High Energy Astrophysics, 2011–12

Course Overview

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www.roe.ac.uk/~pnb/teaching.html

Course Summary

The term ‘High Energy Astrophysics’ can be interpreted in many different ways. In the most narrow sense, it refers to observations involving high energy photons, primarily X-rays and gamma-rays. In a broader and more astrophysical view, it refers to the study of objects such as supernovae, neutron stars, black holes, binary X-ray sources, gamma-ray bursts, active galactic nuclei, radio jets, and clusters of galaxies, which involve extreme astrophysical conditions such as high energies, temperatures, or densities. These objects are characterised by high energy particles, even if the photons that some emit are of much lower energies.

This course examines the many physical processes which are important in the structure and emission of light from extreme astrophysical sources. Starting from Maxwell’s equations, the classical theory of radiation from an accelerated charge is developed, and generalised to the relativistic case. Topic studied then include: synchrotron radiation from relativistic electrons gyrating in a magnetic field; the acceleration of particles to relativistic energies; Compton and inverse Compton scattering; accretion of material onto compact objects; radio galaxies and quasars, and their jets; bremsstrahlung emission from hot gas; cooling flows and the role of black holes in galaxy formation.

Schedule of lectures, problem classes and tutorials

Lectures will be held on Tuesdays and Fridays at 14.00 – 14.50 in the ROE lecture theatre, in weeks 1-11. Problem-solving sessions (which will have a variety of formats) will be inter-mingled with the lectures in these slots. Tutorials will be on Mondays from 15:00 – 17:00 on a fortnightly basis, in weeks 3,5,7,9,11, also in the ROE lecture theatre.

Lecture notes and tutorial sheets will be available on-line at **www.roe.ac.uk/~pnb/teaching.html**. Solutions to tutorials will be posted there 2-3 weeks after the tutorial.

Assessment and Feedback

The course is assessed by an end-of-year examination only. There will be opportunities for feedback on progress in all of the tutorial sessions. In particular, associated with each of the tutorial sheets there is a hand-in question. This is not assessed, but if handed in no later than the Friday lecture preceeding each tutorial then it will be marked and full feedback given during the tutorial on a one-to-one basis.

Recommended Reading

The following books are useful reading for this course:

- Longair M.A., *High Energy Astrophysics Vols 1 & 2*: This is the most suitable book for the course, covering most of the material at an appropriate level.
- Rybicki & Lightman, *Radiative Processes in Astrophysics*: this covers some of the topics at quite an advanced level
- Carroll & Ostlie. *Introduction to Modern Astrophysics*: contains many useful chapters, and also serves as an excellent introduction to astronomy and astrophysics.

Assumed Knowledge

It is assumed that students on this course will be familiar with basic concepts in Electromagnetism, up to and including the derivation of Maxwell's equations. It is also assumed that students are familiar with special relativity, 4-vectors, and Lorentz transformations. Finally, it is assumed that students will have a mathematical background up to and including the level of the Junior Honours "Physical Mathematics" course. This will include full knowledge of vector calculus (div, grad, curl), as well as special functions such as the Dirac delata function, ϵ_{ijk} and Fourier transformations. The sections below provide a recap of the critical features of these that will be required knowledge for the course: students for whom these are not familiar should engage in background reading if they intend to follow the course.

It is not intended to be necessary to have any astrophysics background to follow this course, although students with no astrophysics background may occasionally require a little extra background reading to better understand a particular type of astrophysical source. The Astrophysics 3 course notes, available online through the course notes portal, would provide all necessary background.

A brief resumé of four vectors

This Section provides a resumé of four-vectors and Lorentz transformations, which should be familiar but may serve as a useful reference. Some of the notes are specific to Special Relativity, and to cartesian coordinate systems.

All 4-vectors stem ultimately from the *prototype 4-vector*

$$dx^\mu \equiv (cdt, dx, dy, dz) \tag{1}$$

in cartesian coordinates. μ is an index running from 0 to 3.

Note that this is sometimes written as

$$dx^\mu \equiv (cdt, d\mathbf{x}) \tag{2}$$

where $d\mathbf{x}$ is an ordinary 3-vector in spatial coordinates only; $d\mathbf{x} = (dx, dy, dz)$.

Einstein's postulate that the speed of light is constant in all inertial frames means that

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2) \tag{3}$$

is invariant. It is equal to $c^2d\tau^2$, where τ is the proper time; ie. for a *timelike* interval which can be followed by an observer it is the time interval according to an observer at rest (so $d\mathbf{x} = 0$).

ds^2 may be written as a scalar product betwen two 4-vectors, $dx^\mu \equiv (cdt, dx, dy, dz)$ which is called a 'contravariant' 4-vector, and $dx_\mu \equiv (cdt, -dx, -dy, -dz)$ which is a 'covariant' 4-vector. That is,

$$ds^2 = dx^\mu dx_\mu \tag{4}$$

Summation convention applies for the repeated μ index, from 0 to 3. Summation is always over one upper index and one lower index.

To raise (or lower) an index (ie. convert a contravariant to a covariant 4-vector, or vice versa), multiply a 4-vector by the *metric tensor* (tensors have two or more indices, not just one).

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{5}$$

$\eta^{\mu\nu}$ is the inverse of $\eta_{\mu\nu}$, which, in special relativity in cartesian coordinates, is the same matrix. Thus

$$dx_\mu = \eta_{\mu\nu}dx^\nu; \qquad dx^\mu = \eta^{\mu\nu}dx_\nu. \tag{6}$$

To transform any 4-vector between reference frames, in the 'Standard Configuration' where frame S' moves at speed $v = \beta c$ along the x axis, use the Lorentz transformation matrix Λ^μ_ν:

$$dx'^\mu = \Lambda^\mu_\nu dx^\nu \tag{7}$$

where

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

and $\gamma = (1 - \beta^2)^{-1/2}$. A set of 4 quantities is a (contravariant) 4-vector if it transforms according to this rule.

Other 4-vectors are usually obtained by dividing or multiplying the prototype 4-vector by a *scalar* (invariant) quantity. For example, dividing by the proper time interval $d\tau$ gives the velocity 4-vector:

$$U^\mu \equiv dx^\mu/d\tau \equiv (cdt/d\tau, d\mathbf{x}/d\tau) \tag{9}$$

Note that since $dt/d\tau = \gamma$,

$$U^\mu = \gamma dx^\mu/dt = \gamma(c, \mathbf{v}) \tag{10}$$

where $\mathbf{v}$ is the 3-velocity, $d\mathbf{x}/dt$. The 4-acceleration is similarly

$$a^\mu \equiv d^2x^\mu/d\tau^2 \equiv \gamma dU^\mu/dt = \gamma \left(cd\gamma/dt, d\left[\gamma\mathbf{v}\right]/dt\right). \tag{11}$$

Multiplying the 4-velocity by the (invariant) rest mass gives the 4-momentum:

$$P^\mu = m_0 U^\mu = (E/c, \mathbf{p}), \tag{12}$$

where $\mathbf{p}$ is the 3-vector momentum. Similarly the 4-force is

$$F^\mu = dP^\mu/d\tau = m_0 a^\mu. \tag{13}$$

The scalar product of two 4-vectors $A_\mu B^\mu = A^\mu B_\mu$ is invariant, so, for example

$$P^\mu P_\mu = E^2/c^2 - p^2 = m_0^2 c^2. \tag{14}$$

The last equality comes from evaluating the expression in the rest frame of the particle. This is a very common trick: since the scalar product is invariant, it is the same in any frame, and therefore its value can be evaluated in the frame where it is easiest to calculate.

Note that not all single-value quantities are invariant scalars: only those derived from the scalar product of two 4-vectors. For example, density is not invariant, because of Lorentz contraction.

Relativistically correct equations relate 4-vectors to 4-vectors, or scalars to scalars, or 4-tensors to 4-tensors.

A brief resumé of vector calculus

The vector differential operator, del, is

$$\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \tag{15}$$

Del is a convenient mathematical expression for three specific operators: Grad, Div and Curl.

Grad: $\nabla\phi$ gives a vector quantity, the gradient of a scalar field ϕ.

Div: $\nabla \cdot \mathbf{\Psi}$ gives a scalar quantity, the divergence of a vector field $\mathbf{\Psi}$. It measures the source or sink terms of the vector field, or equivalently the outward flux of a vector field from an infinitesimal volume around a given point.

Curl: $\nabla \wedge \mathbf{\Psi}$ gives a vector quantity, the curl of a vector field $\mathbf{\Psi}$. It describes the degree of rotation of the 3-dimensional vector field.

Grad, Div and Curl can be related to line, surface and volume integrals. If V is a small volume enclosing a point, S is the closed surface bounding this, and $d\mathbf{S}$ is the vector area of a small region of that surface, then Grad, Div and Curl can be defined as

$$\begin{aligned}
\nabla\phi &= \lim_{V\to 0}\left(\frac{1}{V}\oint_S \phi d\mathbf{S}\right) && (16)\\
\nabla\cdot\mathbf{\Psi} &= \lim_{V\to 0}\left(\frac{1}{V}\oint_S \mathbf{\Psi}\cdot d\mathbf{S}\right) && (17)\\
\nabla\wedge\mathbf{\Psi} &= \lim_{V\to 0}\left(\frac{1}{V}\oint_S d\mathbf{S}\wedge\mathbf{\Psi}\right) && (18)
\end{aligned}$$

The definition of Div here leads naturally to Gauss's theorem (the divergence theorem):

$$\int_V \nabla\cdot\mathbf{\Psi} dV = \oint_S \mathbf{\Psi}\cdot d\mathbf{S} \tag{19}$$

An alternative (but equivalent) and often more useful definition of Curl is to consider a plane surface of area A enclosing a point, bounded by a contour C, to which the vector $\hat{\mathbf{n}}$ is a unit normal vector. Then

$$(\nabla\wedge\mathbf{\Psi})\cdot\hat{\mathbf{n}} = \lim_{A\to 0}\left(\frac{1}{A}\oint_C \mathbf{\Psi}\cdot d\mathbf{s}\right) \tag{20}$$

where $d\mathbf{s}$ is the path vector moving around the contour C. This gives Stokes' theorem

$$\int_S (\nabla\wedge\mathbf{\Psi})\cdot d\mathbf{S} = \oint_C \mathbf{\Psi}\cdot d\mathbf{s}. \tag{21}$$

Various combinations of the del operator give

$$\begin{aligned}
\nabla\cdot(\nabla\wedge\mathbf{\Psi}) &= 0 \qquad \text{for all possible vector fields } \mathbf{\Psi} && (22)\\
\nabla\wedge(\nabla\phi) &= 0 \qquad \text{for all possible scalar fields } \phi && (23)\\
\nabla\wedge(\nabla\wedge\mathbf{\Psi}) &= -\nabla^2\mathbf{\Psi} + \nabla(\nabla\cdot\mathbf{\Psi}) && (24)\\
\nabla\cdot(\nabla\phi) &= \nabla^2\phi && (25)
\end{aligned}$$

The 4-vector *gradient operator* is

$$\nabla_\mu \equiv \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \tag{26}$$

Note that this transforms like a covariant 4-vector, so it is given a downstairs index. The 4-vector product

$$\nabla^\mu\nabla_\mu = \Box^2 \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{27}$$

is an invariant scalar operator.

Other useful bits of maths

The Dirac delta function (which is the continuous version of the Kronecker delta) has zero width, infinite height, and encloses unit area underneath. It can be defined in any number of dimensions, and obeys

$$\begin{aligned}\int_{-\infty}^{\infty}\delta(x)dx &= 1 &\quad(28)\\ \int_{-\infty}^{\infty}\delta(x-x_0)f(x)dx &= f(x_0)\quad(1D) &\quad(29)\\ \int\delta(\mathbf{x}-\mathbf{x}_0)f(\mathbf{x})d^3\mathbf{x} &= f(\mathbf{x}_0)\quad(3D) &\quad(30)\end{aligned}$$

The Fourier transform and its inverse are given by:

$$\tilde{\phi}(\mathbf{x},\omega) \equiv \int_{-\infty}^{\infty}\phi(\mathbf{x},t)\exp(i\omega t)dt. \tag{31}$$

$$\phi(\mathbf{x},t) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\phi}(\mathbf{x},\omega)\exp(-i\omega t)d\omega. \tag{32}$$

Fourier representations of the Dirac delta in 1D and 3D are:

$$\delta(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(\pm ikx)dx \tag{33}$$

$$\delta(\mathbf{k}) = \frac{1}{(2\pi)^3}\int\exp(\pm i\mathbf{k}\cdot\mathbf{x})d^3\mathbf{x}. \tag{34}$$

The permutation symbol, ϵ_{ijk}, is defined by:

$$\epsilon_{ijk} = \begin{cases} 1 & \text{if } (i,j,k) \text{ is an even permuation, ie. } (1,2,3),(2,3,1) \text{ or } (3,1,2)\\ -1 & \text{if } (i,j,k) \text{ is an odd permuation, ie. } (1,3,2),(2,1,3) \text{ or } (3,2,1)\\ 0 & \text{if } i=j \text{ or } j=k \text{ or } i=k\end{cases} \tag{35}$$