

Astrophysics 3; Semester 1; Worked Example

2006-7 Degree Exam, Question B1.2

The equations of stellar structure are:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \quad (\text{Continuity})$$

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2} \quad (\text{Hydrostatic Equilibrium})$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \epsilon \rho \quad (\text{Energy Generation})$$

$$\frac{L(r)}{4\pi r^2} = -\frac{4ac}{3\rho\kappa} T^3 \frac{dT}{dr} \quad (\text{Radiative Diffusion})$$

Explain the meaning of the variables in these equations, and briefly explain the origin of the equations. [6]

Assuming the perfect gas law, and using a dimensionless variable approach or otherwise, show that the hydrostatic equilibrium equation implies that the central temperature of the star scales as $T_c \propto M/R$. [4]

Under what conditions is the opacity of a star dominated by electron scattering? Assuming that it is, use the equation of radiative diffusion to derive an expression for the dependence of luminosity on M , T and R , and hence show that $L \propto M^3$. [5]

The radiation from main sequence stars may be well approximated by a black body. Express the luminosity of the star in terms of its radius and effective temperature. Assuming that the effective temperature scales with M and R in the same way as the central temperature does, determine how the radius and effective temperature scale with M . [2]

Explain why the dependencies derived in the previous two parts give rise to the observed properties of the main sequence. [3]