ASTROPHYSICS 3; SEMESTER 1
TUTORIAL 2: Various Estimates

You may need the following physical constants:

$m_{\rm H} = 1.673 \times 10^{-27}\,{\rm kg}$	(mass of the proton)
$m_{\rm He} = 6.643 \times 10^{-27}\,{\rm kg}$	(mass of a Helium nucleus)
$k_{\rm B} = 1.38 \times 10^{-23}\,{\rm J\,K^{-1}}$	(Boltzmann constant)
$h = 6.63 \times 10^{-34}\,{\rm J\,s}$	(Planck's constant)
$\sigma_{SB} = 5.67 \times 10^{-8}\,{\rm W\,m^{-2}\,K^{-4}}$	(Stefan–Boltzmann constant)
$G = 6.67 \times 10^{-11}\,{\rm N\,m^2\,kg^{-2}}$	(gravitational constant)
$c = 3.00 \times 10^{8}\,{\rm m\,s^{-1}}$	(speed of light)
$m_e = 9.11 \times 10^{-31}\,{\rm kg}$	(electron mass)
$\sigma_T = 6.65 \times 10^{-29}\,{\rm m^2}$	(Thomson cross-section)
$\epsilon_0 = 8.85 \times 10^{-12}{\rm N\,m^2\,C^{-2}}$	(permittivity of free space)
$e = 1.60 \times 10^{-19}{\rm C}$	(elementary charge)
$M_\odot = 1.99 \times 10^{30}\,{\rm kg}$	(mass of sun)
$R_\odot = 6.96 \times 10^{8}\,{\rm m}$	(radius of sun)
$L_\odot = 3.86 \times 10^{26}\,{\rm W}$	(luminosity of sun)
$r_{\rm N} \approx 10^{-15}\ {\rm m}$	(approximate size of atomic nucleus)

(a) Estimate the mean density and central temperature of the Sun.

(b) How long would it take the Sun to collapse under its own weight if its internal thermal pressure suddenly vanished?

(c) Estimate the temperature of the Sun's surface, assuming it radiates as a perfect blackbody. What is the wavelength at the peak of the blackbody spectrum for this temperature? What colour does it correspond to?

(d) Consider instead a Supergiant with a surface temperature of $T_{\rm eff} = 26000$ K and luminosity $L = 2.6 \times 10^5\, L_\odot$. Estimate its radius, assuming it shines as a perfect blackbody.

(e) If the average density of the Supergiant is the same as that of the Sun, what would its mass be? In fact its mass is only $25\, M_\odot$. How much smaller is its mean density than the Sun's?

(f) How long would it take a photon to escape from the centre of a Supergiant to the surface if it suffered no collisions?

(g) Estimate the mean free path of a photon between electron collisions in a Supergiant. Use the mean density from (e) above.

(h) Estimate the actual escape time for the photon going from the centre to the surface of the Supergiant, taking into account the number of scatterings the photon has to encounter.

(i) In the classical picture, estimate the velocity that two protons would have to have in order to undergo fusion. What effective temperature does that correspond to?

(j) The Maxwell-Boltzmann equation for the distribution of particle velocities in a thermalised gas is

$$f(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{0.5} \exp\left[\frac{-mv^2}{2kT}\right].$$

At what velocity does this peak in the centre of the sun? By what factor is $f(v)$ lower at the velocity for classical fusion than at this peak?

(k) Estimate the lifetime of the sun assuming it burns 10% of its hydrogen into helium.

Hand-in Question; Modified from Physics of Stars & Nebulae 2003 exam, B1, and Astro-3 2009 exam, B1.2

(a) Briefly explain the meaning of the following terms: *p-p chain*; *CNO cycle*. **[2]**

(b) During their main sequence lifetime, stars convert approximately 10% of their hydrogen into helium through nuclear fusion. Given that the proton mass is 1.6726×10^{-27} kg and the mass of a ^{4}He nucleus is 6.6465×10^{-27} kg, calculate the energy produced during each fusion of four protons into helium. Hence estimate the length of time that the sun will spend on the main sequence. **[5]**

(c) Suppose that the energy transport inside a main-sequence star is done by radiative diffusion. Photons are being generated at the centre of the star. The average number of scatterings a photon has to go through in order to diffuse from the centre to a point at radius r is $N = (r/l)^2$, where l is the mean free path of the photon. Assume that the opacity is dominated by electron scattering. Write l using Thomson scattering cross-section σ_T and electron number density n_e. Show that the diffusion time τ for the photons to diffuse from the centre to the surface of the star at radius R is proportional to M/R, where M is the mass of the star. **[6]**

(d) Assume that the luminosity L of the star is proportional to the radiation energy content of the star divided by the diffusion time above. Show that L roughly scales as $R^4 T_c^4/M$, where T_c is the central temperature of the star. **[4]**

(e) Show that the rough equality between thermal and gravitational energies of a star (the Virial theorem) implies that the central temperature of a star scales as $T_c \propto M/R$. Hence show that $L \propto M^3$. **[3]**