Astrophysics 3; Semester 1; Worked Example

2003-4 Class Exam, Question B2

The energy transport in a star is dominated by radiative diffusion in a layer at radius r where luminosity $L(r)$ at r is given by

$$\frac{L(r)}{4\pi r^2} = -\frac{4acT^3}{3\rho\kappa}\frac{dT}{dr}.$$

The star is in hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{\rho G M(r)}{r^2}.$$

In order for the layer to be stable against convection, the temperature gradient should be

$$\frac{dT}{dr} > \frac{\Gamma - 1}{\Gamma}\frac{T}{P}\frac{dP}{dr}.$$

(a) Show that there is an upper limit on the luminosity $L(r)$ for the stability against convection, and that the upper limit is proportional to $T^4 M(r)/(\kappa P)$. **[7]**

(b) The total pressure P is the sum of the ideal gas pressure $P_{\rm gas}$ and radiation pressure $P_{\rm rad} = \frac{1}{3}aT^4$. Denote the ratio of $P_{\rm gas}$ to P as β, i.e. $\beta \equiv P_{\rm gas}/(P_{\rm gas} + P_{\rm rad})$. Assume that this ratio β is constant throughout the star at any radius. Show that ρ is proportional to T^3, and write the factor T^4/P in the above upper limit of $L(r)$ using only β and a. **[7]**

(c) In the hydrogen and helium ionization zone in the outermost region of the Sun, κ becomes as high as 10^3 m^2 kg^{-1}, while $\kappa \sim 1$ m^2 kg^{-1} at a slightly inner layer outside the ionization zone. Show that the energy transport switches from via radiative diffusion to convection at this ionization zone. Assume $\beta = 0.997$ and $\Gamma = 1.66$. **[6]**