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LETTERS TO NATURE

Detection of a large mass of dust in a radio galaxy at redshift $z = 3.8$

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ELLIPTICAL galaxies are thought to have formed most of their stars in a rapid burst in the early Universe¹, but an unambiguous example of a 'primaeval' elliptical galaxy (one undergoing its first major burst of star formation) has yet to be discovered. High-redshift radio galaxies are among the most promising candidates^{2,3}, because their low-redshift counterparts are identified exclusively with ellipticals, but the presence of an active nucleus complicates the analysis of their evolutionary state from optical-infrared observations^{3–5}. The failure of optical searches to detect primaeval ellipticals^{6–9} suggests that they may be very dusty, prompting us to search for thermal emission from the dust, which will be redshifted to submillimetre wavelengths in our reference frame. Our detection of submillimetre emission from the radio galaxy 4C41.17, reported here, suggests that it contains a large mass of dust, probably located in a dust lane obscuring the centre of the galaxy^{10–14}. The observations are consistent with the recent occurrence of a massive burst of star formation, but probably not the first such episode. We conclude that this galaxy was already in the final stages of its formation at a look-back time of 12–15 billion years.

Building on the discovery¹⁵ of far-infrared luminous starburst galaxies by the Infrared Astronomy Satellite (IRAS), we have embarked on a programme (D.H.H., J.S.D. and S.R., manuscript in preparation) to search for rest-frame far-infrared emission from high-redshift ($z > 2$) radio galaxies by observing that emission in the submillimetre range.

We observed 4C41.17, the most distant known galaxy¹⁰ (that is, the most distant known object with a spatially resolved optical continuum potentially dominated by starlight), as part of this programme on 30 September 1993. During exceptionally dry and stable conditions we were able to integrate continuously for three hours with the ³He bolometer UKT14 (ref. 16) on the

James Clerk Maxwell Telescope (JCMT) through the 800- μ m continuum filter. The observation was made using a beam size of 16.5 arcsec (equivalent to a linear size of 110–230 kpc, for cosmological density parameter $\Omega_0 = 1-0$, and Hubble constant $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), chopping at 7.81 Hz with an east-west throw of 60 arcsec. The result was a flux-density measurement of $S_{800 \mu\text{m}} = 17.4 \pm 3.1 \text{ mJy}$, the only unambiguous detection in a pilot study of six $z > 2$ radio galaxies. Further observations were made at 450 μ m in December 1993 (again in excellent conditions), resulting in a sensitive 3σ limit, $S_{450 \mu\text{m}} < 56 \text{ mJy}$, after a further three hours of integration.

On the basis of its rather blue ultraviolet-optical spectral energy distribution (SED) and complex multi-component morphology, it has been claimed that 4C41.17 may be a genuine example of a primaeval elliptical galaxy¹⁰. The importance of our 800- μ m detection is that it allows us to make a completely independent assessment of the 'evolutionary state' of 4C41.17.

Figure 1 shows the radio-ultraviolet SED of 4C41.17. Neither an extrapolation of the steep radio-lobe spectrum nor the much flatter spectrum of the recently discovered radio core ($\alpha = -0.23$, where α is the spectral index for the flux density, $f_\nu \propto \nu^\alpha$) can account for the observed 800- μ m flux density. Our detection can thus only be explained by thermal emission from dust or by non-thermal emission from an additional synchrotron component. Although our data do not allow us to exclude the second of these possibilities on the basis of the submillimetre spectral index ($\alpha > 2.5$) it is highly implausible that the far-infrared emission of 4C41.17 is due to a synchrotron component. To avoid exceeding the observed flux-density of the radio-core at the observing wavelength $\lambda_{\text{obs}} = 2 \text{ cm}$, this component would need to become self-absorbed at the rest-frame wavelength $\lambda_{\text{rest}} < 510 \mu\text{m}$; such compact ($< 0.015 \text{ pc}$) and energetic synchrotron components have only ever been observed in the flaring 'blazar' class of active galactic nuclei, where they are always accompanied by a series of components with progressively lower turnover frequencies, producing the characteristic flat radio spectrum. It is therefore hard to sustain a synchrotron interpretation of our 800- μ m data, as 4C41.17 would then be unique in containing, in isolation, such an energetic and compact emission region.

It is therefore reasonable to assume that the rest-frame far-infrared emission in 4C41.17 is dominated by optically thin thermal emission from dust, particularly as this assumption has been validated for IRAS-detected galaxies (such as F10214 + 4724; refs 17, 18) and radio-quiet quasars^{19,20}, which, like 4C41.17, have far-infrared peaks in their rest-frame SED. The addition of a 450- μ m limit to the 800- μ m detection restricts the rest-frame dust temperature to $\leq 43 \text{ K}$ (for $\beta = 2$) or $\leq 67 \text{ K}$ (for $\beta = 1$), where β is the grain emissivity index. Adopting a dust temperature $T_{\text{dust}} = 50 \text{ K}$, $\Omega_0 = 1$, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and an average dust opacity (at a rest-frame wavelength of 167 μ m) of $\kappa_d = 3.5 \text{ m}^2 \text{ kg}^{-1}$ (refs 21, 22), yields a dust mass $M_{\text{dust}} \approx 3 \times 10^8 M_\odot$.

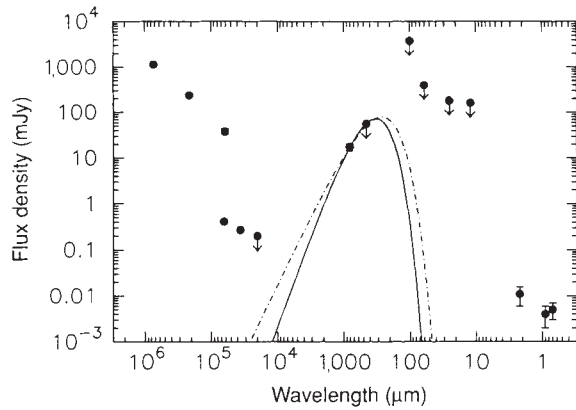


FIG. 1 The observed spectral energy distribution of 4C41.17 from radio to optical (rest-frame ultraviolet) wavelengths. The figure shows the radio flux densities of both the steep-spectrum lobes^{10,12} (at $\lambda = 7.35 \times 10^5$, 2×10^5 and 6.38×10^4 μm), and also the flat-spectrum core¹² (at $\lambda = 6.38 \times 10^4$, 3.6×10^4 and 2×10^4 μm). Extrapolation of either of these radio components to a wavelength of 800 μm produces a non-thermal contribution which is ~ 100 times smaller than the observed 800- μm flux density. It is therefore almost certain that the observed submillimetre flux density is produced by thermal emission from dust. The solid curve shows the emission expected from dust of emissivity index $\beta = 2$ at a temperature of $T_{\text{dust}} = 43$ K normalized to our 800 μm detection. The dashed-dotted curve represents the thermal emission where $\beta = 1$ and $T_{\text{dust}} = 67$ K. Also shown are 3σ IRAS limits at 100 μm and 60 μm (derived from direct determination of the noise level in IRAS maps of the source region), the continuum flux density observed at 2.2 μm (from the line-free K_s image¹⁴) and the (uncorrected) *I*-band and *R*-band flux densities¹⁰.

The dust mass in 4C41.17 is ≥ 10 times greater than is typically found in low-redshift radio galaxies²³; adopting the same model parameters as used by Knapp and Patten²³ ($T_{\text{dust}} = 18$ K (not excluded by our data), $\kappa_d = 1.5 \text{ m}^2 \text{ kg}^{-1}$ at $\lambda_{\text{rest}} = 167 \mu\text{m}$, and $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$) yields a dust mass in 4C41.17 of $4.0 \times 10^9 M_\odot$. The dust mass is also several times greater than that inferred for the most extreme local ($z = 0.44$) cD galaxy 09104+4109 (ref. 24), or radio-loud quasars and galaxies at $z \approx 1$ (ref. 25). Compared to elliptical galaxies which have been studied in the rest-frame, far-infrared 4C41.17 is clearly exceptionally luminous.

Because of the suppression of the resonant Lyman- α line expected in dusty environments, the huge Ly- α luminosities of radio galaxies like 4C41.17 appear at first sight to be inconsistent with such large dust masses. In the light of our JCMT detection we have therefore re-examined the Fabry-Pérot Ly- α image of 4C41.17 (ref. 13) to look for the most likely location of the dust. In fact, there is a clear dip in the surface brightness of the $\sim 15 \times 10$ arcsec Ly- α halo centred 2.2 arcsec west of the Ly- α peak—a dip which the observers chose to associate with a foreground Ly- α absorber, unrelated to the radio galaxy. Given that its position is now known to coincide with that of the recently discovered radio core of the galaxy¹², we prefer to interpret this dip as resulting from absorption of the back half of the Ly- α halo by a dusty torus of approximate dimensions 10×30 kpc (for $\Omega_0 = 1$, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) oriented perpendicular to the radio jet axis, and centred on the radio core (see Fig. 2). This agrees quantitatively with the observed line profiles if the Ly- α halo is expanding, as would be expected if it is driven by the radio-source bowshock¹⁰: the blue wing of the line is similar at the spatial peak of the Ly- α and at the site of the absorber, whereas at the redder line centre, the surface brightness drops by a factor ~ 3 . Moreover, as we show in Fig. 2, this putative dust lane corresponds exactly with pronounced gaps in the rest-frame ultraviolet¹¹ and optical¹⁴ images. Regardless of the detailed geometry, the lack of other holes in the Ly- α image

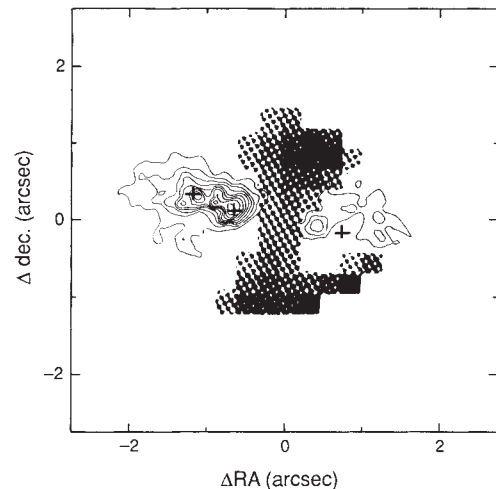


FIG. 2 The probable location of dust in 4C41.17. A grey-scale image of the region of Ly- α obscuration is shown (reproduced from Hippelein and Meisenheimer¹³), superimposed on a suitably scaled and registered (accurate to ~ 0.25 arcsec) contour plot of the Hubble Space Telescope image obtained by Miley *et al.*¹¹ ($\lambda_{\text{rest}} \approx 1,400 \text{ \AA}$). The image is centred on the coordinates of the recently discovered¹² flat-spectrum core of the radio galaxy—right ascension (RA) 06 h 47 min 20.574 s, declination (dec.) $41^\circ 34' 3.85''$ (B1950); for our 800- μm observation the 16.5-arcsec aperture of the JCMT was centred at RA 06 h 47 min 20.74 s, dec. $41^\circ 34' 4.5''$). Our interpretation of this region of Ly- α obscuration as a dust lane in the radio galaxy is supported by the obscuration overlying the position of the radio core, and by its location coinciding almost exactly with a distinct gap in the ultraviolet continuum emission. In addition, it appears to be orientated perpendicular to the radio axis. The three crosses indicate the positions of the radio components B1, B2 and B3¹⁰⁻¹².

gives us confidence that the huge dust mass detected within the 16.5-arcsec JCMT aperture is associated with the inner regions of the radio galaxy itself, and not with a companion or foreground object. Our preferred geometry for the dust in 4C41.17 has potentially far-reaching implications: if all young galaxies have a high dust content, then detections of a Ly- α line may be feasible only when it is produced away from the dusty starburst region by processes associated with the active nucleus.

Is 4C41.17 a primaeval elliptical galaxy? By analogy with IRAS-detected galaxies, and assuming that the dust is heated primarily by young stars, we can use the far-infrared luminosity of 4C41.17 ($L_{\text{FIR}} \geq 5 \times 10^{13} L_\odot$; $\Omega_0 = 1$, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), to estimate the 'current' star formation rate (SFR). A simple scaling between SFR and far-infrared luminosity, calibrated using nearby starbursts such as M82 (ref. 26), yields an estimated SFR of $(2-10) \times 10^3 M_\odot \text{ yr}^{-1}$ ($\Omega_0 = 1-0$). These values agree rather well with the SFR ($\sim 4,300 M_\odot \text{ yr}^{-1}$) inferred from the very young (10^8 yr), dusty model (model D) fit to the ultraviolet-optical SED of 4C41.17 (ref. 10). More generally, star formation rates of $\sim 1,000 M_\odot \text{ yr}^{-1}$ are obviously consistent with an extremely luminous giant elliptical galaxy (with a stellar mass of $\sim 10^{12} M_\odot$; ref. 27) observed during a canonical 1-Gyr formation starburst.

However, any attempt to determine whether a galaxy is primaeval on the basis of its far-infrared luminosity is vulnerable to the same criticisms as arguments based on the shape of the ultraviolet-optical SED. First, although there is no obvious quasar nucleus in 4C41.17, it may well be hidden from view by the very dust detected by our JCMT observation. (In fact, this seems highly likely, given that the dust appears to cover the nucleus as shown in Fig. 2). If this is true, then the dust may be heated by this buried quasar and not by a starburst, in which case the SFR could be very much lower than is deduced by comparison with starburst galaxies. Second, even if correct, both the high

SFR inferred from the far-infrared luminosity of 4C41.17 and the shape of the ultraviolet-optical SED are still consistent with an old galaxy (with formation redshift $z_f > 10$) undergoing a 10^6 -yr starburst involving as little as 1% of the mass of the underlying galaxy.

Similar problems afflict attempts to determine the evolutionary state of 4C41.17 from its peculiar optical morphology. The rest-frame wavelength of the Hubble Space Telescope image reproduced in Fig. 2 is 1,400 Å, and even at $z < 1$ it is now well established that powerful radio galaxies frequently possess a component of ultraviolet light aligned with the radio structure⁴. Our conclusion that the centre of the galaxy is concealed by a dust lane further weakens claims that its multi-modal optical structure is indicative of an elliptical galaxy in the process of formation. It also remains difficult to rule out the presence of a significant population of evolved stars, as even near-infrared observations of 4C41.17 (ref. 14) sample continuum light at a rest-frame wavelength of only 4,600 Å.

In principle, therefore, the most straightforward way to identify whether a galaxy at $z > 3$ is genuinely young is to determine what fraction of its final stellar mass has yet to be converted into stars at the epoch of observation. 4C41.17 certainly contains much more dust than has been found in low-redshift radio galaxies. Although the uncertainties involved in converting this dust mass into a gas mass are large (D.H.H., J.S.D. and S.R., manuscript in preparation), assuming a canonical gas to dust ratio of 500, which is supported by recent CO observations of 10214+4724 (ref. 28), leads to an estimate of the gas mass in 4C41.17 of $M_{\text{gas}} \approx 10^{11} M_{\odot}$. This value is at least consistent with the idea that a significant fraction ($\sim 10\%$) of the galaxy's eventual stellar mass has yet to be converted into stars at $z = 3.8$.

Thus, although our detection of dust in 4C41.17 indicates the occurrence of recent large-scale star-formation activity, it does not prove that 4C41.17 is 'primordial'. Indeed, taken at face value, our results indicate that we are observing a massive starburst in 4C41.17 involving ~ 5 – 10% of its eventual stellar mass, as would be expected for a giant elliptical galaxy in the final stages of formation. None of the available data on 4C41.17 can yet distinguish whether star formation in 4C41.17 began at $z \approx 4$, or whether the bulk of its stellar content was formed at much higher redshifts. It remains entirely possible that the general population of elliptical galaxies was formed at $z > 10$, and that in high-redshift radio galaxies at $z \approx 3$ – 4 we are observing the final spectacular stages in the construction of luminous cD galaxies by mergers. □

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Density and size of comet Shoemaker–Levy 9 deduced from a tidal breakup model

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ALTHOUGH comets have been studied throughout most of recorded history, a detailed understanding of their internal properties is still lacking. Recent observations¹ of the split comet Shoemaker–Levy 9—actually a spectacular string of cometary fragments that resulted from the tidal disruption of a single parent body as it passed close to Jupiter^{2–5}—have therefore stimulated much interest, as they provide an unprecedented opportunity to investigate the physical properties of comets more generally^{6–8}. I report here simulations of the tidal breakup of the parent comet, which I assume to have been an assemblage of a large number of spherical components bound together only by gravity. Following the initial tidal disruption of the assemblage, the particles coalesce rapidly by mutual gravitation into a chain of larger fragments, the morphology of which depends critically on the density of the components. By comparing the size, number and distribution of the simulated fragments with observations of Shoemaker–Levy 9, I determine an average comet density of about 0.5 g cm^{-3} and a parent comet diameter of about 1.8 km.

In contrast to the common view of a solid body⁹, I model the comet as an agglomeration of individually competent components—'snowballs'—bound together only by mutual gravitational attraction. The depiction of comets as 'flying rubble piles' has enjoyed increasing support^{6,10,11} and comets with multiple nuclei are not exceptional^{12–14}. There are probably other cohesive forces between components, but I assume that these are small compared to gravitational binding.

In the simulations, the spherical components interact gravitationally except when they touch. The touching, or collision, of two components is handled as a non-adhesive dissipative scattering, that is, the velocities are suddenly changed in such a way that momentum is conserved, but some of the kinetic energy is converted to heat. The simulation is a detailed calculation of the gravitational interaction and collisions of the components—it is not a hydrodynamic calculation.

A further simplification, which greatly accelerates computation, is to assume that the radius r_0 and density ρ of each component is the same. Under this assumption, the equation of motion in the vicinity of the comet's centre of mass is well approximated by

$$\ddot{\mathbf{r}}_i = G \left[m_0 \sum_{i \neq j} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \frac{2M(\mathbf{R} \cdot \mathbf{r}_j)\mathbf{R}}{R^5} \right] \quad (1)$$

where G is the universal gravitation constant, M is the jovian mass, m_0 is the component mass, \mathbf{r}_i is the radius vector of the i th component from the comet's centre of mass, and \mathbf{R} is the radius vector of the comet's centre of mass from the jovian