frac{f_d}{(\theta_{d,max} - \theta_{d,min})} \times \Theta(\theta - \theta_{d,min}) \Theta(\theta_{d,max} - \theta)$$

Heaviside step function
↓

f_d - Probability of finding an object in the bin labelled d

* Mathematical description of a d-dimensional histogram

Hierarchical Bayesian Inference



Higher probability



Lower probability