# Astrophysics 3, Semester 1, 2011–12

## Physics of Nebulae (1): The ISM and HII regions


**Philip Best**

**Room C21, Royal Observatory; pnb@roe.ac.uk**

**www.roe.ac.uk/∼pnb/teaching.html**


### 1 An overview of the interstellar medium

The interstellar medium (ISM) is a component of great significance in astronomy. It is:

(1) the birthplace of stars
(2) the stellar graveyard – leading to 'metal' enrichment (Nebulae)
(3) the dynamic arena of stars: HII regions, winds, supernova remnants
(4) a significant part of Galaxy: $M_{\rm gas} \simeq 0.05 M_{\rm Galaxy}$

Its major components are as follows (you'll study these much more in Semester 2):

1. **Neutral Hydrogen** ($H^0$ or HI regions). This is revealed by 21cm radiation. Neutral hydrogen constitutes $\gtrsim 90\%$ of total mass of ISM. It typically has $T \simeq 80$ K with $n_H \simeq 3\times10^6$ m$^{-3}$, although denser clouds are colder ($T \simeq 30$ K). It is concentrated along the spiral arms of Galaxy.

2. **Molecular Hydrogen** ($H_2$). $H_2$ forms in the densest regions of HI clouds, where $T \simeq 30$ K and $n(\rm H_{tot})$ reaches $10^9 - 10^{12}$ m$^{-3}$.

3. **HII Regions** ($H^+$). These are located around hot, blue stars (O & B). In these regions, typical densities are $n_e \simeq 10^8 - 10^{10}$ m$^{-3}$ near O stars, and $n_e \simeq 3 \times 10^5$m$^{-3}$ near B stars and planetary nebulae. Temperatures are typically $T \simeq 8000$ K. HII regions contribute about 10% of the total mass of the ISM, and are concentrated along the Galaxy's spiral arms.

4. **Dust**. The ISM contains small particles or grains (mostly carbonates and silicates), of radii $< 1\mu$m. These are revealed by extinction, as we will discuss below: the apparent magnitudes of stars in HI clouds are typically reduced by 4 magnitudes, because of the dust content of the clouds. $M_{\rm dust} \simeq M_{ISM}/160$.

The **abundances** of the ISM are about 70% H and 30% He by mass – similar to Sun. 1–2% of the mass is in **metals** (C, N, O, Fe, Si, Na, Mg, etc.) but with different relative ratios compared with the Sun. This is believed to be due to condensation of some of the metals into dust grains – some elements solidify more easily than others.

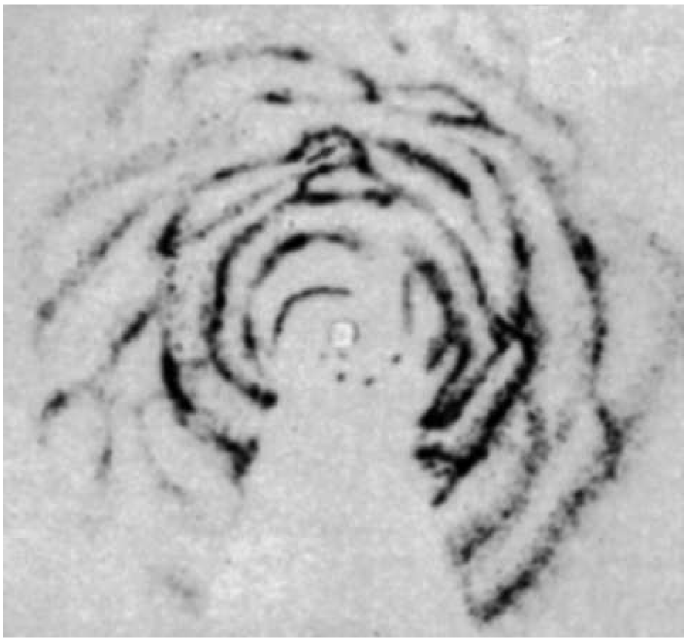


Figure 1: The distribution of neutral hydrogen in the Milky Way, as measured by the 21cm emission line. The location of the Sun (8 kpc from the centre) is shown by the arrow in the upper centre. The concentration of neutral hydrogen towards spiral arms is very clear.

## 2 Extinction

The interstellar dust grains absorb light from stars on the way to the Earth. For a path element $ds$, we can write that

$$dI_\lambda = -\rho\kappa_\lambda I_\lambda ds, \tag{1}$$

where $\kappa_\lambda$ is the *absorption coefficient* (in m$^2$ kg$^{-1}$; $\rho\kappa_\lambda$ has units of m$^{-1}$), and the subscript $\lambda$ indicates that the amount of flux reduction – or *extinction* – will in general depend upon the wavelength of the light. This, and the amplitude of $\kappa$ depend on the nature of the absorbing material. We can rewrite this as

$$\frac{dI_\lambda}{I_\lambda} = -\rho\kappa_\lambda ds \tag{2}$$

or

$$d(\ln I_\lambda) = -\rho\kappa_\lambda ds = -d\tau_\lambda \tag{3}$$

where $\tau_\lambda$ is called *optical depth*. Then we can see that along a path $s$ we get

$$I_\lambda(s) = I_\lambda(0)e^{-\rho\kappa_\lambda s} = I_\lambda(0)e^{-\tau_\lambda(s)}. \tag{4}$$

The amount of light falls exponentially with the optical depth. We often measure the amount of the absorption in magnitudes (called the **absorption coefficient**), such that $m_{\rm observed} = m_{\rm true} + A_\lambda$. As such,

$$A_\lambda = -2.5 \log_{10} \left( \frac{I_\lambda(s)}{I_\lambda(0)} \right) = -2.5 \log_{10} e^{-\rho \kappa_\lambda s} = 1.09 \rho \kappa_\lambda s = 1.09 \tau_\lambda. \tag{5}$$

If $\kappa_\lambda$ is independent of $\lambda$, then obscuration will just make things fainter at all wavelengths, without affecting the colour of astrophysical objects. In fact, extinction is found to vary with $\lambda$: within the optical/infrared range, the extinction is larger at shorter wavelengths, so stars gets redder. Therefore this effect is called **interstellar reddening**. We characterise this reddening by the **colour excess**, which is the change in colour between a given wavelength (usually the $B$-band) and the $V$-band, due to differential extinction.

$$E(\lambda - V) = A_\lambda - A_V \tag{6}$$

The variation of the extinction with wavelength constrains models for the dust grains, which are believed to be a mixture of carbon and silicate grains. A common approximation in the optical regime is to say that the extinction scales as $A_\lambda \propto 1/\lambda$, although this ignores the UV bump and the faster drop-off towards the infrared (cf. Figure 2). These features show where the wavelength of light becomes comparable to the size of the dust grains. If the grains were very much smaller than the wavelength, we would expect the $A_\lambda \propto 1/\lambda^4$ law of *Rayleigh scattering*, as seen in the Earth's atmosphere. The larger dust grains also differ in an important way: they genuinely *absorb* photons, rather than simply *scattering* them in a different direction. Either effect dims an object seen through a screen of dust, but we will see later in HII regions that the difference between absorption and scattering can be important.

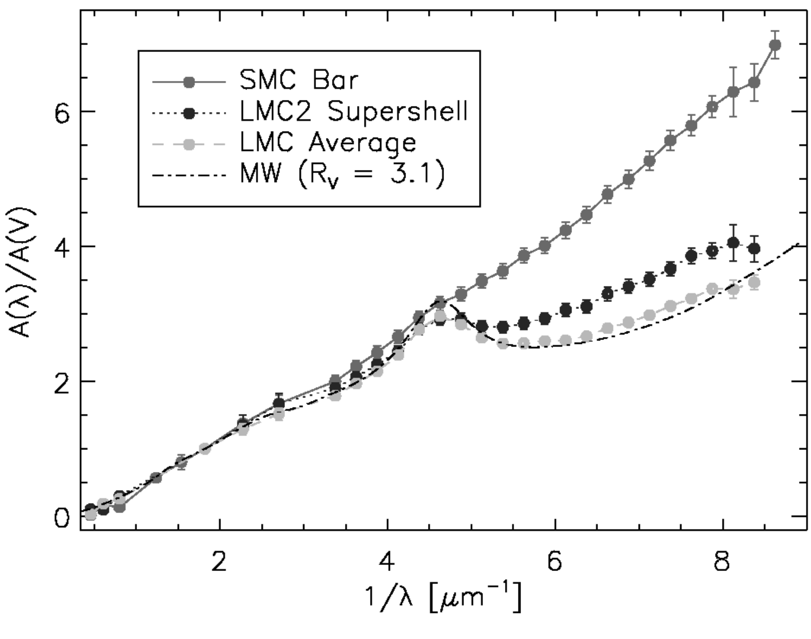



Figure 2: The extinction $A_\lambda/A_V$ against $1/\lambda$ in microns for the Milky Way and the Magellanic Clouds. This has an almost power-law form through the optical band ($1 \lesssim (1/\lambda)/\mu\mathrm{m} \lesssim 4$), but a bump at ultraviolet wavelengths ($\approx 220$nm).

### 2.1 De-reddening interstellar extinction

If one can determine the *differential* reddening between the $B$ and $V$ bands, $E(B - V)$, then the *actual* reddening the $V$–band can be easily deduced:

$$E(B-V) = A_B - A_V = A_V\left(\frac{A_B}{A_V} - 1\right) \quad (7)$$

which for $A_\lambda \propto 1/\lambda$ gives

$$E(B-V) = A_V\left(\frac{\lambda_V}{\lambda_B} - 1\right) = A_V\left(\frac{550nm}{430nm} - 1\right) = 0.28A_V. \quad (8)$$

Therefore $A_V \simeq 3.5E(B-V)$. A lot of research effort is still invested into determining what is often labelled $R$, the ratio of *total-to-selective* extinction (i.e. $R \equiv A_V/E(B-V)$), but the current 'best-bet' result ($R = 3.2 \pm 0.2$) is still consistent with the simple result $R = 3.5$ deduced above.

In order to determine $E(B-V)$ you must either know the intrinsic $B-V$ colour of the object, or you can determine it from a two colour diagram. In the $(U-B)$ vs $(B-V)$ plane (Figure 3), for example, the reddening vector will have a specific direction:

$$\frac{E(U-B)}{E(B-V)} = \frac{A_U - A_B}{A_B - A_V} = \frac{A_B\left(\frac{A_U}{A_B} - 1\right)}{A_V\left(\frac{A_B}{A_V} - 1\right)}. \quad (9)$$

For $A_\lambda \propto 1/\lambda$ this is

$$\frac{E(U-B)}{E(B-V)} = \frac{\lambda_V}{\lambda_B}\frac{\left(\frac{\lambda_B}{\lambda_U} - 1\right)}{\left(\frac{\lambda_V}{\lambda_B} - 1\right)} = \frac{550}{430}\frac{\left(\frac{430}{370} - 1\right)}{\left(\frac{550}{430} - 1\right)} = 0.74. \quad (10)$$

Using this known slope, any star can be de-reddened back to the unreddened S-shaped stellar locus, allowing both $E(B-V)$ (and hence $A_\lambda$) and the spectral type to be determined (cf. Figure 3).

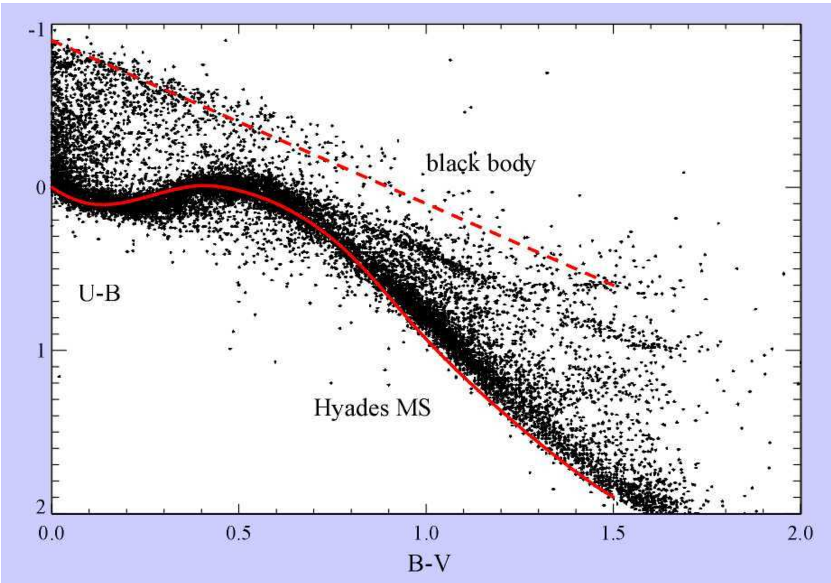
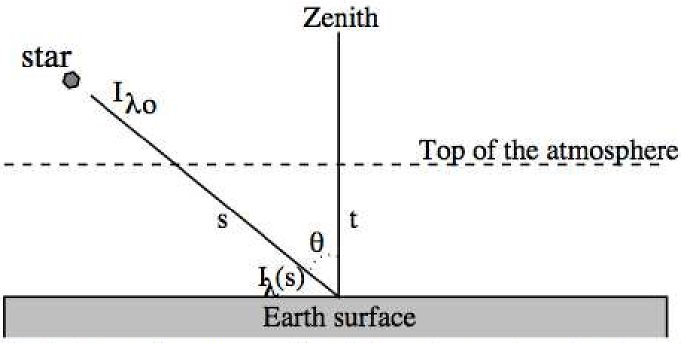



Figure 3: A $U-B$ vs $B-V$ diagram for the Hyades cluster. The dashed line shows the locus for a perfect black body, whilst the S-shaped grey line shows the expected locus of the main sequence.

### 2.2 Aside: Extinction by the Earth's Atmosphere




The lightbeam of a star passes through the earth atmosphere along the path, s, which is inclined with respect to the zenith direction by the angle theta. Along path, s, the intensity is reduced due to absorption. The absorption increases if s becomes longer for larger zenith distances.

Figure 4: Atmospheric extinction

It is not only interstellar dust that suppresses the light seen at ground-based telescopes from astrophysical objects, but also the Earth's atmosphere. The path length $s$ that the light takes through the atmosphere depends on zenith angular distance $\theta$ (i.e. $\cos\theta = t/s$ where $t$ is the vertical thickness of the atmosphere), therefore

$$ds = \sec\theta dt. \tag{11}$$

The optical depth is given by

$$\tau_{\lambda,s} = \int_0^s \rho\kappa_\lambda ds = \sec\theta \int_0^t \rho\kappa_\lambda dt = \sec\theta \cdot \tau_\lambda, \tag{12}$$

where $\tau_\lambda$ is optical depth of the atmosphere at the zenith at wavelength $\lambda$. The quantity $\sec\theta$ is called **airmass** (it is actually a more complicated function of $\theta$ for large $\theta$), so that the atmospheric extinction optical depth along the line of sight is $\tau_\lambda \times$ airmass. The intensity of the incoming light above the atmosphere $I_\lambda^0$ goes through the extinction so that

$$I_\lambda(\theta) = I_\lambda^0 e^{-\tau_\lambda \sec\theta}. \tag{13}$$

Obviously we can reduce $\tau_\lambda$ by going to high altitude. Since $\theta$ is trivial to measure, we can work out the atmospheric extinction in principle by observing the same star twice in the same night, at different angles $\theta$ (i.e. different times). Then we solve the two simultaneous equations to deduce $\tau_\lambda$ as

$$\tau_\lambda = \frac{\ln I_{\lambda,1} - \ln I_{\lambda,2}}{\sec\theta_2 - \sec\theta_1}. \tag{14}$$

If we use this $\tau_\lambda$ in the previous equation, we can deduce $I_\lambda^0$, the light intensity outside the atmosphere, which is usually what we want. We can also use this to distinguish between the effects of atmospheric and interstellar extinction.

## 3 HII Regions

### 3.1 Photoionisation

Incoming photons with enough energy to lift an electron out of its Coulomb potential well into the continuum are said to photoionise the electron: this is the **photoelectric effect**. For hydrogen, the ionisation potential is

$$I_H = 13.6 \text{ eV}. \tag{15}$$

Consider a star with a temperature $T_* \simeq 30\,000$ K. The average energy of a photon from the star

$$\langle \epsilon_* \rangle \simeq kT_* = 4 \times 10^{-19} \text{ J} = 0.2 I_H \tag{16}$$

is thus too small to ionise hydrogen. But recall that the spectrum of a star is a black body, which has a long high energy tail, so there will be some photons with sufficient energy to ionise hydrogen. What will be the effect of these ionising photons on the surrounding neutral gas (HI)?

If the average number of photons emitted per second with sufficient energy to ionise hydrogen is $S_*$ then in a time $t$, $N_\gamma = S_* t$ photons will be produced. These will be all used up in ionising the surrounding gas. If $n_H$ is the number density of hydrogen atoms (number of H atoms per unit volume), then

$$N = \frac{4\pi R^3}{3} n_H \tag{17}$$

hydrogen atoms will be ionised where

$$N = N_\gamma = S_* t \quad \Rightarrow \quad R_I = \left( \frac{3}{4\pi} \frac{S_* t}{n_H} \right)^{1/3}. \tag{18}$$

So as time passes, an ever-increasing ionised sphere will surround the star. This is called an **HII region**. For an O6 star, with $L \simeq 20 L_\odot$, the number of ionising photons emitted per second is $S_* \simeq 10^{49}$ s$^{-1}$, and the typical surrounding density for a cloud where young O stars form is $n_H \simeq 10^{10}\,\text{m}^{-3}$. Thus after $t = 10\,000$ years, $R_I \simeq 4 \times 10^{16}\,\text{m} \simeq 1.4$ pc.

In the above, we assumed $n_H$ was uniform. This was an unnecessary assumption. More generally, if the HII region has already grown to a radius $R$, then in a short time $\Delta t$ it will grow an additional amount $\Delta R$ given by

$$4\pi R^2 \, \Delta R \, n_H(R) = S_* \Delta t, \tag{19}$$

and so we have

$$\frac{\Delta R}{\Delta t} \to \frac{dR}{dt} = \frac{S_*}{4\pi R^2 \, n_H(R)}, \tag{20}$$

for any $n_H(R)$. This tells us how fast the HII region grows. The clouds surrounding young stars tend to be denser at their centres (where the stars form), and decrease in density outward. Suppose $n_H = n_0 (R/R_0)^{-3/2}$, then

$$\int_0^{R_I(t)} 4\pi R^2 n_0 \left( \frac{R}{R_0} \right)^{-3/2} dR = \int_0^t S_* \, dt = S_* t \tag{21}$$

(if $S_* = \text{constant}$). Therefore,

$$S_* t = 4\pi n_0 R_0^{3/2} \int_0^{R_I(t)} R^{1/2} dR$$
$$= \frac{8\pi}{3} n_0 R_0^{3/2} R_I^{3/2} \tag{22}$$

and so

$$R_I(t) = \left( \frac{3}{8\pi} \frac{S_* t}{n_0 R_0^{3/2}} \right)^{2/3} \tag{23}$$

We see now that instead of growing like $R_I(t) \propto t^{1/3}$ as in the uniform density case, the HII region now grows faster, as $R_I(t) \propto t^{2/3}$. This is because the gas density of the cloud is decreasing with $R$ – the cloud is thinning out – and so the star is able to ionise a greater volume of cloud for the same number of photons. The sensitivity to the density gradient can in part explain why an HII region is so filamentary in its appearance (cf. Figure 5). We shall see, however, that this is too simple a view of an HII region, and only describes its very early stages.

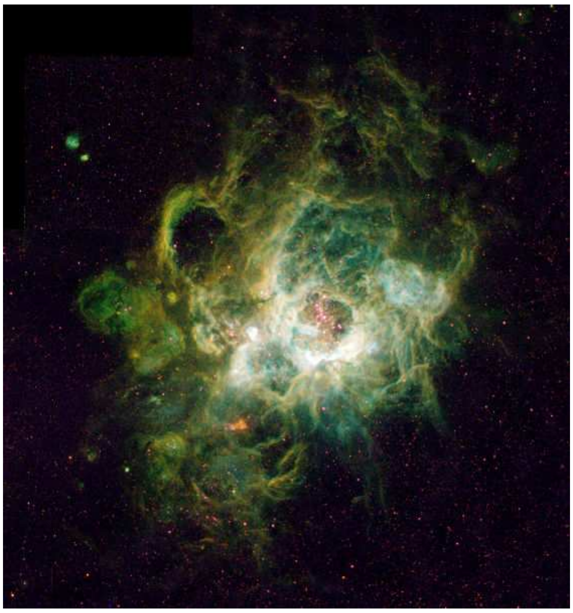


Figure 5: The giant HII region NGC604 in the Triangulum Galaxy.

### 3.2 Radiative Recombination

When we look at an HII region, most of what we see is line emission from hydrogen. This is dominated by the $n = 3 \rightarrow n = 2$ Balmer H$\alpha$ line at $\lambda = 6563$Å: how did an electron get into the $n = 3$ state? Through **radiative recombination**, which is the capture by a positive ion (or proton in the case of H) of a free electron, with the consequent release of radiation. If the electron has initial energy $E_e$, the amount of energy released during the recombination to a level of energy $E_n$ is

$$E_\gamma = E_e + |E_n|. \tag{24}$$

Once captured, the electron can make its way down by spontaneous transitions to levels of lower energy, resulting in a complex cascade of transitions (cf. Figure 6). These inevitably lead to the production of Balmer ($n > 2 \rightarrow n = 2$) photons if the nebula is optically thick to Lyman series photons (as it usually is).

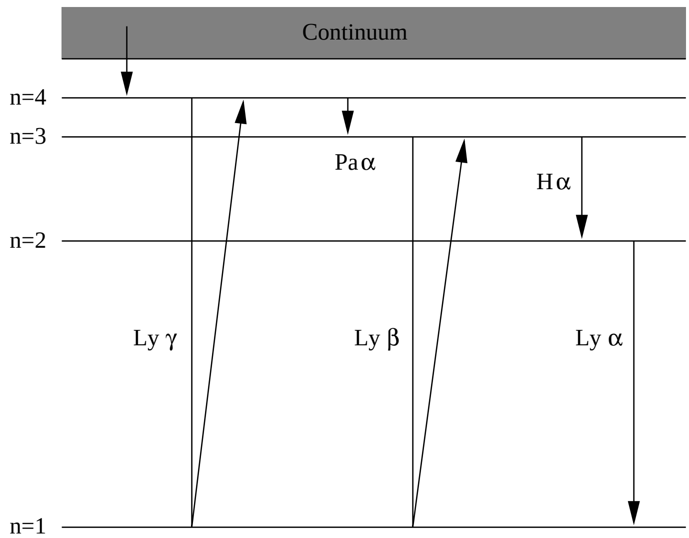



Figure 6: The recombination cascade in hydrogen. A continuum electron is absorbed, placing the atom in the $n = 4$ excited state in this example. Decay direct to the ground state produces a Ly$\gamma$ photon, which is immediately re-absorbed by another atom. The path to the ground state instead produces a sequence of Balmer and lower-energy photons, plus Ly$\alpha$.

Most of the hydrogen that's neutral is in the ground $n = 1$ state. That means that any Lyman photons will be immediately re-absorbed (by a nearby H atom in the $n = 1$ state). The result is that Ly$\alpha$ photons undergo **resonant scattering** in the nebula, only escaping by a very slow diffusive process. In a typical HII region, the odds are high that on its effectively long journey through the HII region, a Ly$\alpha$ photon will eventually hit a dust grain and be absorbed by it. As a consequence, very few Ly$\alpha$ photons can be seen from HII regions.

Another consequence of the recombinations is that hydrogen atoms that were once ionised recombine, producing a new neutral atom which will then be ionised again. As a consequence, there will always be some neutral hydrogen atoms in the HII region. This also means the full amount of ionising photons won't make it to the edge of the HII region (since some are used to re-ionise this recombined neutral hydrogen). This slows down the advance of the ionisation front, and can even stop it altogether. Let's see how this can happen.

Suppose we consider a particular free electron. How long will it take to meet a proton and so recombine? Obviously the rate of collisions will be proportional to the density of protons: doubling the number of protons in a box of fixed volume will double the chance of the electron recombining with a proton during the same interval of time. Thus the rate of recombinations per electron is

$$\mathcal{R} = \alpha n_p, \tag{25}$$

where we call the proportionality constant $\alpha$ the **radiative recombination coefficient**. The time it will take the electron to recombine is then

$$\boxed{t_{\rm rec} = \frac{1}{\mathcal{R}} = \frac{1}{\alpha n_p},} \tag{26}$$

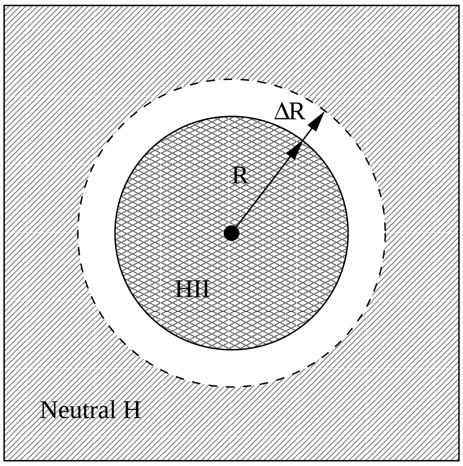



Figure 7: The geometry for the growth of an HII region. If the ionised region is of radius $R$, the time taken to ionise a further shell of thickness $\Delta R$ depends on the rate of production of ionising photons, minus the rate at which they are required to balance recombinations within $R$.

where $t_{\rm rec}$ is called the **recombination time**. Typically $\alpha \simeq 3 \times 10^{-19}$ m$^3$s$^{-1}$, although it depends on temperature. Then for $n_p = 10^{10}$ m$^{-3}$, $t_{\rm rec} \simeq 3 \times 10^8$ s $\simeq 10$ yr.

Now, if there are $n_e$ electrons per unit volume in the HII region (of radius $R_I$), the total recombination rate is

$$\mathcal{R}_{\rm tot} = \frac{4\pi}{3} R_I^3 n_e \, \alpha n_p, \tag{27}$$

where the first factor is the total number of electrons in the HII region. But this can't exceed the rate at which the star is producing ionising photons $S_*$. Thus the HII region must stop growing when the rate of ionising photons balances the total rate of recombinations, $S_* = \mathcal{R}_{\rm tot}$. This balance gives

$$\boxed{S_* = \frac{4\pi}{3} R_S^3 \alpha n_e n_p,} \tag{28}$$

where $R_S$ is called the **Strömgren radius**.

For a pure hydrogen nebula, $n_e = n_p$ by ionisation balance, and this is $\simeq n_H$, the original total H-atom density, since the gas is highly ionised. So

$$R_S \equiv \left( \frac{3}{4\pi} \frac{S_*}{\alpha n_H^2} \right)^{1/3}. \tag{29}$$

To put numbers into this, consider again the O star with $S_* = 10^{49}$ s$^{-1}$ in a cloud of density $n_H = 10^{10}$ m$^{-3}$. Then $R_S \simeq 4.3 \times 10^{15}$ m $= 0.14$ pc.

How long does it take for the ionisation radius to reach $R_S$? Consider an advancing shell as before: the number of hydrogen atoms in a shell of thickness $\Delta R_I$ at radius $R_I$is

$$n_H(R_I) \times 4\pi R_I^2 \, \Delta R_I. \tag{30}$$

The number of ionising photons produced by the star in the time available to ionise the atoms is $S_* \Delta t$. But we know not all the photons will make it all the way to $R_I$ because some will be absorbed on route by the newly recombined hydrogen atoms. How many?

In ionisation equilibrium, every newly recombined atom is compensated by a new photoionisation. So we only need count up the total number of recombinations inside the sphere:

$$\frac{4\pi}{3} R_I^3 n_e \left(\alpha n_p\right) \Delta t. \tag{31}$$

The number of photons that reach $R_I$ is thus reduced by this amount, needed to keep the parts at $R < R_I$ ionised, and so:

$$4\pi R_I^2 \, n_H(R_I) \, \Delta R_I = S_* \Delta t - \frac{4\pi}{3} R_I^3 \, \alpha n_H^2 \, \Delta t \tag{32}$$

(again taking $n_e = n_p \simeq n_H$), or

$$\boxed{4\pi R_I^2 n_H(R_I) \frac{dR_I}{dt} = S_* - \frac{4\pi}{3} R_I^3 \alpha n_H^2(R_I)} \tag{33}$$

This is a key result, which gives the rate of advance of the ionisation front. Now we can see explicitly that $dR_I/dt = 0$ when

$$R_I = R_S = \left( \frac{3}{4\pi} \, \frac{S_*}{\alpha n_H^2} \right)^{1/3}, \tag{34}$$

so this is the maximum size of the HII region.

Equation 33 can be solved directly to see how the HII region grows with time. Re-arranging Equation 34 to get an expression for $S_*$, and substituting this into Equation 33 gives

$$4\pi R_I^2 n_H(R_I) \frac{dR_I}{dt} = \frac{4\pi}{3} R_S^3 \alpha n_H^2 - \frac{4\pi}{3} R_I^3 \alpha n_H^2(R_I) \tag{35}$$

which can be simplified to

$$3R_I^2 \frac{dR_I}{dt} = (R_S^3 - R_I^3) \alpha n_H \tag{36}$$

Note that there are two characteristic quantities of the situation, defined by the parameters $n_H$ & $S_*$: the Strömgren radius $R_S$ and the recombination time $t_{\rm rec} = 1/(\alpha n_H)$. Let us therefore define dimensionless units of:

$$\lambda \equiv R_I/R_S; \qquad \tau \equiv t/t_{\rm rec}. \tag{37}$$

Multiplying Equation 36 through by $t_{\rm rec}/R_S^3$ and re-writing it in terms of these dimensionless units gives

$$3\lambda^2 \frac{d\lambda}{d\tau} = 1 - \lambda^3. \tag{38}$$

The left-hand side is just $d\lambda^3/d\tau$, so the solution is clearly

$$\lambda^3 = 1 - \exp(-\tau), \tag{39}$$

or

$$\boxed{R_I(t) = R_S \left[1 - \exp(-t/t_{\rm rec})\right]^{1/3}.} \tag{40}$$

### 3.3 Temperatures in nebulae

How do we know the temperature of the gas in an HII region? This is measured via **collisional excitation**, which is the excitation of an atom or ion from its ground state to a higher energy (excited) state caused by the collision with a free electron:

$$X(n=1) \quad \overset{\text{excitation by e}^-}{\longrightarrow} \quad X(n>1) \quad \overset{\text{decay}}{\longrightarrow} \quad X(n=1)+\gamma. \tag{41}$$

By looking at the intensity of lines produced by the spontaneous decay following the excitation, we can measure the temperature of the electrons. This works because the kinetic energies of electrons in thermal equilibrium follow a Boltzmann distribution:

$$f(E) \propto e^{-E/kT}. \tag{42}$$

If it takes an energy $E_{n'} - E_1$ to excite an atom from the $n = 1$ state to the $n'$ state, the number of electrons with sufficient energy will be $\propto \exp -\left(E_{n'} - E_1\right)/kT$. Thus, the larger $E_{n'} - E_1$ is, the dimmer the line $n' \to n = 1$ will be, since there will be fewer transitions to the higher $n'$ levels. Specifically, consider two such levels: $n'$ and $n''$ where $E_{n''} > E_{n'}$, Then the ratio of collisions to the $n''$ level to those to the $n'$ level will scale like the corresponding densities of electrons with the required energies:

$$\frac{f\left(E_{n''}-E_1\right)}{f\left(E_{n'}-E_1\right)} = \frac{e^{-(E_{n''}-E_1)/kT}}{e^{-(E_{n'}-E_1)/kT}} = e^{-(E_{n''}-E_{n'})/kT}. \tag{43}$$

If $I_{n'',1}$ and $I_{n',1}$ represent the intensities of the lines that result, then

$$\boxed{\frac{I_{n',1}}{I_{n'',1}} \propto \frac{f\left(E_{n'}-E_1\right)}{f\left(E_{n''}-E_1\right)} = e^{\Delta E/kT},} \tag{44}$$

where $\Delta E = E_{n''} - E_{n'}$.

Temperatures in nebulae are frequently estimated using the OIII (twice ionised oxygen). As shown in Figure 8 the ground state $2p$ electrons can be collisionally excited to either the $2s$ or $2d$ level, and then decay. The emission line ratio

$$\frac{I_{\lambda 4959} + I_{\lambda 5007}}{I_{\lambda 4363}} \propto e^{\Delta E/kT} \tag{45}$$

is then used to measure the temperature of the nebulae.

Typical measured temperatures lie in the range $6000 < T < 12\,000$ K. What determines the temperature?

**Heating:** Heat is added to the HII region by the photoionisation process. The typical energy of a photo-ejected electron is $kT_*$, where $T_*$ is the effective black-body temperature of the star. So the total rate of heat input by the star is:

$$G = S_* \times kT_*. \tag{46}$$

**Cooling:** The nebula loses energy due to radiative recombinations. Every time an electron recombines, an amount of energy $kT_e$ (on average) is radiated away. If this is the only way in which the nebula loses energy, then the total rate of energy loss is the total recombination rate times $kT_e$:

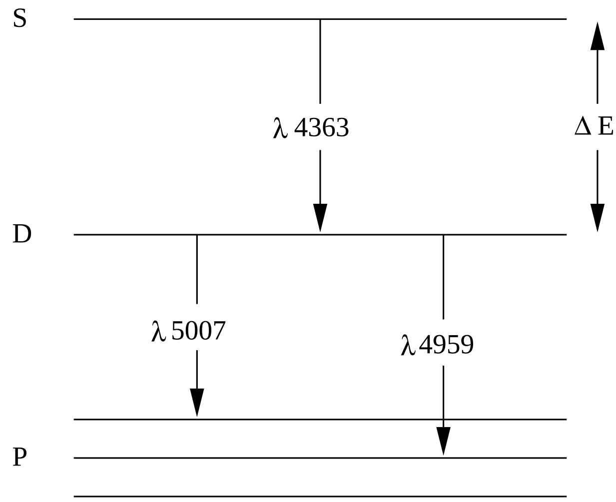



Figure 8: The energy level diagram for the lowest terms of OIII (twice ionised oxygen). The ground state consists of two electrons in the $2p$ state, but fine-structure interactions split this according to the total electron spin and (mainly) total angular momentum (denoted S,P,D etc.). The relative intensities of the two sets of lines shown (one a singlet, the other a doublet) is set by the relative occupancies of the upper levels, separated by $\Delta E$.

$$L_R = \frac{4\pi}{3} R_I^3 \alpha n_H^2 \times kT_e \tag{47}$$

In equilibrium, we have $L_R = G$ and $R_I = R_S$, where $(4\pi R_S^3/3)\,\alpha n_H^2 = S_*$ (Strömgren sphere). So

$$L_R = \frac{4\pi}{3} R_S^3\, \alpha n_H^2\, kT_e = S_* kT_e = G = S_* kT_* \tag{48}$$

Thus, we find

$$\boxed{T_e = T_*.} \tag{49}$$

But $T_* \simeq 30\,000 - 50\,000$ K typically (O & B stars), which is much higher than measured. So what went wrong?

### 3.4 Forbidden-line cooling

The problem with the above argument is that it neglects the energy emitted from the nebula by processes other than hydrogen recombination. The general name for the total loss of energy by radiation is **cooling**. This is a bad name, since it implies diffusion of heat, but we are stuck with it. In discussing the use of species such as OIII as temperature diagnostics, we introduced the idea of collisional excitation, and such excitation can contribute to the cooling in a variety of ways:

(1) Neutral H: emits Ly$\alpha$ $\lambda$1216 plus H$\alpha$ $\lambda$6563; Ly$\alpha$ re-radiated in IR by dust;
(2) OII ($O^+$): emits $\lambda$3726, $\lambda$3729; ($\lambda$2470);
(3) OIII ($O^{++}$): emits ($\lambda$4363), $\lambda$4959, $\lambda$5007 (as we saw earlier);
(4) NII ($N^+$): emits $\lambda$6548, $\lambda$6583

We might expect the collisional excitations of HI to dominate because it is so abundant, but the trouble is it turns out that the electrons are quite cool, and they lack the energy to excite HI (since

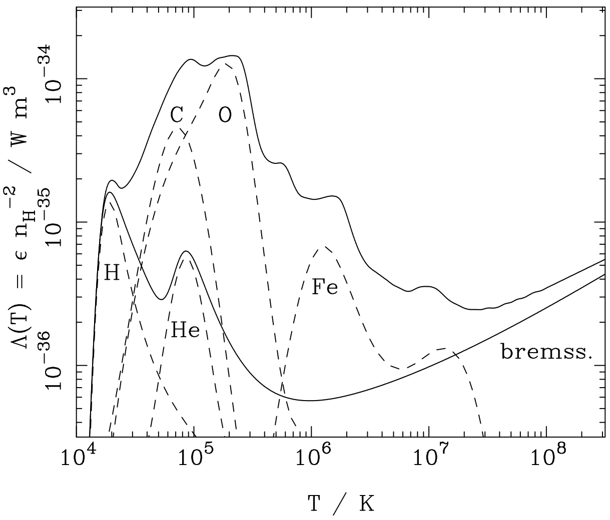



Figure 9: The cooling curve, defined such that the volume emissivity of the plasma, $\epsilon$ is $\epsilon \equiv \Lambda(T)n_H^2$. The two curves show results for hydrogen+helium only, and including Solar metals. The emissivity falls below $10^4$ K, as the plasma becomes neutral. Above that temperature, collisional excitation of ions dominates, with bremsstrahlung or free-free radiation taking over for $T \gtrsim 10^8$ K.

the energy of a UV photon is required). In any case, most of the hydrogen is ionised, rather than neutral. It turns out that the main **coolant** for HII regions is the emission from OII ($O^+$) ions. This is demonstrated in the **cooling curve**, $\Lambda(T)$, for an interstellar medium, shown in Figure 9. The cooling curve is defined by expressing the volume emissivity, $\epsilon$ (the power radiated from unit volume) in terms of a function of temperature multiplied by the square of the hydrogen density (since for any process involving the collision or interaction of particles, the rate of energy radiation scales with the square of the density). Thus,

$$\boxed{\epsilon = \Lambda(T)\, n_H^2.} \tag{50}$$

As can be seen in Figure 9, at high temperatures ($T \gtrsim 10^8$ K) the dominant cooling process is **bremsstrahlung**, or free-free radiation: radiation emitted from electrons accelerated by the electrostatic field of ions. At lower temperatures, the ionic collisional processes dominate, and the cooling curve has a sharp peak at $T \simeq 3 \times 10^5$ K associated with cooling by Oxygen. The rate of energy loss due to collisional excitation of $O^+$ can be calculated as

$$L_{\mathrm{O}^+} = \frac{4\pi}{3} R_I^3 \mathcal{L}_{\mathrm{O}^+}, \tag{51}$$

where $\mathcal{L}_{\mathrm{O}^+}$ is the cooling rate of $O^+$ per unit volume; this scales with both the the density of the colliding particles ($n_e\, n_{\mathrm{O}^+}$), and temperature. It evaluates approximately to

$$\mathcal{L}_{\mathrm{O}^+} = 1.1 \times 10^{-35} \left( \frac{n_{\mathrm{O}^+}}{n_{\mathrm{O}}^{\mathrm{tot}}} \right) \frac{n_H^2}{T_4^{1/2}} e^{-3.9/T_4} \;\; \mathrm{J\,m^{-3}s^{-1}}. \tag{52}$$

Here, $T_4 \equiv T_e/10^4\,\mathrm{K}$, and we have used high ionisation of hydrogen plus the known cosmic abundance of oxygen to replace the natural collisional term $n_e\, n_{\mathrm{O}^+}$ by one $\propto n_H^2$.

Balancing radiative losses with photoionisation heating then gives

$$L_{0^+} = \frac{4\pi}{3} R_S^3 \, \mathcal{L}_{\mathrm{O}^+} = G = S_* k T_* = \frac{4\pi}{3} R_S^3 \, \alpha n_H^2 \, k T_* \tag{53}$$

$$\boxed{\Rightarrow \quad \mathcal{L}_{\mathrm{O}^+} = \alpha n_H^2 k T_*.} \tag{54}$$

The hydrogen recombination coefficient can be approximated by

$$\alpha = 2 \times 10^{-16} \, T_e^{-3/4} \ \ \mathrm{m^3 s^{-1}}, \tag{55}$$

so that, taking $n_{\mathrm{O}^+}/n_{\mathrm{O}}^{\mathrm{tot}} \simeq 1$,

$$\mathcal{L}_{\mathrm{O}^+} = 1.1 \times 10^{-35} \frac{n_H^2}{T_4^{1/2}} e^{-3.9/T_4} = 2 \times 10^{-19} T_4^{-3/4} n_H^2 k T_* \tag{56}$$

$$\boxed{\Rightarrow \quad T_4^{1/4} e^{-3.9/T_4} = 2.5 \times 10^{-7} T_*.} \tag{57}$$

Solving this equation, we find that the nebula does indeed maintain itself at a temperature well below that of the ionising star, in agreement with observations:

| $T_*$/K | $T_e$/K |
|---|---|
| 20 000 | 7450 |
| 40 000 | 8500 |
| 60 000 | 9300 |

### 3.5 Forbidden lines and densities

Collisionally excited species are useful not only for measuring and understanding the temperatures in nebulae, but also for measuring electron number densities. This is because another name for a line such as the $\lambda$3727 doublet of OII is a **forbidden transition**. What this means is that, owing to the symmetry of the wavefunctions of the upper and lower states, the transition rate between the energy levels involved in the lines is very low. For example, the D level in OIII from which the $\lambda$4959 & $\lambda$5007 lines is produced has a lifetime of about 30 s (a **metastable state**). This is very much longer than the corresponding figure for **permitted** transitions, such as the hydrogen series: for example, the $n = 2$ level decays spontaneously to $n = 1$ in a lifetime of about $10^{-9}$ s, producing a Ly$\alpha$ photon.

This long lifetime of forbidden lines is useful, because the line is fragile: if the excited ion is disturbed before it can decay, the corresponding transition will not be produced. The natural way to achieve this is **collisional de-excitation**: the same collisions with electrons that excite ions can also depopulate the excited levels if they occur too frequently. The collision rate for a given ion is proportional to the electron number density, with a temperature-dependent coefficient $\Gamma(T)$. The production of transitions down from a level with lifetime $\tau$ therefore requires

$$\boxed{\Gamma(T) n_e < \tau^{-1}.} \tag{58}$$

Clearly, there is a **critical density** at which $n_e = (\Gamma\tau)^{-1}$; above this, the transition is quenched and the spectral line does not occur. Table 1 shows critical densities for some of the oxygen lines, computed at $T = 10^4$ K (not terrible temperature sensitive). We see that the intensity ratio for the $\lambda3726$ & $\lambda3729$ OII lines is an excellent way to measure the density, if it is around $10^9\,\mathrm{m}^{-3}$ (cf. Figure 10).

We can also see why these transitions are unlikely to be observed on Earth: very low-density gas is needed. For some time, the OIII $\lambda4959/\lambda5007$ doublet was thought to result from a new element: **nebulium**. The correct explanation was only given by Ira Bowen in 1927.

Table 1: Critical densities for nebular forbidden lines

| Ion | line | critical $n_e/\mathrm{m}^{-3}$ |
|---|---|---|
| OII | $\lambda3726$ | $1.6 \times 10^{10}$ |
| OII | $\lambda3729$ | $3.1 \times 10^{9}$ |
| OIII | $\lambda4959$ | $7.0 \times 10^{11}$ |
| OIII | $\lambda5007$ | $7.0 \times 10^{11}$ |

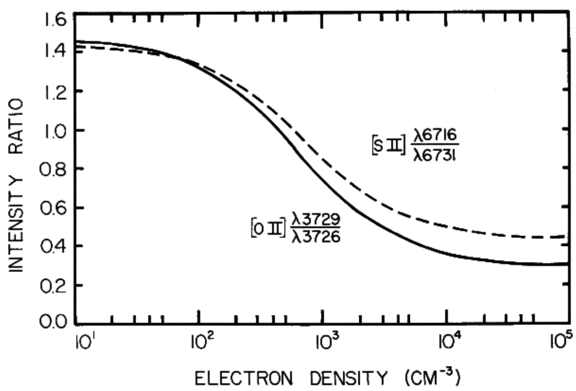



Figure 10: The variation of the OII and SII intensity ratios as a function of density, at a temperature of $10^4$K. [From Osterbrock's Astrophysics of Gaseous Nebulae book; NB: density axis is in $cm^{-3}$ instead of $m^{-3}$.]