ASTROPHYSICS 3; SEMESTER 1
SOLUTIONS FOE TUTORIAL 1: Observational Astronomy

1. (a) The distance to the Sun is 1 AU = 1.496×10^{11} m. The luminosity L equals $L = 4\pi R^2 F = 4\pi(1.496 \times 10^{11})^2 \times 1410$W m^{-2} = 3.96×10^{26}W

   If we move the sun to 1pc, the luminosity is the same, so flux × (Distance)^2 = const. Therefore, the flux at that distance is 1410 × (1AU in m)^2/(1pc in m)^2 = 3.3×10^{-8} W m^{-2}.

   (b) The subtended solid angle is $\Omega = \pi\theta^2$ where θ is the angular radius in radians. The angular radius of the Sun is 0.25 degrees = $0.25 \times (\pi/180)$ radians = 4.36×10^{-3} radians. Therefore the subtended solid angle is $\pi(4.36 \times 10^{-3})^2$ = 6.0×10^{-5} sr.

   The intensity averaged over the solid angle $\bar{I}$ is $\bar{I}$ = Flux F/solid angle $\Delta\Omega$. More accurately, if we go back to the definition of flux and intensity, $F = \int I \cos\theta d\omega = \bar{I} \int \cos\theta d\omega \simeq \bar{I} \cdot \Delta\Omega$, where θ is the angle from the normal to the photon-collecting area at the Earth and in this case $\cos\theta \simeq 1$. Thus, we have $\bar{I} = 1410/6 \times 10^{-5} = 2.4 \times 10^7$ W m^{-2} sr^{-1}

   (c) Since Mercury is 0.387 times closer, the solar flux will be $(1/0.387)^2$ times higher, which is 9414 W m^{-2}. The angular size of the sun will be (1/0.387) times larger, so 0.5/0.387 = 1.29deg. The subtended solid angle scales as angular radius squared, so is $(1/0.387)^2$ times larger, so $6.0 \times 10^{-5}/0.387^2 = 4.0 \times 10^{-4}$ sr. Both of the flux and subtended solid angle are larger by the same factor — *the intensity of the solar radiation is the same there!*

   (d) If Pluto is in thermal equilibrium then the energy it absorbs from the sun equals the energy it radiates. The solar flux at Pluto is given by $F\times$ (1 AU / r_P in AU)^2. The surface area of Pluto as seen from the sun is πR_P^2, so the rate at which energy is absorbed is $F\times$ (1 AU / r_P in AU)$^2 \times \pi R_P^2$. Assuming Pluto behaves like a black body then its luminosity is $4\pi R_P^2 \sigma T^4$. Equating these gives $T = 45$K.

2. (a) Magnitude $m = m_{\rm inst} - 2.5log_{10}$(countrate), where $m_{\rm inst}$ is the magnitude of a star that produces 1 count per second. In this case, $10.7 = V_{\rm inst} - 2.5log_{10}(1350/10)$, and hence $V_{\rm inst} = 16.03$.

   (b) Using $V_{\rm inst}$ we now have $m = 16.03 - 2.5log_{10}(75/300) = 17.54$.

   Note that you could instead go directly from the first star measurements, since $m_1 - m_2 = -2.5log_{10}(\text{countrate}_1/\text{countrate}_2)$, and therefore $10.7 - m_2 = -2.5log_{10}((1350/10)/(75/300))$.

   (c) The flux zero-point ($f_{\rm ZP}$) is the flux density of an object of zero magnitude, ie. $m = -2.5log_{10}(f/f_{\rm zp})$. Hence, $f = f_{\rm zp} \times 10^{-m/2.5}$. The colours $B - V = 1$ and $B - R = 1.5$ mean that $B = 18.54$ and $R = 17.04$. Putting these in, $f_B = 6.6 \times 10^{-12} \times 10^{-18.54/2.5} = 2.5 \times 10^{-19}$ W m^{-2}Å^{-1}, and similarly $f_V = 3.5 \times 10^{-19}$ W m^{-2}Å^{-1} and $f_R = 2.6 \times 10^{-19}$ W m^{-2}Å^{-1}. The flux density in V is the highest.

   More generally, from $f = f_{\rm zp} \times 10^{-m/2.5}$, the colours $B-V = 1$ and $B-R = 1.5$ mean that if the flux zero points were the same in each band then the flux in B would be $10^{1/2.5} \approx 2.5$ lower than in V, and a factor $10^{1.5/2.5} \approx 4.0$ lower than in R. These factors can then be multiplied by the flux zero point differences to obtain the answer without needing to substitute in the exact magnitudes.

3. (a)The parallax is 0.002 arcsec, therefore the distance $D = 1/0.002 = 500$ parsecs. The absolute magnitude is defined as the apparent magnitude at a distance of 10 parsec. At that distance the star would have been 50 times closer, so would be 50^2 brighter. From $m_1 - m_2 = -2.5log_{10}(f_1/f_2)$ the magnitude would be $2.5\log_{10} 2500$ smaller. Thus, $M_V = 2.5 - 2.5\log(500/10)^2 = -6.0$.

   Or, simply from the distance modulus $m_V - M_V = 5\log D - 5 = 8.5$, $M_V = -6.0$.

   This is 6.6 magnitudes more luminous than an A0 main sequence star. The only stars with such high luminosities are supergiants.

   (b) The luminosity is increased by 50,000. Therefore the magnitude will go down by $2.5\log 50000 = 11.7$. This is for both the apparent and absolute magnitudes! Hence $M_V = -17.7$ and $V = -9.2$.

   (c) The expansion velocity of the ringlike structure is 10,000 km/sec, meaning that the radius increases by 20,000 km/sec. After 1 day, the ring has a diameter of $20,000 \times 24 \times 60 \times 60 = 1.73 \times 10^{12}$ metre. The distance is 500 pc $= 1.54 \times 10^{19}$ meter. Therefore the angular size is $1.73 \times 10^{12}/1.54 \times 10^{19} = 1.1 \times 10^{-7}$ radians $= 0.023$ arcseconds. This is too small to be resolved since the atmosphere tend to blur objects out to a size of typically $1''$ (this is known as *seeing*).

4. This is calculated very simply using the Wien's Displacement law:

$$T\lambda_{\rm max} = 2900\mu{\rm mK}.$$

   Betelgeuse $\lambda_{\rm max} = 1.160$ μm VERY RED
   Ross 128 $\lambda_{\rm max} = 1.036$ μm RED
   Sirius B $\lambda_{\rm max} = 0.363$ μm VERY BLUE
   Polaris $\lambda_{\rm max} = 0.468$ μm BLUE

5. The Planck function $B_\nu = 2h\nu^3 c^{-2}/(e^{h\nu/kT} - 1)$ gives the intensity which doesn't change over distance, and it is also isotropic, i.e. doesn't have a (θ, ϕ) dependence. Therefore, the flux density F_ν is given by B_ν $\times$ solid angle.

   The solid angle is equal to $\pi\theta^2$, where θ is the angular radius is radians given by $\theta = R/D$, where R is the physical radius of the star and D is the distance to the star. Hence, $F_\nu = B_\nu \pi R^2/D^2$, and here we have $\pi R^2/D^2 = 1 \times 10^{-17}$. $D = 5$pc (i.e. 1/0.2 from the parallax), and thus $R = 2.8 \times 10^5$km.

6. (a) The best time to observe is when the sun is 12 hours away from the system, ie when the sun is at RA=2.6. By definition the sun has RA=0 on March 21st, and increases by 2 hours per month. Therefore best time is $2.63/2 = 1.3$ months after 21st March, ie. around 30th April.

   (b) The system is northern, so the WHT is better.

   (c) Offset in declination = 39:15:24 – 39:14:56.3. This is $84 - 56.3 = 27.7$ arcsec [where 84 arises from 60+24, due to difference in the arcminute coordinate]. Offset in right ascension is 14:37:42.27 - 14:37:39.83 = 2.44 seconds. In arcsec this is $15 \times 2.44 \times \cos({\rm Dec}) = 28.3$ arcsec. Total offset between the two stars is $(27.7^2 + 28.3^2)^{1/2} = 39.6$ arcsec. Star 2 is to the south (lower Dec) and west (lower RA) of star 1. On a sketch (where N is up and East to the left) that places star 2 down and to the right (by roughly equal amounts) relative to star 1.