ASTROPHYSICS 3; SEMESTER 1
TUTORIAL 5: The Interstellar Medium

1. (a) Describe the two processes which change the colour of a Galactic star, between the moment the light leaves the star's surface and the moment it enters the ground-based telescope. Explain how these two processes can be distinguished.

   (b) A star of spectral class A0 has an observed $V - R$ colour of 0.50 and an apparent magnitude of V=5.0. Assuming an effective wavelength in the V and R bands of 5500Å and 6500Å respectively, and that the magnitude of extinction is proportional to λ^{-1}, what is the absorption in V-band towards this star?

   (c) The absolute magnitude in V of an A0 main sequence star is typically M_V=+0.6. Estimate the distance to the star.

2. A star produces ionising photons at a rate of $S_* = 2 \times 10^{45}\mathrm{s}^{-1}$, and is surrounded by a large hydrogen cloud.

   (a) Ignoring recombinations, derive an expression for the rate at which the ionised region expands if the density of the surrounding gas can be described as $n_\mathrm{H} = n_0(R/R_0)^{-2}$, where R is the distance from the star centre, R_0 is a charactestic radius, and n_0 is the hydrogen number density at radius R_0.

   (b) Using the radiative recombination coefficient, $\alpha = 3 \times 10^{-19}\mathrm{m}^3\mathrm{s}^{-1}$, derive an expression for the rate of recombinations within a shell of thickness ΔR at radius R from the star. Hence, show that the total rate of recombinations within the HII region is given by

$$\mathcal{R} = 4\pi\, n_0^2\, \alpha\, R_0^4 \left[\frac{1}{R_*} - \frac{1}{R_\mathrm{I}}\right]$$

   where R_* is the radius of the star and R_I is the radius of the ionisation front of the HII region.

   (c) Using the results of parts (a) and (b), derive an expression for the rate of advance of the ionisation front in the presence of recombinations, assuming $R_\mathrm{I} \gg R_*$. Taking values of $n_0 = 10^8\mathrm{m}^{-3}$, $R_0 = 10^{14}\mathrm{m}$, and $R_* = 2 \times 10^9\mathrm{m}$, how large will the HII region be after 10^4 years?

   (d) Show that there is a critical density, n_0, above which essentially no HII region develops. In this case, use the steady-state balance between recombination and ionisation to derive an expression for the size of the HII region. If $n_0 = 10^9\mathrm{m}^{-3}$, and other parameters are unchanged, calculate the maximum size of the HII region that will form.

Hand-in Question; 2009 Astrophysics 3 exam, B1.3

B1.3 (a) Explain why we observe emission lines of Hydrogen from the nebulae around massive stars. Why is the Hα Balmer line usually the strongest hydrogen line observed? **[3]**

(b) The temperature of the nebula around Star A is such that the ratio of fluxes of the Hα and Hβ emission lines (at wavelengths 656.3nm and 486.1nm respectively) emitted from the nebula is theoretically predicted to be $f_{H\alpha}/f_{H\beta} = 2.8$. However, the observed emission line fluxes are $f_{H\alpha} = 5.2 \times 10^{-15}\,\mathrm{W\,m^{-2}}$ and $f_{H\beta} = 8.4 \times 10^{-16}\,\mathrm{W\,m^{-2}}$. Explain how and why dust in the interstellar medium can produce this effect. **[2]**

(c) Show that the extinction in magnitudes at the wavelength of the Hα line is given by

$$A_{H\alpha} = -2.5\log_{10}\left(\frac{f_{H\alpha,\mathrm{obs}}}{f_{H\alpha,\mathrm{true}}}\right),$$

where $f_{H\alpha,\mathrm{obs}}$ is the observed flux of the $H\alpha$ emission line, and $f_{H\alpha,\mathrm{true}}$ is what it would have been in the absence of interstellar dust extinction. A similar expression will hold for $A_{H\beta}$. Assuming that the extinction law in the optical waveband can be approximated as $A_\lambda \propto \lambda^{-1}$, use these expressions and the observed and theoretical line ratios given in (b) above to show that the extinction at the wavelength of the Hα line is $A_{H\alpha} = 2.45$ magnitudes. Hence calculate the flux that the line would have been observed to have in the absence of extinction. **[6]**

(d) Star A has ionised a 'Strömgren Sphere' around it. Remembering that the rate at which each free electron undergoes recombination is $\alpha_{\mathrm{rec}} n_p$ (where $\alpha_{\mathrm{rec}} \approx 3 \times 10^{-19}\mathrm{m^3\ s^{-1}}$ is the radiative recombination coefficient and n_p is the number density of protons), and assuming that the nebula around the star has a constant density (initially n_H hydrogen atoms per $\mathrm{m^3}$), derive an expression for the number of recombinations inside the Strömgren sphere (radius R_S). Assuming that each recombination leads to the production of one Hα photon, show that the expected Hα flux from the nebula is

$$f_{H\alpha} = \frac{h\nu_{H\alpha} R_S^3 \alpha_{\mathrm{rec}} n_H^2}{3D^2},$$

where D is the distance to the star, and $h\nu_{H\alpha}$ is the energy of each Hα photon. **[6]**

(e) The Strömgren sphere around Star A is observed to have an angular radius of 10 arcsec, and the density is observationally determined to be $n_H \approx 10^9\mathrm{m^{-3}}$. Calculate the distance to the star. **[3]**