Astrophysics 3; Semester 1; Worked Example

Chandrasekhar limit

A star of mass M_* and radius R_* is in hydrostatic equilibrium. We can show that the gravitational potential energy Ω can be written in terms of pressure integration as

$$\Omega = -3 \int_0^{R_*} P \cdot 4\pi r^2 dr.$$

On the other hand, for a uniform density case, we can show that

$$\Omega = -\frac{3GM_*^2}{5R_*}.$$

(a) Suppose that the star is supported by the pressure from relativistic degenerate electron gas, which is given as $P = Kn_e^{4/3}$ where K is a constant. The electron number density n_e is related to the mass density ρ as $n_e = \rho/(\mu_e m_p)$. Assuming uniform density and using the two relations given above, show that the mass of the star M_* is a constant for a given μ_e.

(b) Using $K = 2.45 \times 10^{-26}$ J m, calculate M_* in units of the Solar mass $M_\odot$ for the case of $\mu_e = 2$.