Beyond Gaussian Fisher Matrix Forecasts

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Motivation:

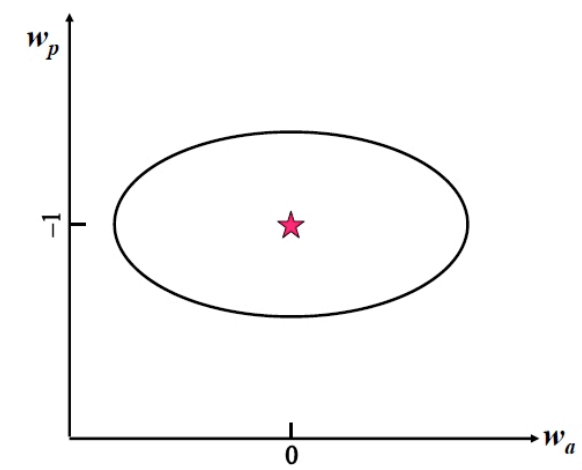
End of the first Golden Age of cosmology:
LSS: PSCz + 2dFGRS + SDSS +...
CMB: COBE + Boomerang + WMAP + ...
SN Ia: HST + SDSS-II + ...
Shear: CTIO + Combo-17 + COSMOS(1/2) + CFHTLens + ...

Where do we go next?
Plenty of ideas:
DE + MOG + INFLATION+...?

PLANCK+...	LOFAR+ ALMA+...	Wigglez+ BOSS+...	DES+ Panstarrs1+...
CMBPol+...	SKA+...	EUCLID+ JDEM+...	LSST+ Panstarrs4+...

How do we select the few from the many?
DE task force figure of merit (FOM) + ...

$$FoM = \frac{1}{\sigma_{w_0}\sigma_{w_a}} \rightarrow F^{-1}_{w_0 w_a}$$



The DETF figure of merit, which is defined to be the reciprocal of the area in the w_0–w_a plane that encloses the 95% C.L. region, is also proportional to $[\sigma(w_p)\times\sigma(w_a)]^{-1}$.

Forecasting cosmological parameters:

Fisher matrix formalism: Taylor expand the likelihood around its maximum....

$$F_{\alpha\beta} = -\left\langle \frac{d \log L(\{x_i\}|\{\theta_i\})}{\partial\theta_\alpha \partial\theta_\beta} \right\rangle$$

For Gaussian Likelihood function Fisher matrix becomes (Tegmark, Taylor & Heavens 1997):

$$F_{\alpha\beta} = \frac{1}{2}\mathrm{Tr}\left[C^{-1}C_{,\alpha}C^{-1}C_{,\beta}\right] + \mu_{,\alpha}C^{-1}\mu_{,\beta}$$

For LSS the lowest order statistic of interest is the Correlation function or the Power spectrum

$$\xi(r_{12}) = \langle\delta(r_1)\delta(r_2)\rangle \leftrightarrow P(k_1) = V_\mu \left\langle \delta_{\mathbf{k}_1}\delta_{\mathbf{k}_2}\right\rangle \delta^K_{k_1,k_2}$$

For the case of the power spectrum and Gaussian Random Field we have:

$$\mu_i = \langle P(k_i)\rangle \;\; ; \quad C_{ij} = \langle \delta P(k_i)\delta P(k_j)\rangle = \frac{2}{N_k} P_i^2 \delta^K_{ij}$$

Second term in Fisher matrix grows proportionally to the number of modes

$$\Rightarrow F_{\alpha\beta} \approx \mu_{,\alpha}C^{-1}\mu_{,\beta} = \sum_{i,j} P(k_i)\frac{\partial \log P(k_i)}{\partial\alpha} C^{-1} P(k_j)\frac{\partial \log P(k_j)}{\partial\beta}$$

Going beyond the Gaussian Fisher matrix....

Is the likelihood any where near a Gaussian?

The power in single mode is given by:

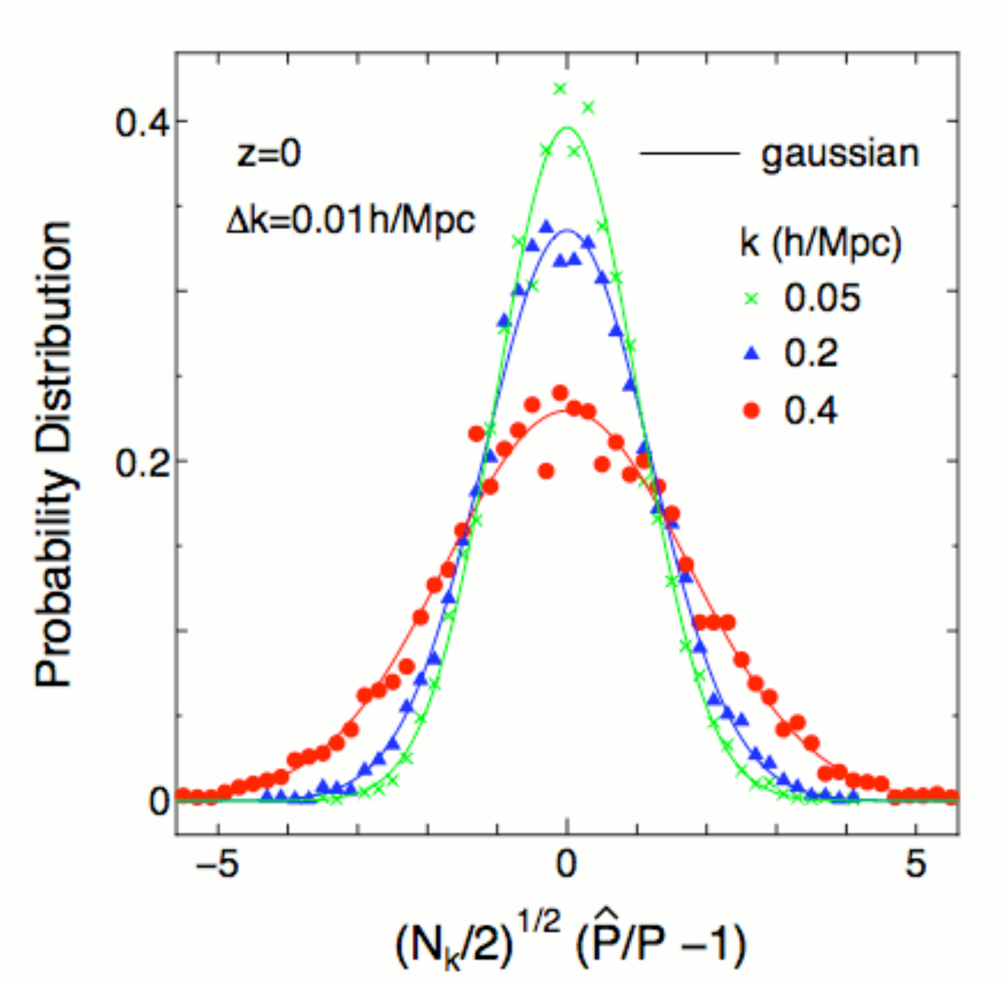
$$P(\mathbf{k}) = V_\mu \, |\delta(\mathbf{k})|^2$$

For a Gaussian random field we would expect exponential pdf

$$p(P)dP = \exp\left[-P/\langle P\rangle\right]/\langle P\rangle$$

However, what we measure is the estimator:

$$\hat{P}(k) = \frac{V}{N_k}\sum_i |\delta(k_i)|^2$$



(Takahashi et al 2009)

For a Gaussian random field the estimator distribution is, a chi-square:

$$p(\hat{P}) = \frac{1}{\Gamma[N_k/2]}\frac{1}{\hat{P}}\left[\frac{N_k}{2}\frac{\hat{P}}{\langle P\rangle}\exp\left[-\frac{\hat{P}}{\langle P\rangle}\right]\right]^{N_k/2}$$

Gaussian for large N

Beyond linear theory:

There are two major effects that complicate this simple model

(I) Nonlinear evolution couples Fourier modes and `information flows'

(II) Discreteness (sampling) corrections introduce additional terms to the covariance

=> Fully non-Gaussian covariance matrix

$$\overline{C}^d[k_i,k_j] = \frac{1}{V_\mu}\overline{T}[k_i,k_j] + \frac{2}{N_k}\left[\left(\overline{P}(k_i)+\frac{1}{\bar{n}}\right)\right]^2\delta^K_{k_i,k_j} + \frac{4}{N}\overline{B}(k_i,k_j) + \frac{2}{\bar{n}N}\left[\overline{P}[k_i,k_j]+\overline{P}(k_i)+\overline{P}(k_j)\right] + \frac{1}{\bar{n}^2 N}$$

(Scoccimarro et al 99)
(Meiksin & White 99)
(Smith 2009)

The N-body Simulations:

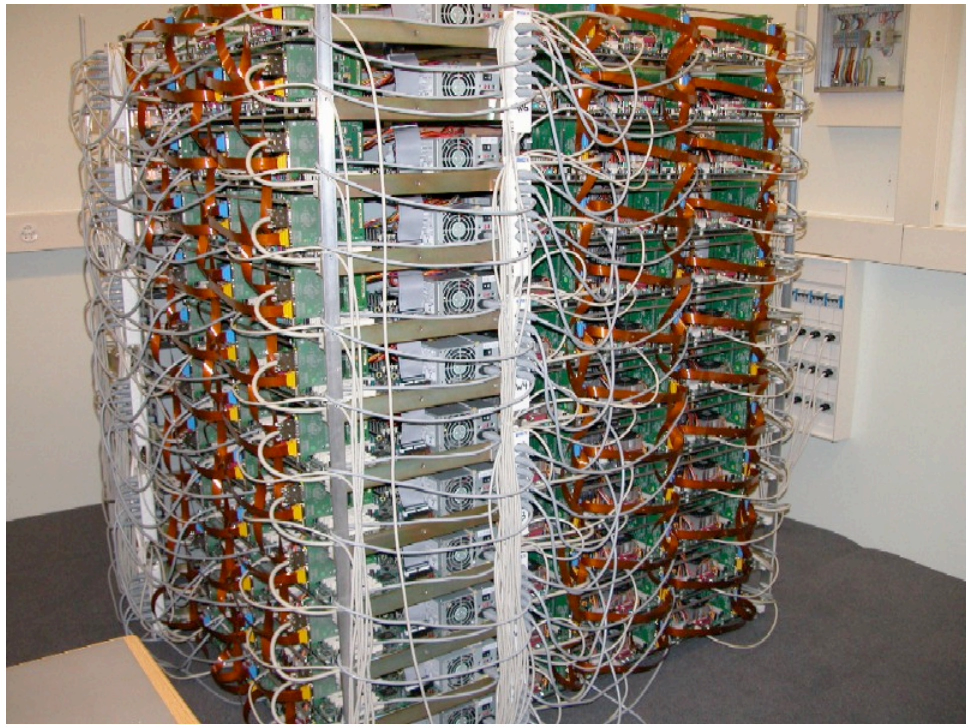
zHORIZON-I Simulations:

Ensemble of 40 simulations of cubical patch of the LCDM Universe, with cosmological parameters close to WMAP3 cosmology...

$$V = 1.5^3[\mathrm{Gpc}/h]^3,\ N = 750^3,\ \Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.8, n_s = 1.0$$

Using: GADGET-2, with 2LPT ICs, and CMBFAST Tfs.
Run on 128 to 256 processors of the zBox2/3 clusters

zBOX2

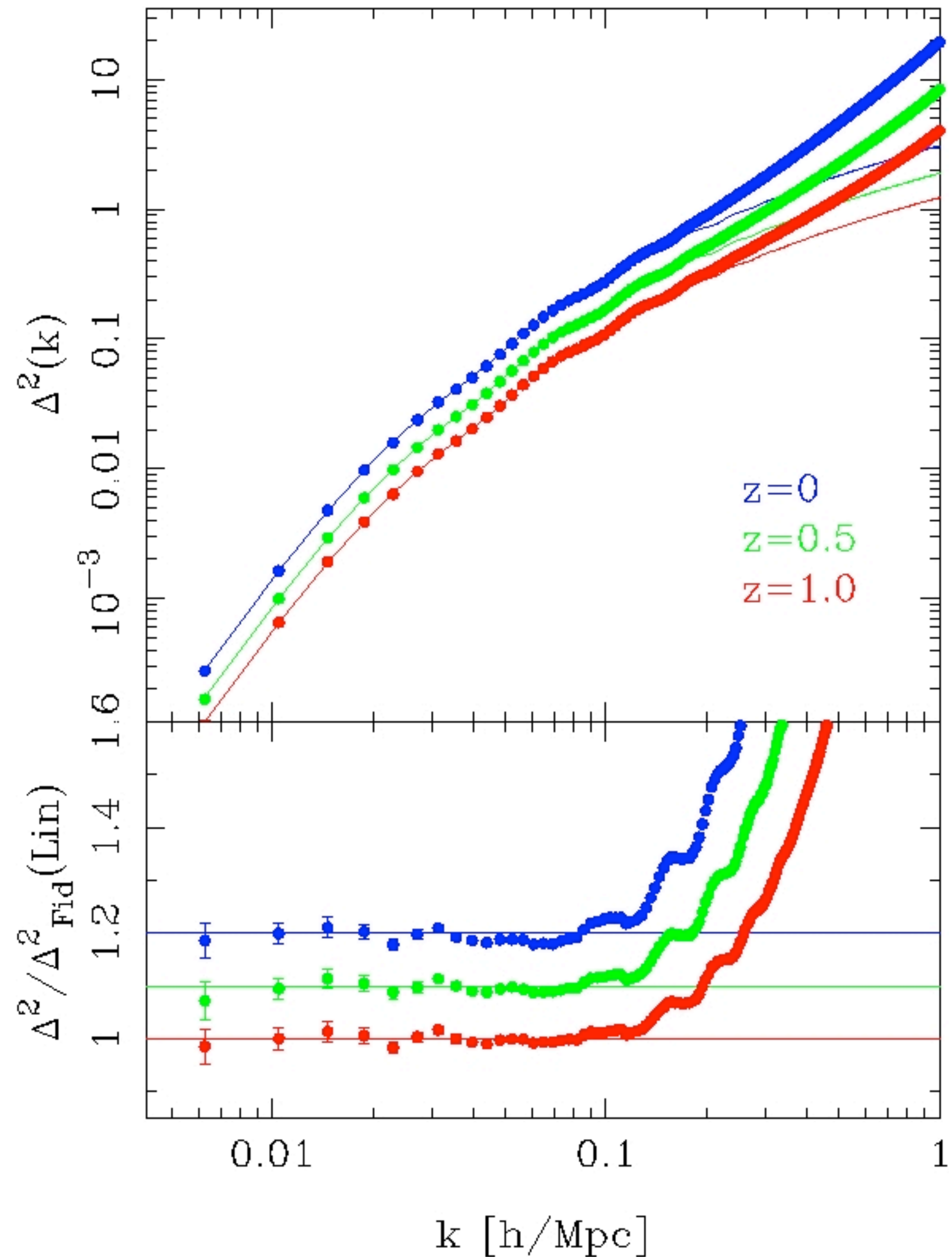


zBOX3

Evolution of Fiducial model:

Matter power spectrum

$$\Delta^2(k) = \frac{4\pi}{(2\pi)^3} k^3 P(k)$$



N-body Simulations II:

zHORIZON-Variants

: 8 cosmological models, 4 realizations per model.

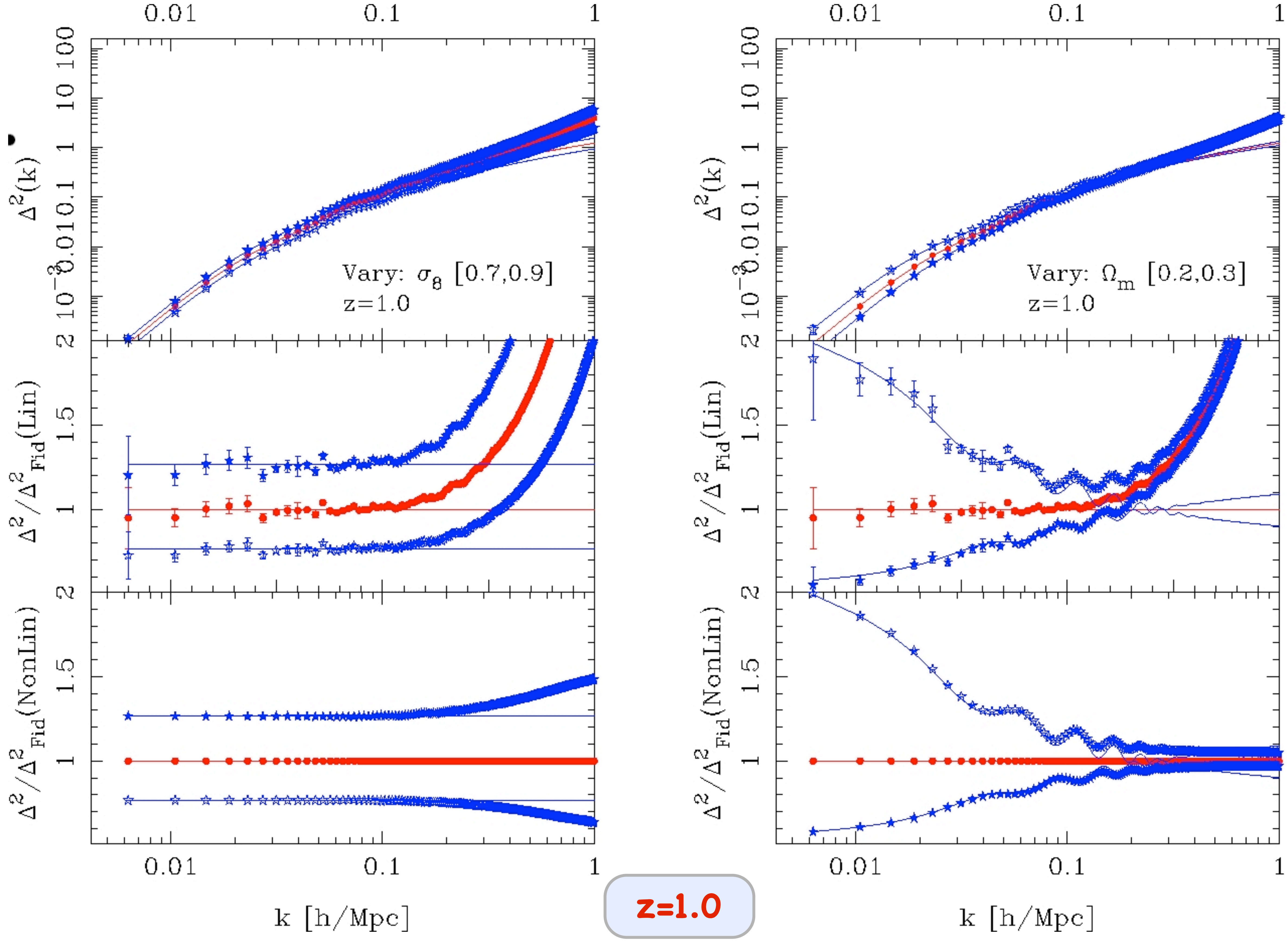
: Modification to a single cosmological parameter, all others held fixed

: Numerical parameters identical

: Initial Gaussian Random Field matched between models

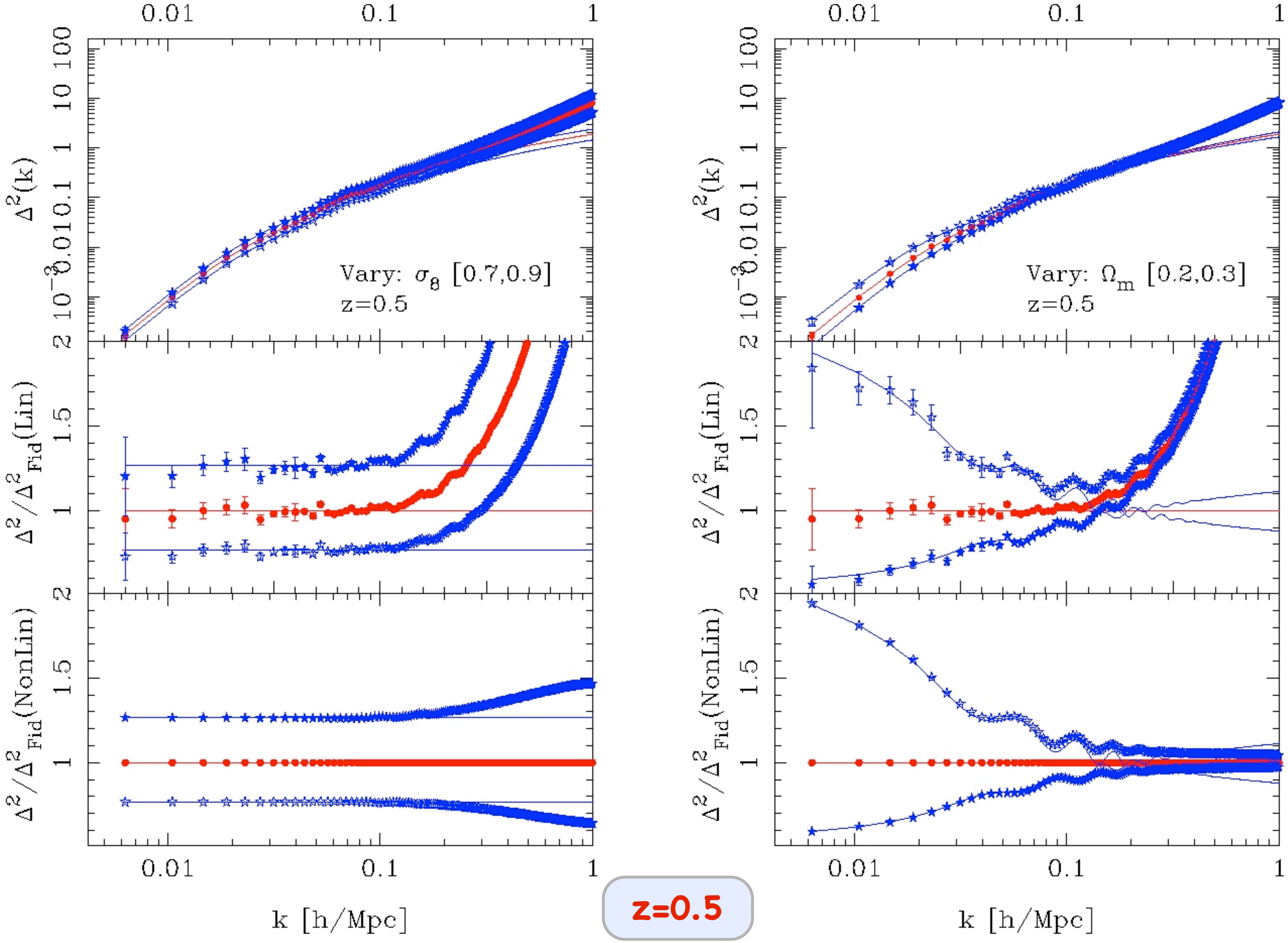
$$
\begin{aligned}
\mathrm{vIa} &= \{\Omega_m = 0.20,\ \Omega_{\rm DE} = 0.80, \sigma_8 = 0.8, n_s = 1.00, w_0 = -1.0\} \\
\mathrm{vIb} &= \{\Omega_m = 0.30,\ \Omega_{\rm DE} = 0.70, \sigma_8 = 0.8, n_s = 1.00, w_0 = -1.0\} \\
\mathrm{vIIa} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.7, n_s = 1.00, w_0 = -1.0\} \\
\mathrm{vIIb} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.9, n_s = 1.00, w_0 = -1.0\} \\
\mathrm{vIIIa} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.8, n_s = 0.95, w_0 = -1.0\} \\
\mathrm{vIIIb} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.8, n_s = 1.05, w_0 = -1.0\} \\
\mathrm{vIVa} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.8, n_s = 1.00, w_0 = -1.2\} \\
\mathrm{vIVb} &= \{\Omega_m = 0.25,\ \Omega_{\rm DE} = 0.75, \sigma_8 = 0.8, n_s = 1.00, w_0 = -0.8\}
\end{aligned}
$$

Evolution of variational cosmologies:



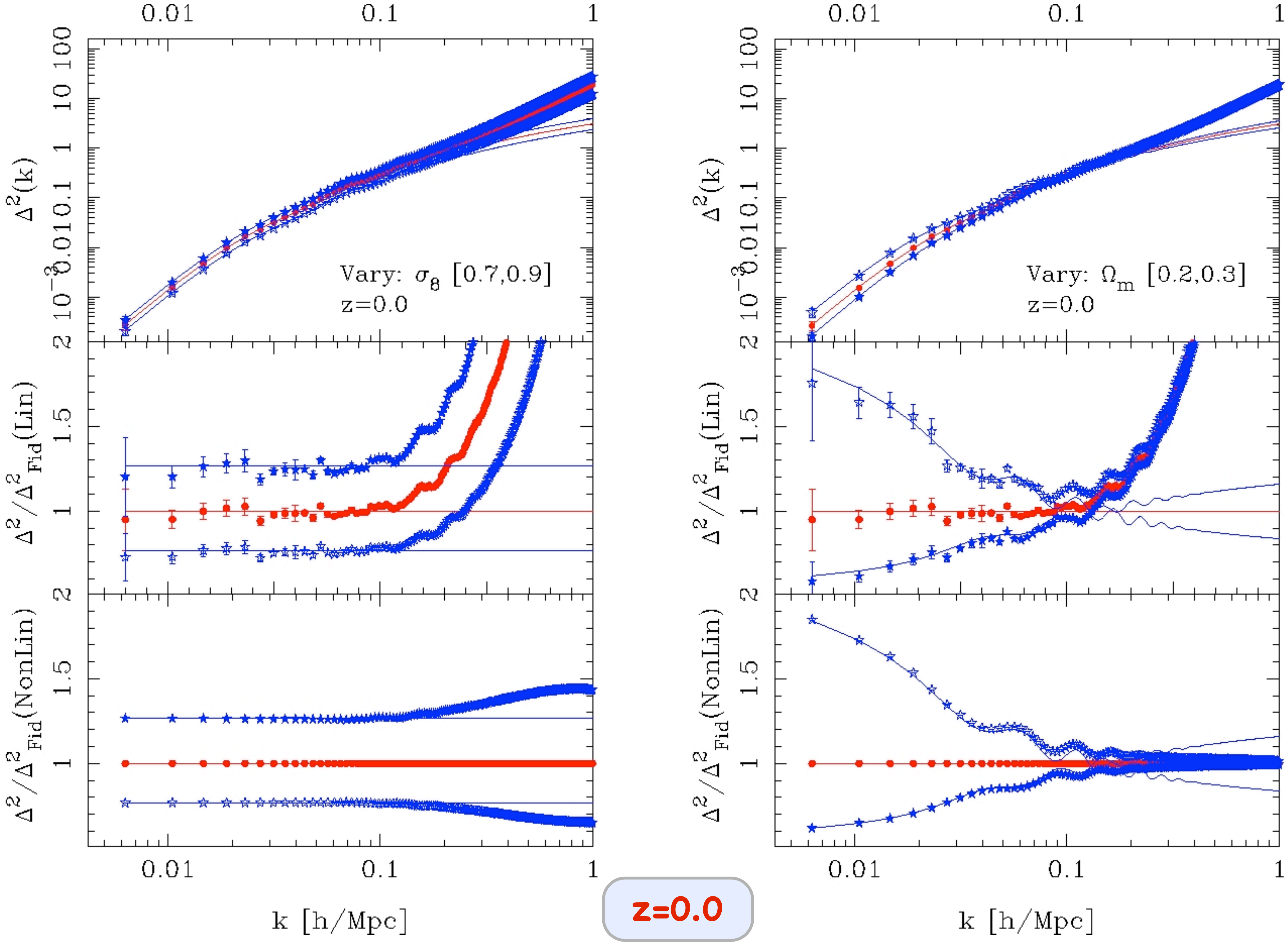
z=1.0

Evolution of variational cosmologies

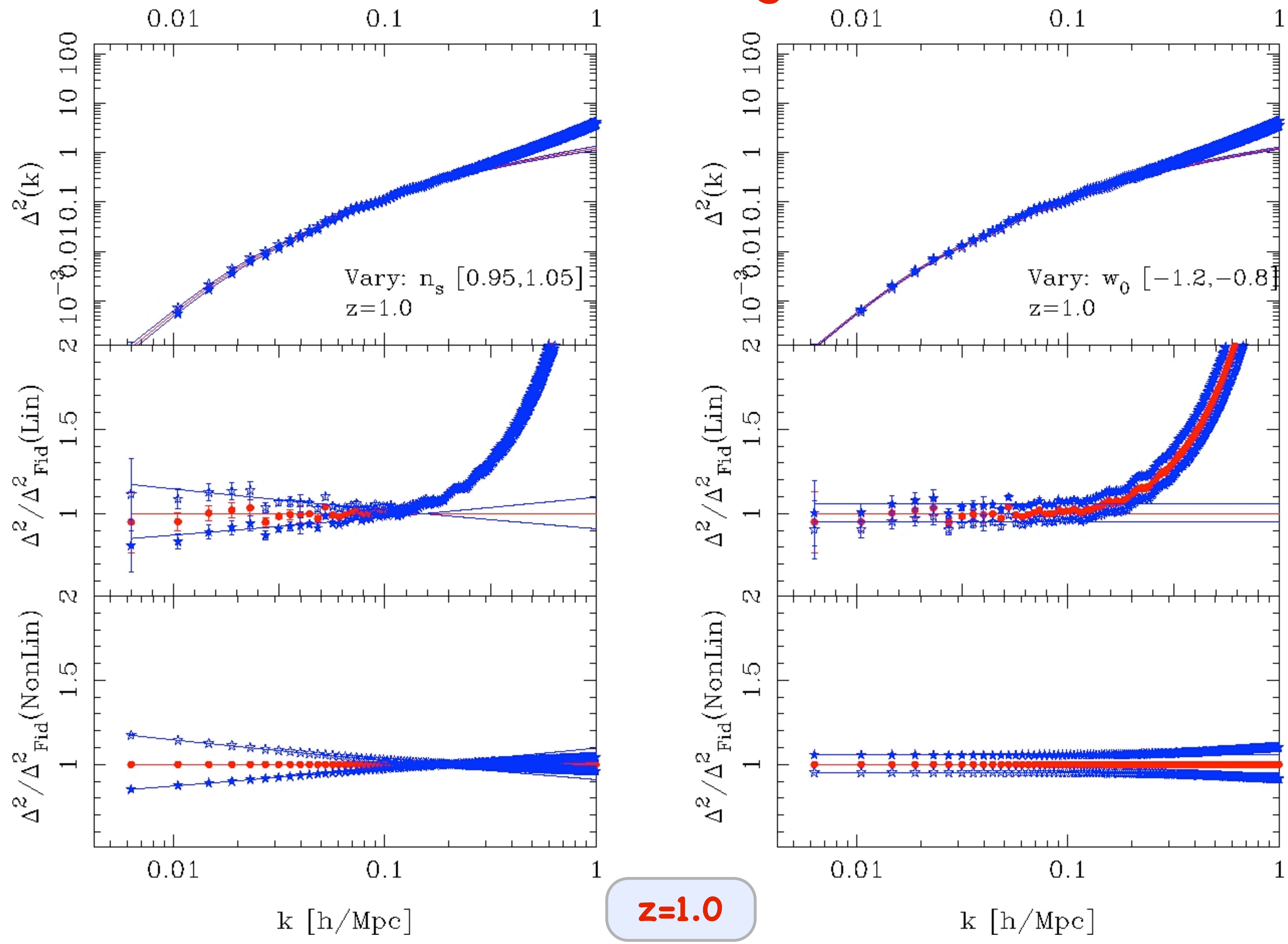


z=0.5

Evolution of variational cosmologies

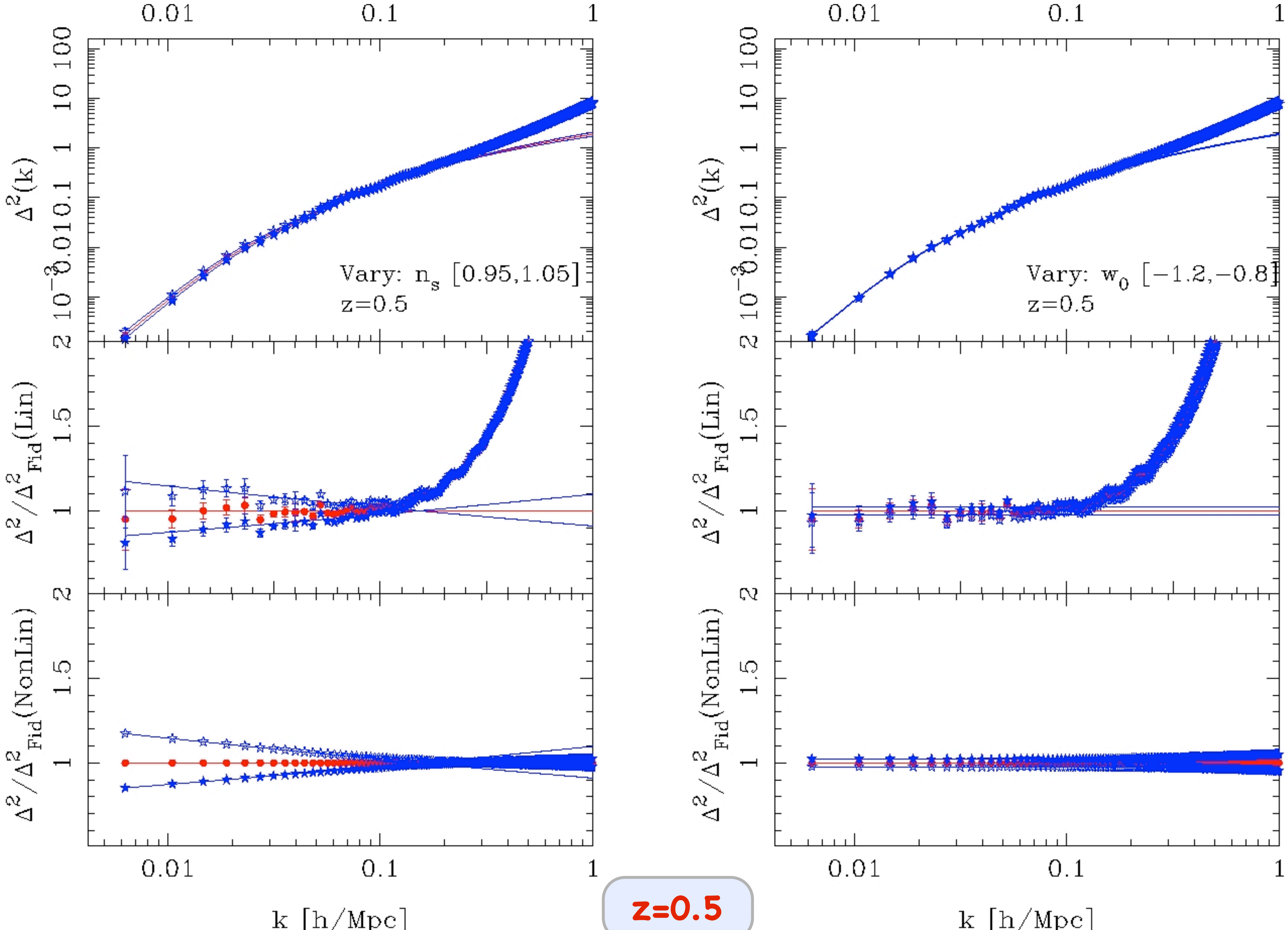


Evolution of variational cosmologies



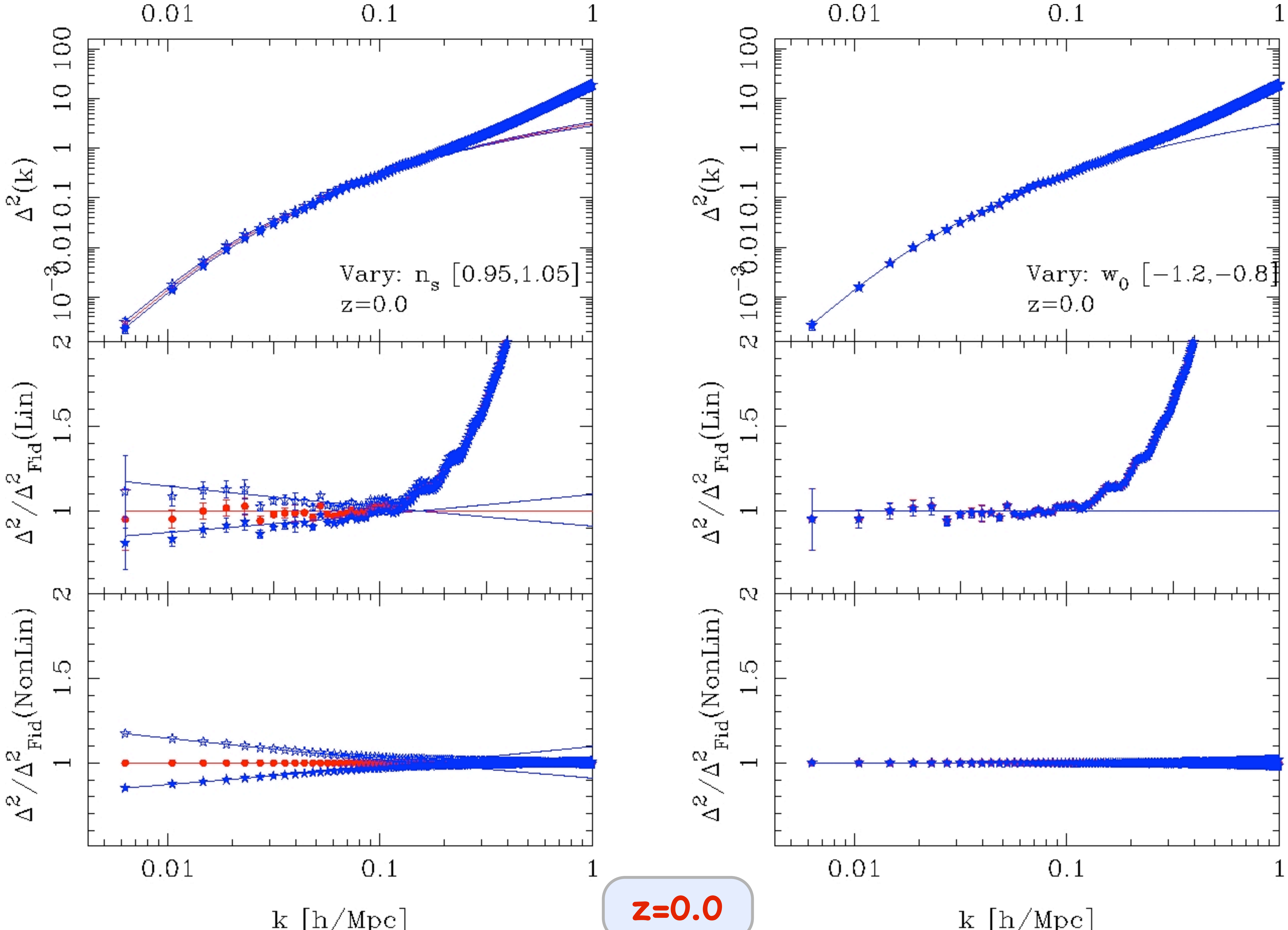
z=1.0

Evolution of variational cosmologies



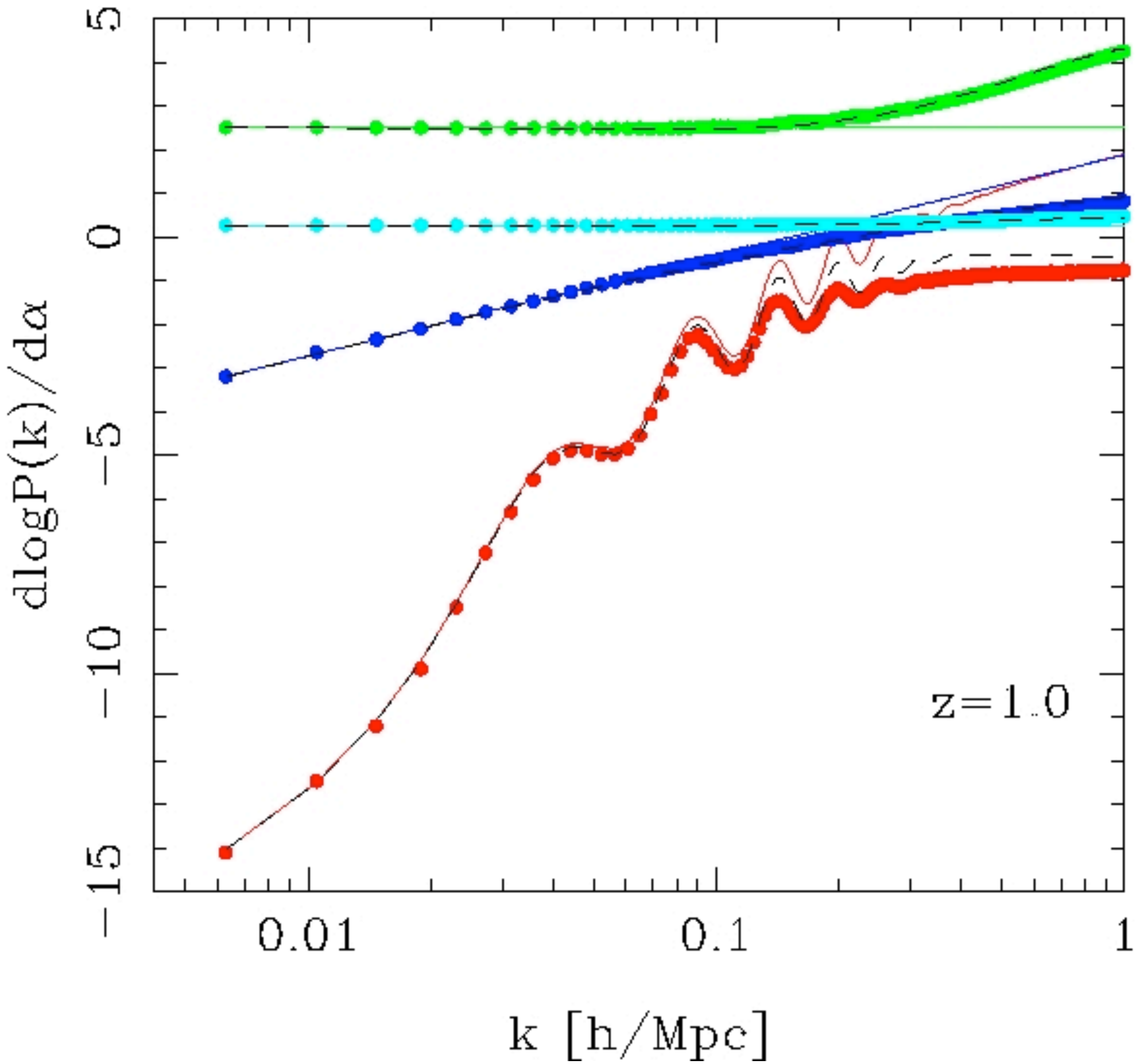
z=0.5

Evolution of variational cosmologies



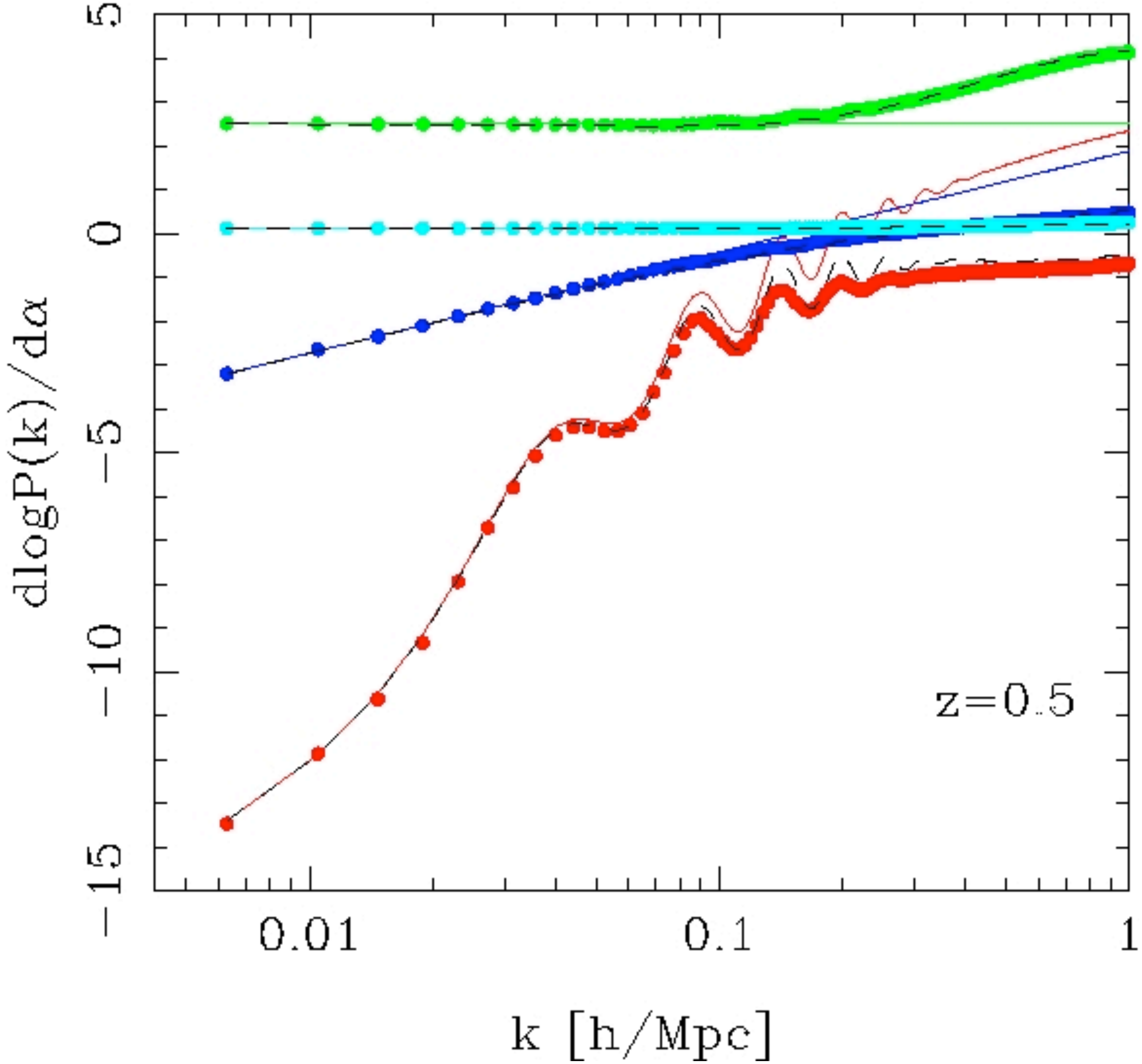
z=0.0

Fisher matrix derivatives:



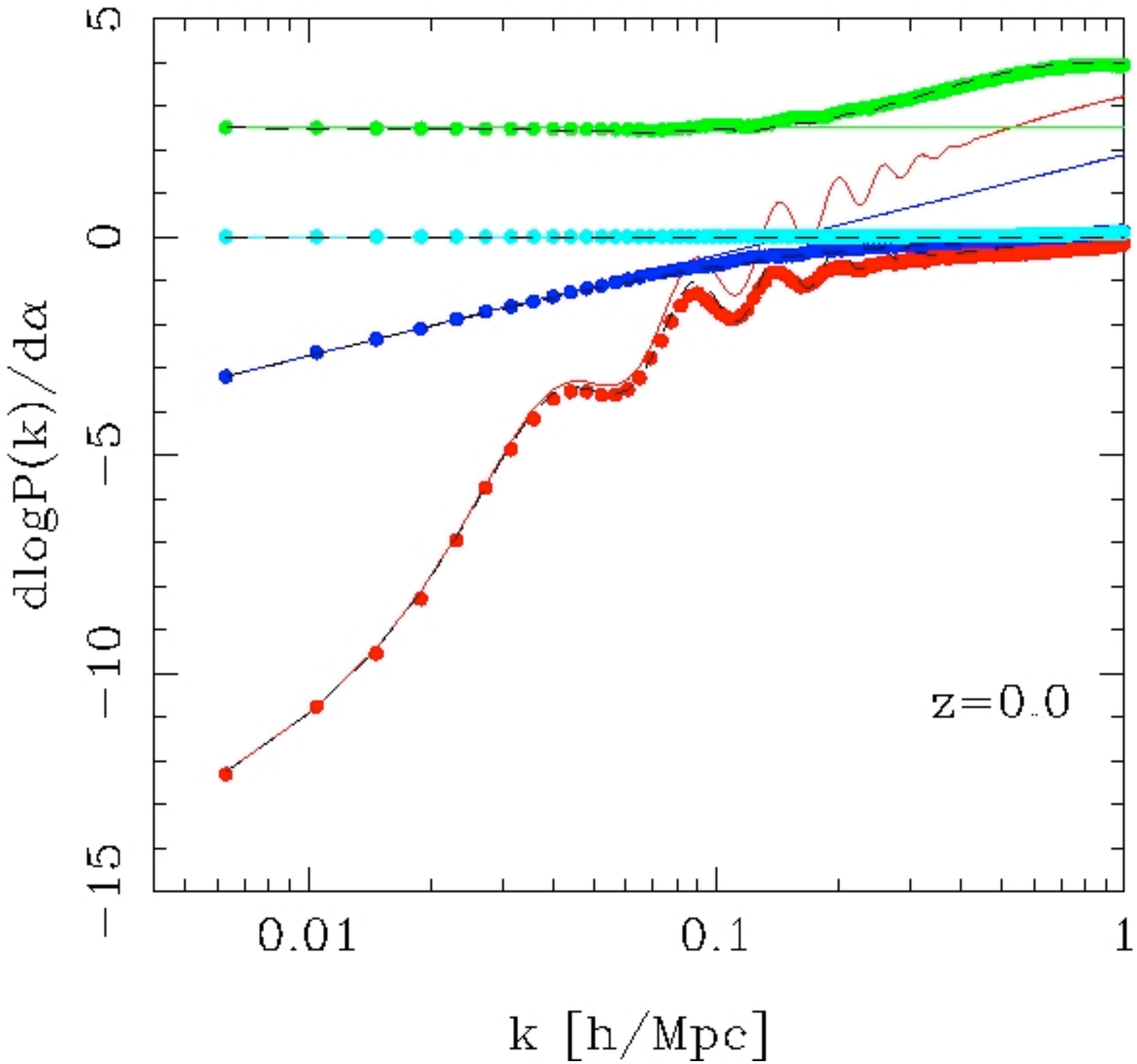
z=1.0

Fisher matrix derivatives:



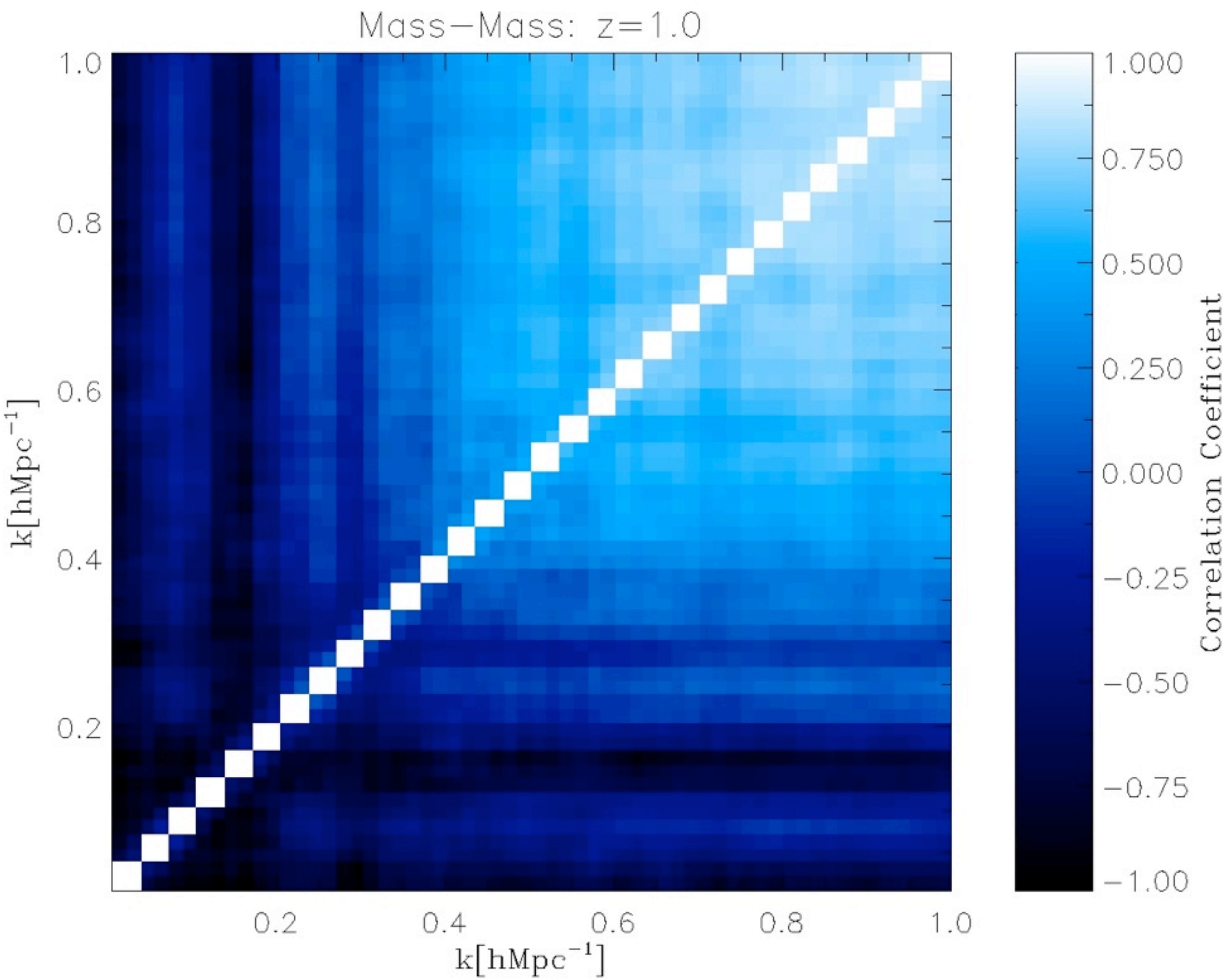
z=0.5

Fisher matrix derivatives:



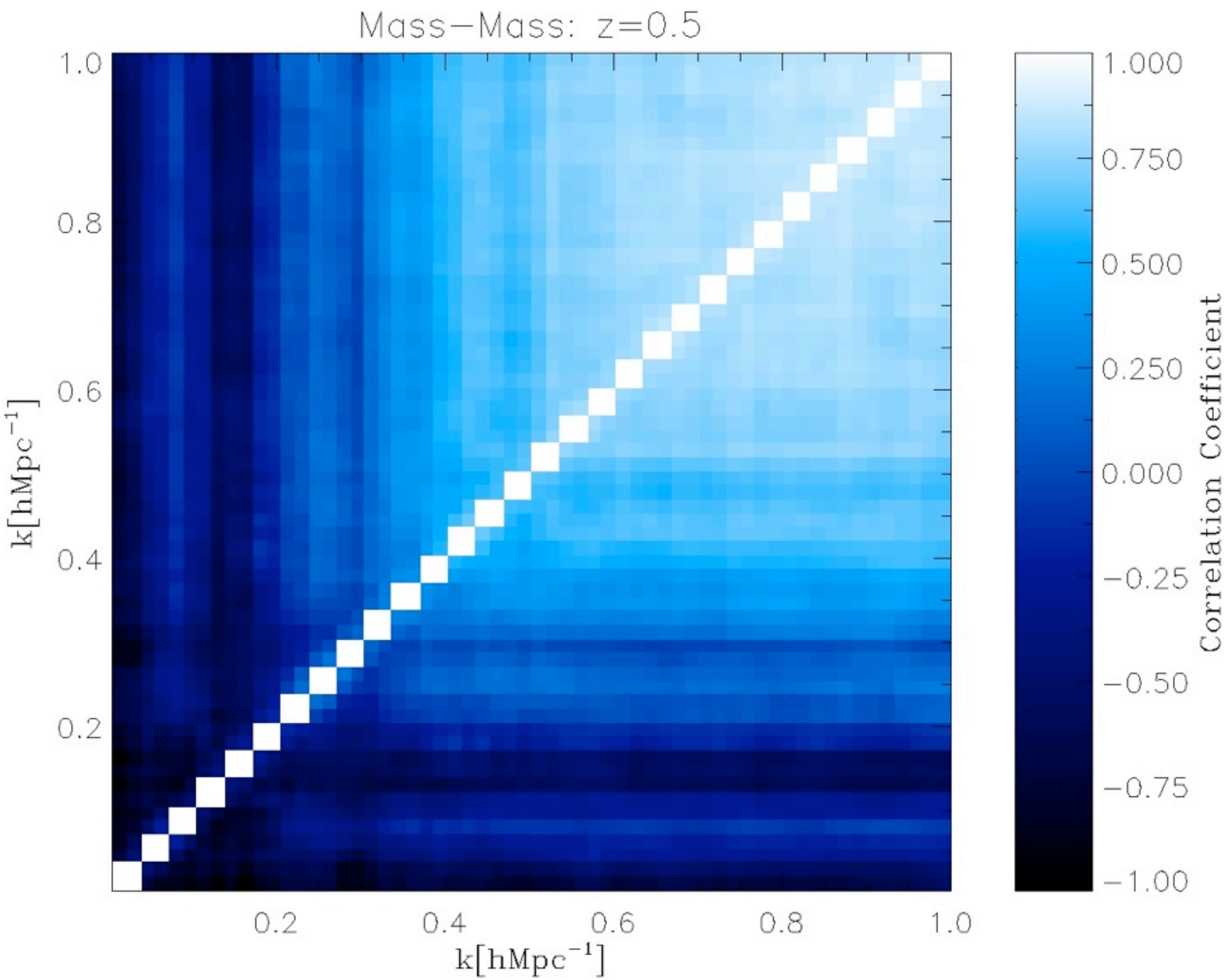
z=0.0

Power spectrum Correlation matrices:



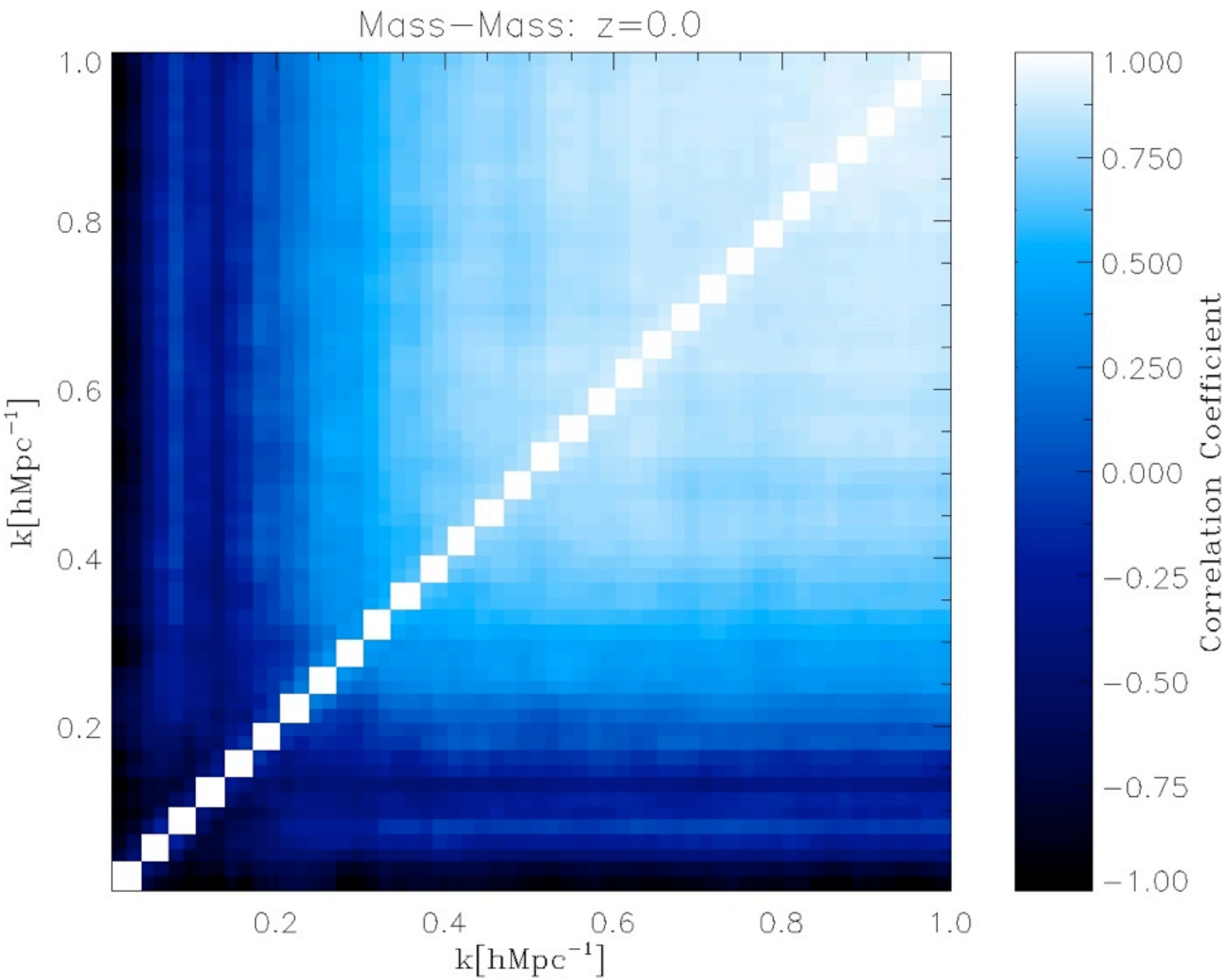
z=1.0

Power spectrum Correlation matrices:



z=0.5

Power spectrum Correlation matrices:



z=0.0

What about the Fisher information?

Simple survey case:

1: Suppose we have 3 galaxy surveys tracing structure at:
low (z=0) + intermediate (z=0.5) + high redshift (z=1)

2: Suppose all surveys cover same volume: V=3 [Gpc/h]^3 &
galaxies are densely sampled (galaxy bias=1 & nP>>1)

3: We measure 4 parameters from the data: $\{\Omega_m, \sigma_8, n_s, w_0\}$

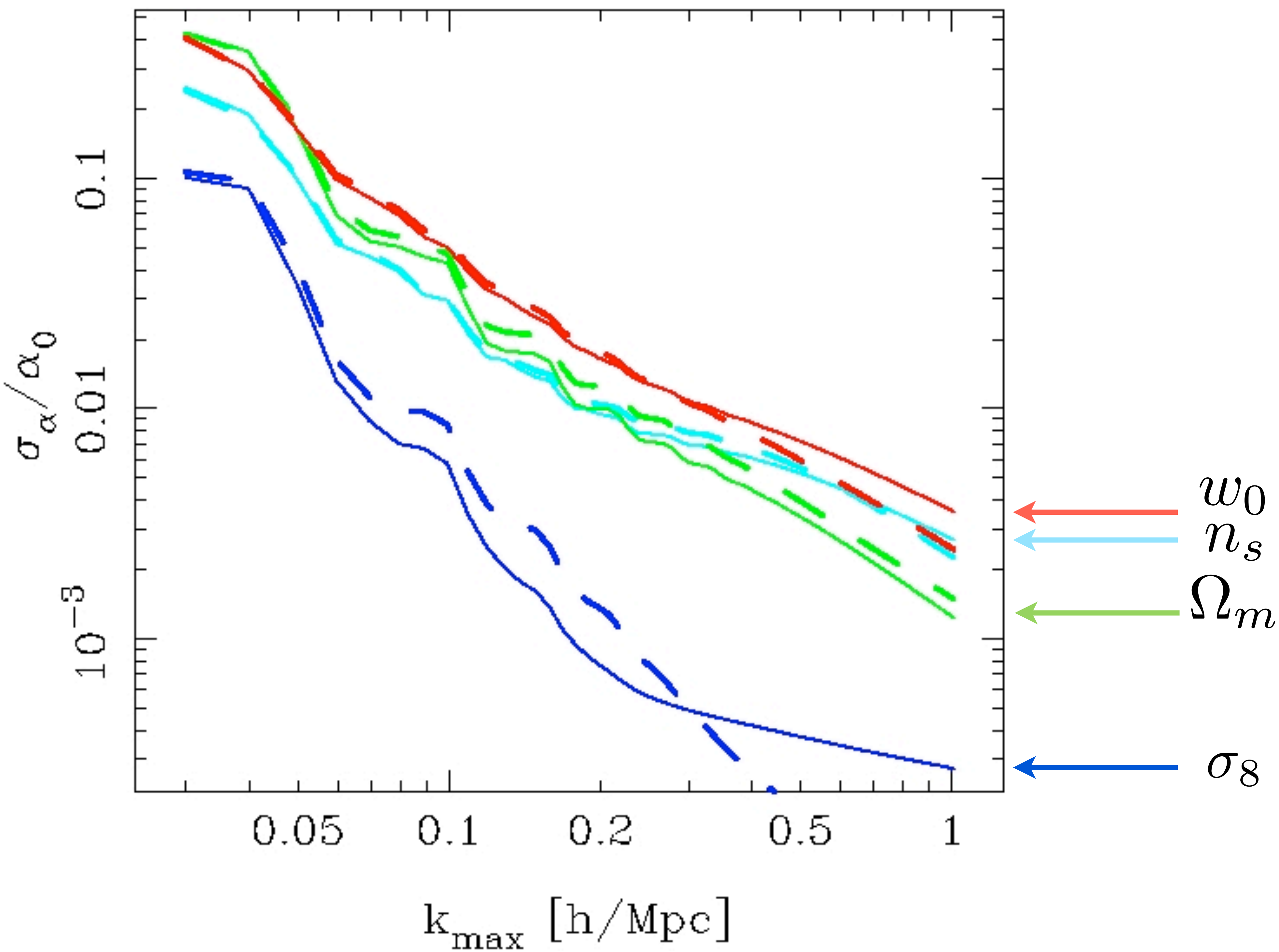
Assuming survey volumes are uncorrelated, then we add Fisher matrices
=> Total Fisher information (Important for constraining Dark Energy!)

$$F^{\rm TOT}_{\alpha\beta} = F_{\alpha\beta}(z=0.0) + F_{\alpha\beta}(z=0.5) + F_{\alpha\beta}(z=1.0)$$

1-D Marginalized errors:

(Solid lines) => Linear P(k) + Gaussian Covariance matrix

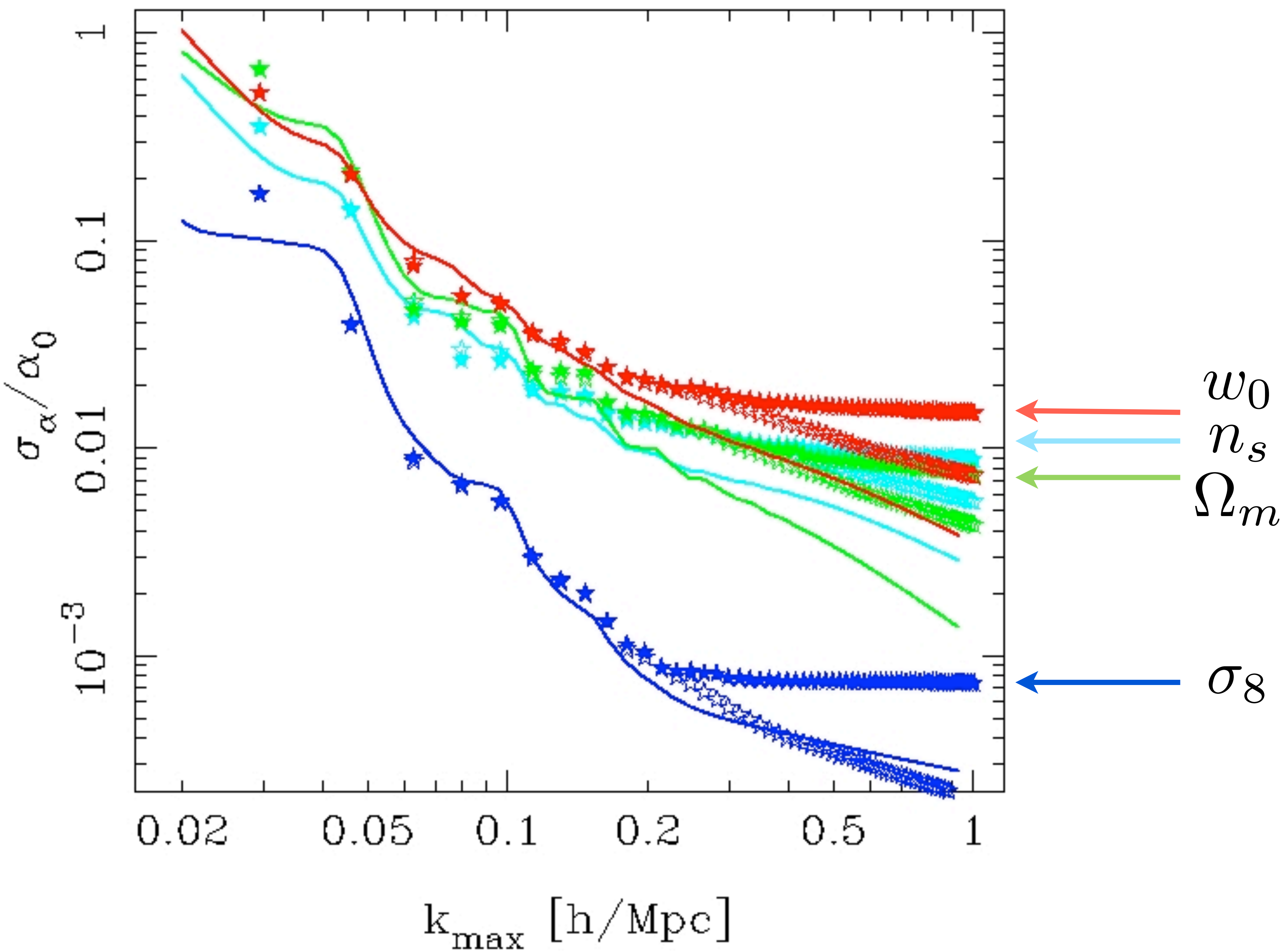
(Dash lines) => halofit P(k) + Gaussian Covariance matrix



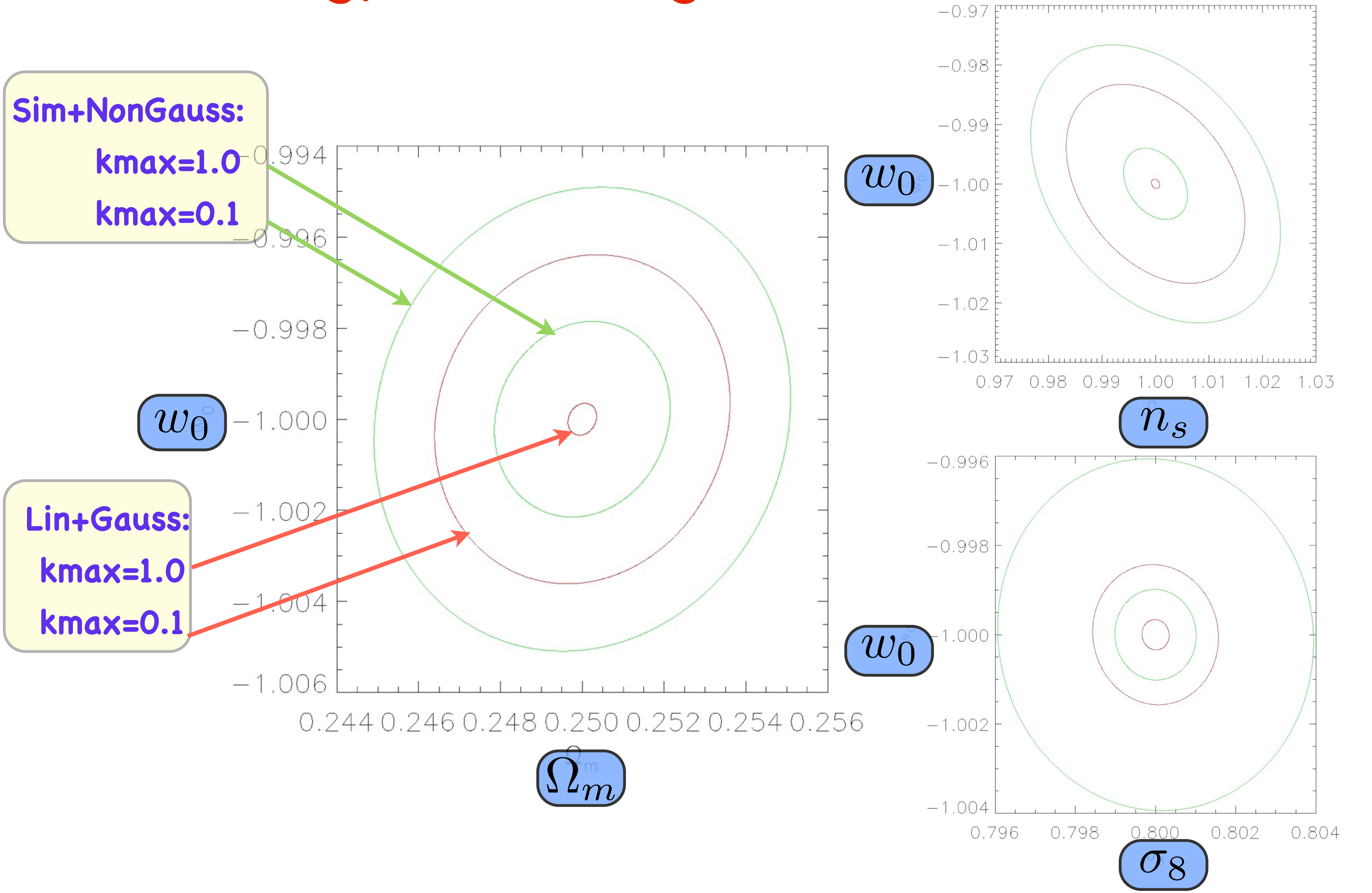
1-D Marginalized errors:

(Solid points) => measured P(k) + derivatives + Non-Gaussian Covariance matrix

(Open points) => measured P(k) + derivatives + Diagonal elements of Non-Gaussian Covariance matrix



Dark Energy: 2-D Marginalized errors:



Summary & Conclusions:

1: Fisher matrix formalism allows us to optimize future galaxy surveys

2: One must explore assumptions

3: We modeled Fisher matrix using simulations

4: Nonlinear evolution can erode information in P(k)
=> non-Gaussian covariance matrix
=> nonlinear P(k) derivatives

5: Parameter errors saturated at much lower k than currently thought (see also, Takada & Jain 2009, Takahashi et al 2009, Sato et al 2010)

Future directions: => Information flows into the higher order moments!

Forecasting cosmological parameters:

Linear Theory + Gaussian Case:

$$\Rightarrow F_{\alpha\beta} \approx \sum_{i,j} P(k_i)\frac{\partial \log P(k_i)}{\partial \alpha} C^{-1} P(k_j)\frac{\partial \log P(k_j)}{\partial \beta}$$

$$= \frac{1}{2}\sum_i N(k_i)\left[\frac{\bar{n}P(k_i)}{1+\bar{n}P(k_i)}\right]^2 \frac{\partial \log P(k_i)}{\partial \alpha}\frac{\partial \log P(k_j)}{\partial \beta}$$

$$= \frac{1}{2}\frac{V_\mu}{(2\pi)^3}\int d^3k\left[\frac{\bar{n}P(k)}{1+\bar{n}P}\right]^2 \frac{\partial \log P(k_i)}{\partial \alpha}\frac{\partial \log P(k_j)}{\partial \beta}$$

(Tegmark 1997)

Predictions for the marginalized errors on parameters,
and their covariances can then be obtained by inverting the Fisher matrix:

$$\mathrm{MVB} \Rightarrow \begin{cases} \sigma_{\alpha\alpha} = \sqrt{[F^{-1}]_{\alpha\alpha}} \\ \sigma_{\alpha\beta} = \sqrt{[F^{-1}]_{\alpha\beta}} \end{cases}$$

(For a great review see Heavens 2009)