The Determination of the Curvature of Space-Time in de Sitter's World.

By Knut Lundmark, Ph.D.

1. In a very interesting paper in *M.N.*, **84**, 363, 1924, and in several letters to the editor of *Nature*,* Dr. Silberstein has derived from de Sitter's cosmology a formula for the Doppler shift, $\frac{d\lambda}{\lambda}$, of a star in inertial relative motion to the observer.

The formula may be written

$$\frac{d\lambda}{\lambda}=\left(1-\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}\left\{1\pm\left[1-\cos^2\frac{r}{R}\Big/\left(1-\frac{v_0^2}{c^2}\right)\right]^{\frac{1}{2}}\right\}-1,$$

where v_0 = the individual velocity of a star at a past or future epoch,
c = velocity of light,
r = distance of the star from observer,
R = the curvature radius of space-time.

The expression $\left(1-\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}$ as suggested by astronomical observations will be $\leq 1{\cdot}0000444$, and for most of the stars $\leq 1{\cdot}000001$. Taking account of this fact, the formula can be simplified to

$$\frac{d\lambda}{\lambda}=\pm\sin\frac{r}{R},$$

or for small or moderate values of $\frac{r}{R}$,

$$\frac{d\lambda}{\lambda}=\pm\frac{r}{R}.$$

Dr. Silberstein makes use of this formula for computing R by using a few of the observed radial velocities for globular clusters. Before entering on a discussion concerning the value of R thus derived, we will for a moment look into the matter of the nature of the spectral shifts observed for globular clusters and spiral nebulæ, as it may not be without interest to know if the spectral displacements are of the Doppler nature.

2. The velocities for these objects are mainly due to the wonderful spectrographic work performed at the Lowell Observatory by Dr. V. M. Slipher. It is well known that the velocities are very high and in many respects of similar nature for the two classes of objects, except that the spiral nebulæ show a very large spectral shift towards the red. Some astronomers seem to have been rather sceptical as to the meaning of the spectral displacements observed, and several times the question has been raised: Are the displacements observed of the Doppler character? *i.e.* is $\frac{d\lambda}{\lambda}$ constant within the limits of the errors of measurement? This is

* *Nature*, 1924 March 8, April 26, and June 7.

certainly the case, and in illustration we may examine the following measures of a spectrogram of the Andromeda Nebula, which I am allowed to quote through the courtesy of Professor Wright at the Lick Observatory.

λ.	$\frac{\Delta\lambda}{\lambda}$.	λ.	$\frac{\Delta\lambda}{\lambda}$.
$\mu\mu$		$\mu\mu$.	
486	0·0012	427	0·0011
473	10	423	10
455	11	410	14
438	11	405	6
438	16	405	11
434	15	403	14
433	13	398	5
429	0·0017	397	0·0010

The mean is $0{\cdot}00116 \pm 0{\cdot}00008$, and the shift is evidently a Doppler one. The same applies to the velocities of the globular clusters.

3. Another question is, whether such a large Doppler shift represents motion in the line of sight alone or is caused in other ways? The validity of the Doppler principle has been proved by laboratory experiments only for velocities smaller than 1 km./sec. or so. The measures of stellar spectrograms giving such velocities as can be computed from the laws of gravitational astronomy (*e.g.* velocity in orbit of *Venus*, *Earth*, and *Mars*, radial velocities of comets) have proved the correctness of the Doppler formula for velocities as high as 100 km./sec., and thus it seems allowable to assume that the displacements found for globular clusters and spiral nebulæ are due to motions of the objects in the non-relativistic sense or to motions and the above-mentioned effect of the curvature of space-time.

If we treat the 18 velocities known for globular clusters and the 43 for spiral nebulæ as real motions, and compute the apex for the *Sun's* motion relatively to the two systems by solving equations of the form

$$X \cos \alpha \cos \delta + Y \sin \alpha \cos \delta + Z \sin \delta + K - V = 0,$$

where K is a systematic correction to the velocities and V the observed radial velocity, we obtain the results:

Globular Clusters.		Spiral Nebulæ.
$A = 20^h{\cdot}4$	$A = 21^h{\cdot}0$	$A = 20^h{\cdot}3 \pm 3^h{\cdot}0$ (m.e.)
$D = +60°$	$D = +64°$	$D = +75° \pm 30°$
$V_0 = 305$ km./sec.	$V_0 = 304$ km./sec.	$V_0 = 651 \pm 135$ km./sec.
$K = +31$ km./sec.	$K = 0$ (assumed) *	$K = +793 \pm 88$ km./sec.

* The mean errors of the solution cannot be given here, but we may quote the mean errors derived at a solution using 16 globular clusters (*Publ. A.S.P.*, 35, 318, 1923), from which we find $A = 21^h{\cdot}3 \pm 6^h{\cdot}0$, $D = +64° \pm 22°$, $V_0 = 266 \pm 112$ km./sec., and $K = 0$.

Thus the *Sun* has a different motion relatively to the systems of globular clusters and spiral nebulæ from that relative to the nearest stars ($A = 18^h$, $D = +30°$, $V_0 = 20$ km./sec.). The explanation may simply be that *our local system as a whole* has the motion found above relatively to the huge systems of globulars and spirals. When using the nearest stars for a determination of the apex we do not get this motion, but the motion of our *Sun* relative to the local system. If we include in our solution the nearest stars as well as more distant objects, we obtain a value for the apex and the velocity V_0 intermediate between the values obtained by using separately very close and remote objects. This is in harmony with the fact* that Dyson and Thackeray obtained a progression in the position of the solar apex when grouping the Groombridge stars according to apparent magnitude; that is to say, when going from nearer to more distant stars. The progression in question shows a change of the position of the apex towards higher declinations, and increases the right ascension and the value of V_0.†

It is of interest to note that our motion relatively to the spirals is directed towards galactic longitude $75° \pm 17°$, or nearly at right angles to the direction (gal. long. $= 325° - 335°$) where many modern researches place the centre of our galactic system. This fact might suggest that our local cluster is revolving around this centre or moving in an orbit with a period of roughly $3 \cdot 10^9$ years.

A support for the view that the systematic motions of the spiral nebulæ and globular clusters derived above might be a real phenomenon not showing any effect of the curvature of space-time is perhaps obtained from using the proper motions of the spiral nebulæ. The best values we have for these are undoubtedly the ones derived by van Maanen from measuring 882 "nebular points" in 7 spirals.‡ Using the values for μ_α and μ_δ according to Dr. Smart's re-discussion,§ and solving Airy's equations,

$$x \sin \alpha - y \cos \delta = \mu_\alpha \cos \delta$$
$$x \cos \alpha \sin \delta + y \sin \alpha \sin \delta - z \cos \delta = \mu_\delta,$$

we find the following value for the apex:

$$\begin{aligned} A &= 1^h{\cdot}7 \pm 4^h{\cdot}7 \text{ (m.e.)} \\ D &= +63° \pm 45° \\ g &= +0''{\cdot}0074 \pm 0''{\cdot}0055. \end{aligned}$$

Of course considerable uncertainty is involved in this result on account of the scantiness of the material at hand. Still it may be more

* *M.N.*, 65, 428, 1905.

† *Kungl. Svenska Vetenskapsakademiens Handlingar*, Bd. 60, No. 8, 1920.

‡ The proper motions derived by Curtis for 66 spirals and by the present writer for 82 spirals, to be mentioned later in this paper, are not considered to be real by either author. The values obtained give an upper limit for the size of the mean proper motion, and thus are useful as also giving an upper limit for the size of the mean parallax.

§ *M.N.*, **84**, 333, 1924.

than a chance that we obtain the same direction for the apex from the proper motions as from the radial velocities. As the former are not affected by a curvature of space-time, the conclusion would be that the spectral shifts observed for globular clusters are due to real motion and show no influence of the world-curvature. On account of the uncertain material we leave the question open until more information is accumulated.

The apex computed from the radial velocities of spiral nebulæ seems to be fairly well determined, inasmuch as calculations performed for a few years ago only, using 15 or 18 objects, gave essentially the same results.*

4. Turning to the question of the determination of the curvature of space-time from the radial velocities of the globular clusters, it may at first be noticed that there is an essential difference between the views held by Weyl and Eddington and the ones held by Silberstein. According to the former authors there are no terms in the expression for the Doppler shift which will explain approaching velocities, whereas according to Dr. Silberstein both plus and minus occur in the formula, which thus accounts as well for receding as for approaching velocities. As Dr. Silberstein promises to discuss this question at length in a coming edition of his book, *The Theory of Relativity*, we may take his views for granted, and see what result can be obtained by using the material at hand for a determination of R.

Dr. Silberstein has not given, and will probably not be able to give, any justification for the use of the velocities of the globular clusters for a determination of R. These objects are probably among the most distant celestial objects we know at present, but how do we know that they are so far away that the effect of the curvature of space-time outweighs the effect of the real motions of the clusters themselves? In fact, the smallness of the K term for the globular clusters, found above to be +31 km./sec., while for the spiral nebulæ this term is nearly +800 km./sec., suggests that the former objects are comparatively near as compared with the latter, and that the globulars are little if at all affected by the slowing down of atomic vibrations in distant objects in de Sitter's world which might be erroneously interpreted as a motion of recession.

But granted the globular clusters are distant enough to permit a determination of the R, it does not seem possible at present to get an accurate determination from the material at hand. It is to be regretted that Dr. Silberstein in his articles has not used the whole material. Of the 16 velocities so far published he uses only 7. He seems to have been aware of the existence of at least 10 cluster velocities, but he says: "In view of the said large p.e. of the latter (the velocities of clusters), only those are worth considering the radial velocities of which are not much less than a hundred km./sec." Of course, there is no good reason for selecting such an arbitrary limit for excluding objects *which do not give a rather constant value of the R.* Furthermore, the velocities of globular clusters have certainly a much smaller mean

* *Kungl. Svenska Vetenskapsakademiens Handlingar*, Bd. 60, No. 8, 1920; *Astr. Nachr.*, 209, 369, 1919.

error than 100 km./sec. Taking the two objects measured both at the Lowell Observatory and at Mount Wilson, we have :

	Sanford.	Slipher.
N.G.C. 5024	−200 km./sec.	−175 km./sec.
5272	−140	−125

indicating a rather small probable error.

5. As the spectrograms of the spiral nebulæ are of much the same nature as those of the globular clusters, we may obtain an idea concerning the uncertainty in deriving cluster or nebular velocities from the following small table giving the results of several different determinations of the radial velocities of two spiral nebulæ :

N.G.C. 224.		N.G.C. 1068.	
Radial Velocity. km./sec.	Authority.	Radial Velocity. km./sec.	Authority.
−329	Pease and Adams.	+ 910	Moore.
−297	,, ,,	+ 940	Lundmark.*
−300	Slipher.	+ 765	Pease.
−350	Wolf.	+1060	Slipher.
−304	Wright.	+1150	,,
−316±10 (m.e.)		+1080	,,
		+1135	,,
		1005±53 (m.e.)	

In the case of N.G.C. 1068 there seems to be a rather high mean error in the radial velocity, but the peculiarities of the spectrum suggest that the differences found by different observers may be partly real.

The mean error in the velocities of globular clusters and spiral nebulæ does not seem to be higher than 25–30 km./sec. Of course, it is not justifiable to exclude the values of velocities even below that limit in computing R, as the mean of the small velocities may very well have a real significance, and thus give an important contribution to the determination of R. The very fact that small radial velocities exist among the distant clusters shows that the R must be greater than the value found by Dr. Silberstein from his *selected* values of existing radial velocities.

Using the 16 velocities of globular clusters together with two as yet unpublished values, kindly communicated in a letter by Dr. Slipher, we find the values for R given in Table I. For the distances of the globular clusters values are used derived by Shapley and by the present writer using different methods.† It is clear from an inspection of the table that the rather large dispersion in R found by us cannot be explained by the uncertainty in the relative parallaxes of the clusters.

* Remeasured on the request of Dr. Moore.

† *Mount Wilson Contr.*, pp. 151–157, 1918; *Astrophys. Journ.*, **48–49**, 1918; *Kungl. Svenska Vetenskapsakademiens Handl.*, Bd. 60, No. 8, 1920.

TABLE I.

The Curvature Radius of Space-Time as derived from Globular Clusters.

Object.	Radial Velocity.	Distance in Parsec. Acc. to Shapley.	Distance in Parsec. Acc. to Lundmark.	R in km. From Shapley's Parallaxes.	R in km. From Lundmark's Parallaxes.
N.G.C. 1851	+300	1·72 . 10⁴	1·41 . 10⁴	3·6 . 10¹²	2·9 . 10¹²
1904	+300	2·56	2·50	5·3	5·2
5024	−190	1·89	1·49	6·2	4·8
5272	−135	1·39	0·80	6·4	3·7
5904	+ 10	1·25	0·75	77·3	46·4
6093	+ 70	2·00	1·41	17·7	12·5
6205	−300	1·11	0·67	2·3	1·4
6218	+160	1·23	0·89	4·8	3·4
6229	−100	4·35	7·14	26·9	44·3
6266	− 50	1·52	1·43	18·8	17·7
6273	+ 30	1·59	1·56	32·8	32·2
6333	+225	2·50	2·27	6·9	6·2
6341	−160	1·23	1·03	4·8	4·0
6626	0	1·85	1·92	∞	∞
6934	−350	3·33	3·70	5·9	6·5
7078	− 95	1·47	1·19	9·6	7·7
7089	− 10	1·56	1·01	96·5	62·5
7099	−125	1·72	1·30	8·5	6·4
Small Magellanic Cloud	+168	3·10 *	...	11·4	...
Great Magellanic Cloud	+278	3·45	2·78 †	7·7	6·2

The graph shows that there is little or no correlation between V and r, in contrariety to what is to be expected from the theory of Dr. Silberstein. Computing the correlation coefficient, r, we find the following value:

$$r = 0{\cdot}255 \pm 0{\cdot}227,$$

which cannot be considered to have a real significance.

The parallaxes for globular clusters derived by Shapley and by the writer involve the supposition that the apparent diameters of the clusters are very intimately related to the parallaxes or distances, and if Dr. Silberstein's theory is correct, they also are closely correlated to the apparent diameters. Computing the correlation between the apparent diameters and radial velocities, we find:

$$r = -0{\cdot}141 \pm 0{\cdot}223 \text{ (the writer's diameters).}$$
$$r = -0{\cdot}185 \pm 0{\cdot}229 \text{ (Shapley's diameters).}$$

As the dispersion in R is $26{\cdot}1 \,.\, 10^{12}$ km. and thus considerably higher than what could be expected from the dispersions in V and r,

* *Harv. Circ.*, No. 255, 1924. † *Observatory*, 1924 Sept.

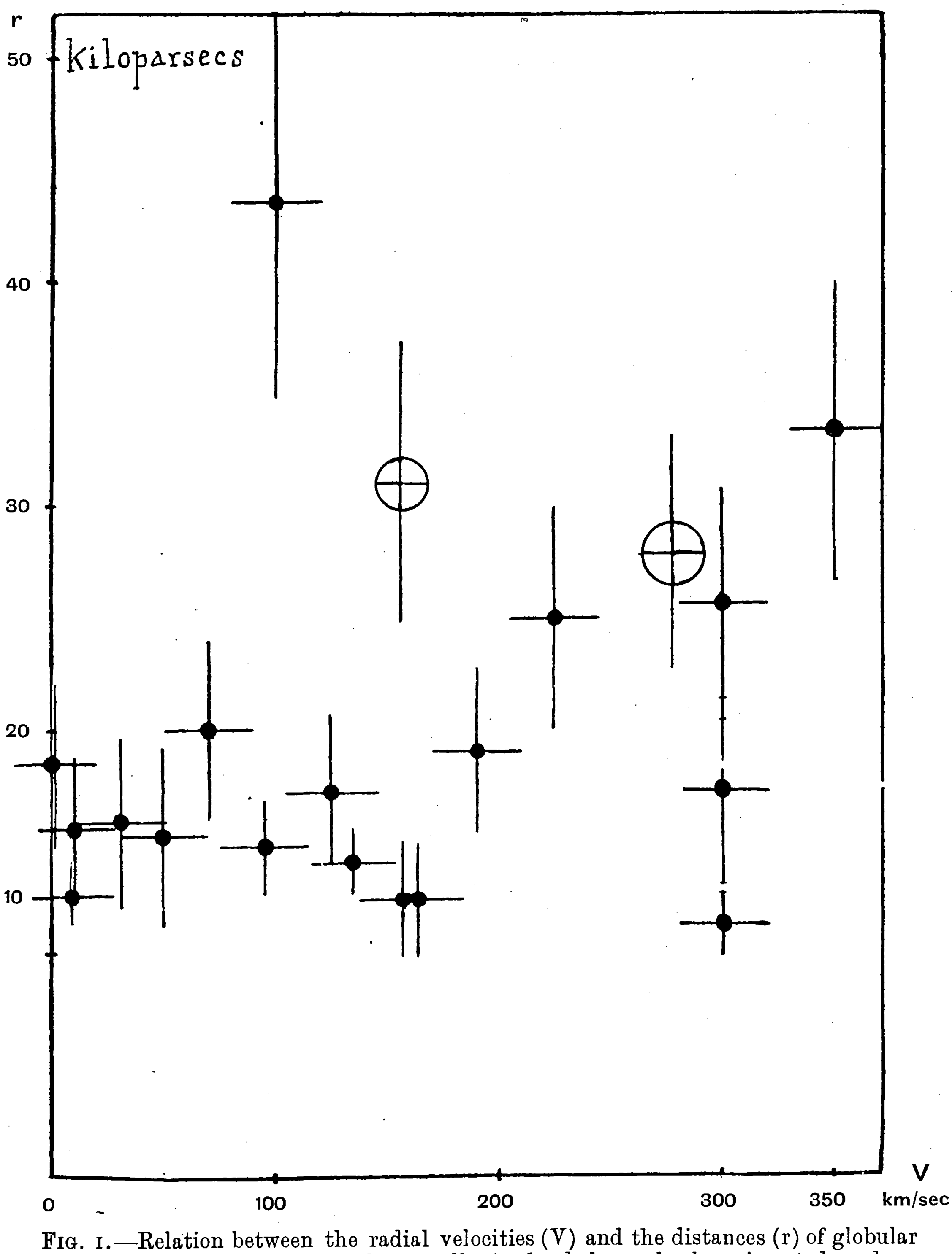

Fig. 1.—Relation between the radial velocities (V) and the distances (r) of globular clusters. The values for the Magellanic clouds have also been inserted, and are denoted by circles. The crosses indicate the size of the estimated probable error in either direction. This diagram, as giving all the known cases, supersedes that of Dr. Silberstein in *Nature*, 1924 June 7.

it does not seem that the curvature of space-time, at least for the present, can be determined with any accuracy by using the displacements in the spectra of globular clusters.

But if we assume that a value for R can be obtained, it will certainly be considerably larger than that of $6 . 10^{12}$ km. adopted by Silberstein, and which he considers worthy of confidence. From Table I. the mean value is found to be $19{\cdot}7 . 10^{12}$ km., using Shapley's distances, which should be preferred to the writer's on account of the more extensive material on which they are based. We have excluded N.G.C. 6626, as no stress should be laid on the fact that this cluster gives infinity for the curvature radius. The actual value may well be $80 . 10^{12}$ km. or higher, but the very small velocity of the object favours a higher value for R than that adopted by Dr. Silberstein.

6. Dr. Silberstein applies his R to the problem of determining at what distance from the *Sun*, r^*, de Sitter's "repulsion" is just balanced by the gravitational attraction. He computes the r^* at 3·45 parsecs, and thinks there are several stars close to this "neutral" sphere. As this result, if correct, is of an unusual importance for our conceptions of the stellar system, we will see if the result can be confirmed.

TABLE II.

The Stars having a Parallax $\geq 0''{\cdot}20$.

	Name.	α (1900).	δ (1900).	π Abs. Red.
		h m	° ′	″
1	Groombr. 34 *	0 12·7	+43 27	0·282
2	[+55°·100	26·6	+55 27	0·200] †
3	v. Maanen's star	43·9	+ 4 55	0·255
4	τ Ceti	1 39·4	−16 28	0·324
5	Fox's star	2 50·3	+52 5	0·254
6	ε Eridani	3 28·2	− 9 48	0·318
7	o^2 Eridani *	4 10·7	− 7 49	0·198
8	Cord. Z.V. 243	5 7·7	−44 59	0·309
9	Sirius *	6 40·7	−16 35	0·374
10	Procyon *	7 34·1	+ 5 29	0·313
11	Groombr. 1618	10 5·3	+49 58	0·208
12	W.B. $10^h{\cdot}234$	14·2	+20 22	0·197
13	22 H Camelop.	57·9	+36 38	0·409
14	Innes' star	11 12·0	−57 2	0·337
15	AOe 11677	14·8	+66 23	0·215
16	A.G. Berl. 5003	13 40·7	+15 26	0·216

* Double star or physical system.

† Excluded, as the parallax found by Peters needs confirmation.

TABLE II.—*continued.*

	Name.	α (1900).	δ (1900).	π Abs. Red.
		h m	° ′	″
17	Proxima Cent.	14 22·9	−62 15	0·761
18	α Centauri *	32·8	−60 25	0·763
19	AOe 17415	17 37·0	+68 26	0·238
20	Barnard's star	52·9	+ 4 25	0·555
21	70 Ophiuchi *	18 0·4	+ 2 31	0·196
22	Σ 2398 *	41·7	+59 29	0·292
23	Altair	19 45·9	+ 8 36	0·214
24	δ Pavonis	58·9	−66 26	0·203 †
25	279 G Sagittarii *	20 4·6	−36 21	0·209
26	61 Cygni *	21 2·4	+38 15	0·310
27	Lac. 8760	11·4	−39 15	0·233
28	ε Indi	55·7	−57 12	0·269
29	Krüger 60 *	22 24·4	+57 12	0·256
30	Lac. 9352	59·4	−36 26	0·282
31	Cord. 32416	23 59·5	−37 51	0·219

The absolute parallaxes have been used, and van Maanen's reductions for systematic errors have been adopted.

At first it will be noticed that there are not, as Dr. Silberstein states, 19 stars having a parallax $\geq 0''{\cdot}20$, but 30 or 31, as will be seen from the accompanying Table II. Our knowledge concerning the southern sky is as yet incomplete, and, besides, if we had a complete survey of radial velocities we certainly should find several stars with large parallaxes among those having high radial motions and moderate proper motions. Anyhow, the stars known at present within a radius of 5 parsecs do not show such a crowding as required by Dr. Silberstein. Not counting components of double stars or distant companions except *Proxima Centauri*, we find that there are 12 stars between the limits $0''{\cdot}255$ and $0''{\cdot}340$, or 3·92 and 2·94 parsecs. If the 30 stars filled uniformly a sphere of 5·10 parsecs, the number of stars which would fall within these limits would be 8·0. Thus there is scarcely any crowding that cannot just as well be accounted for by our imperfect knowledge of the parallaxes of the stars within this sphere. If we assume, which is very reasonable, that the unknown stars falling within this sphere are more likely to have a parallax smaller than $0''{\cdot}255$ than larger, we see that the very slight tendency to a crowding at present suggested may easily be obliterated.

As from the complete material of radial velocities for globular clusters a much higher value for R is suggested than that derived by Silberstein, we find that the value for r^* should be 11·29 parsecs, which corresponds to a parallax of $0''{\cdot}091$. In order to see if there is any

* Double star or physical system.
† Included on the strength of the spectral and proper-motion parallax.

crowding of stars around this value, we have grouped the known parallaxes according to the following table:—

Parallax.	Number of Trigonometric Parallaxes.	Number of Spectroscopic Parallaxes.	A(π).	A′(π).
0″·191 – 0″·200	2	2	2	2
0·181 – 0·190	4	3	6	5
0·171 – 0·180	5	2	11	7
0·161 – 0·170	5	0	16	7
0·151 – 0·160	2	7	18	14
0·141 – 0·150	8	4	26	18
0·131 – 0·140	8	7	34	25
0·121 – 0·130	5	7	39	32
0·111 – 0·120	9	11	48	43
0·101 – 0·110	14	17	62	60
0·091 – 0·100	17	33	79	93
0·081 – 0·090	19	24	98	117
0·071 – 0·080	39	37	137	154
0·061 – 0·070	60	62	197	216
0·051 – 0·060	84	129	281	345

The material used is obtained for the trigonometric parallaxes from our card catalogue containing about 1750 stars measured for parallax, and for the spectroscopic parallaxes from the catalogues of Adams (and co-workers), Rimmer, Abetti, Shapley (and co-workers), Edwards, Lundmark and Luyten, which altogether contain nearly 3000 stars. Absolute mean parallax-values have been used.

The figures A(π) and A′(π) giving the number of stars having a parallax larger than π can easily be used for computing the number of stars to be expected within the different shells if the stars are uniformly distributed within the sphere having a radius of 20 parsecs.

There does not seem to be any crowding around the value $\pi = 0''{\cdot}091$, and although the parallaxes $\geq 0''{\cdot}20$ are certainly more completely known than are the parallaxes between $0''{\cdot}20$ and $0''{\cdot}10$, still the fairly regular distribution of the known parallaxes does not suggest that further scrutiny will detect such a crowding as expected on Dr. Silberstein's theory.

7. As the value of R derived from the globular cluster material is rather uncertain, we will examine other classes of distant objects where it might be possible to find an effect of the curvature of space-time. Such classes of rather distant objects are the Cepheids, the Novæ, the O stars, the eclipsing variables, certain classes of red stars, and the spiral nebulæ. The distances of these objects are generally sufficiently large for the curvature of space-time to be perceptible if it exists at the

distances of the globular clusters. Besides, there ought to be a progression in the numerical values of the radial velocities with the distance if we have a group of bodies with a considerable range in the individual distances, such as the Cepheids.

(*a*) *Cepheids.*—The distances of these stars, first derived by Hertzsprung and later revised by Shapley by using the data for proper motion in connection with the period-luminosity law, are the foundation for the adopted distances of globular clusters. As the radial velocities have been used by deriving the mean parallax without applying any correction for a possible influence of the curvature of the space-time, it may not be allowable to compute the R from the distances thus derived.* Of course the same remark refers to the use of the distances of the globular clusters, as they are founded on the mean parallax of Cepheids.

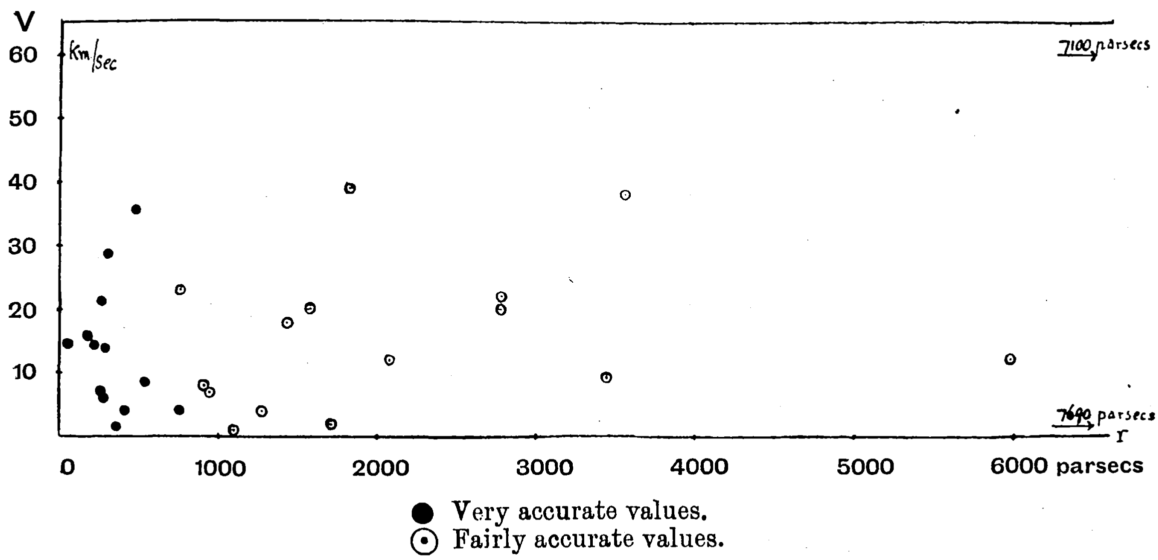

FIG. 2.—Radial velocities of Cepheids plotted against their distances. Two stars fall outside the diagram, as indicated by the arrows.

Now, the direct measured parallaxes for Cepheids seem to confirm the order of magnitude of the distance, and, besides, the influence of the curvature of space-time in the radial velocities is probably not very large when we deal with objects comparatively near. Thus we think we are justified in using the Cepheids for a determination of R.

For the 94 Cepheids having a period longer than one day the mean distance computed from the catalogue of Shapley is found to be 1940 parsecs.† Plotting the radial velocities known and the corresponding distances, we see that no progression is indicated in the former, as ought to be the case if R is a constant.

* Strictly speaking, the mean parallax of the Cepheids was derived by using the τ and υ components of their proper motions compared with the value of the velocity relative to the stellar system, or V_0, derived from other classes of stars. But Shapley has shown that in the radial velocities of Cepheids there is no systematic motion left. A computation of the apex of the Cepheids using the known radial velocities gives a value for V_0 in substantial agreement with the one adopted by Shapley.

† *Mount Wilson Contr.*, 153, 1918; *Astrophys. Journ.*, 48, 1918.

Grouping the 30 radial velocities available for Cepheids according to distance, we get:

$\bar{r}$. parsecs.	$\bar{V}$. km./sec.	Number of Objects.	R. km.
196	14·8	5	$0·9 . 10^{12}$
333	10·9	5	1·9
792	15·9	5	3·1
1360	14·8	6	5·7
2586	18·8	5	8·5
6030	16·6	4	22·5
1700	14·0	30	7·5

Even if the first three values be excluded, the progression in R is not favourable to Dr. Silberstein's views, and the conclusion seems to be that the spectral displacements of the Cepheids scarcely show any effect of the curvature of space-time.

(*b*) *The Novæ.*—The distances of the *Novæ* have been determined by the writer in several papers * on the reasonable assumption that the mean absolute maximum magnitude has a fairly constant value. The mean absolute magnitude at maximum, which evidently is more constant than the original or the final absolute magnitude of a Nova, has been derived by several different methods according to the following table:—

M_{max}.	Weight.	
−6·2	2	Direct or indirect determinations of parallaxes for individual *Novæ*. Altogether 39 values for 11 objects have been used.
−6·5	1	From the "dip" of *Novæ*, caused by the location of our Sun being north of the galactic plane.
−8·2	1	From the concentration of *Novæ* in the central regions of our stellar system (Sagittarius-Scorpius), the distance of these being known from Shapley's work on globular clusters.
−4·2	½	From comparisons with the galactic distribution of other classes of stars.
−7·1	1	From proper motions and radial velocities (τ components).
−6·9	1	,, ,, ,, (υ components).
−9·1	1	,, ,, ,, (parallactic motion).
−7·0±0·56		

The parallaxes of the 50 *galactic Novæ* known at present can be computed from the formula

$$\pi = 10^{-(2·4+0·2m_{max})},$$

where m_{max} is the apparent magnitude at maximum brightness.

* *P.A.S.P.*, **35**, 95, 1923; **34**, 207, 225, 1922: *Pop. Ast.*, **30**, 621, 1922. The most extensive account of the writer's results concerning the determination of the distribution of Novæ in space is given in a paper in Swedish, published in the Journal of the Swedish Astronomical Society (*Popular Astronomisk Tidskrift*, H. 3–4, 1922).

The three last methods in the above-given table are liable to be in error, as no correction for a possible influence of the curvature of space-time has been applied to the radial velocities. But if we take the mean from the first four methods we find $\bar{M}_{max} = -6{\cdot}8$. As these methods do not imply any use of radial velocities for distant objects, we think the parallaxes of *Novæ* are free from any systematic error arising from the effect of the curvature of space-time.

As to the radial velocities for *Novæ*, reliable determinations exist for only 8 objects. The values given in the following table have as a rule been obtained from the five components of the H and K lines, but in three cases (η *Argus*, *T Coronæ*, and *P Cygni*) from several lines in the absorption spectrum. The very large displacements of the bright emission bands have not been considered to represent the motion of the star itself, and we think most astronomers agree with us in this view.

The reliability of the radial velocities thus used is illustrated by the fact that a solution for the apex gives $A = 17^{h}{\cdot}0$, $D = +12°$, $V_0 = -11{\cdot}72$ km./sec., or very close to the direction found for most classes of stellar objects. If we use the displacements of the emission bands the mean distance of *Novæ* would be 1,000,000 parsecs, a result which in itself seems incredible and which is not in harmony with other evidences mentioned above concerning the space-distribution of novæ.

There does not seem to be any progression in the radial velocities with the distance, but of course the material is meagre. The mean distance for 50 novæ is found to be 9600 parsecs, and, excluding two uncertain cases, 8600 parsecs. As the mean radial velocity is 13 km./sec., we obtain

$$R = 41 \,.\, 10^{12} \text{ km.}$$

If we compute R from the individual values of V and r we obtain

	r. parsecs.	V. km./sec.	R. km.
Nova Persei 2	110	+ 6	$1{\cdot}1 \,.\, 10^{12}$
Nova Aquilæ 3	320	− 13	1·6
Nova T Coronæ	670	− 10	4·1
Nova Geminorum 2	170	+ 10	1·1
Nova η Argus	625	− 24	1·6
Nova Ophiuchi 4	7100	+ 6	73·0
Nova Cygni 1920	120	+ 17	0·4
Nova P Cygni	470	− 17	1·7

The parallaxes of the objects *Nova Persei* 2, *Nova Aquilæ* 3, and *Nova T Coronæ* ought to be very accurate, as they have been determined by several independent methods; on the other hand, their distances are rather small for a determination of R, and therefore the value derived from the mean distance and the mean radial velocity is to be preferred.

(*c*) *The O Stars.*—These stars being giants or rather supergiants have been assumed to have a constant absolute magnitude. There are 50 stars for which the proper motion is known, and 27 for which the radial velocity has been determined. Using these data for a determination of the mean parallax, we have found $\bar{\pi} = 0''\cdot00063$, corresponding to a value of $-4\cdot44$ for the absolute magnitude.

This value of the absolute magnitude may at first seem to be rather high, but it gets support from another method. In some cases there are involved in the galactic bright irregular nebulæ O stars as well as early B stars, and it seems to be a rule without exception that in such clusters of "nebulous" stars the O stars are considerably brighter than are the B stars. From eight such nebular clusters, where it can be

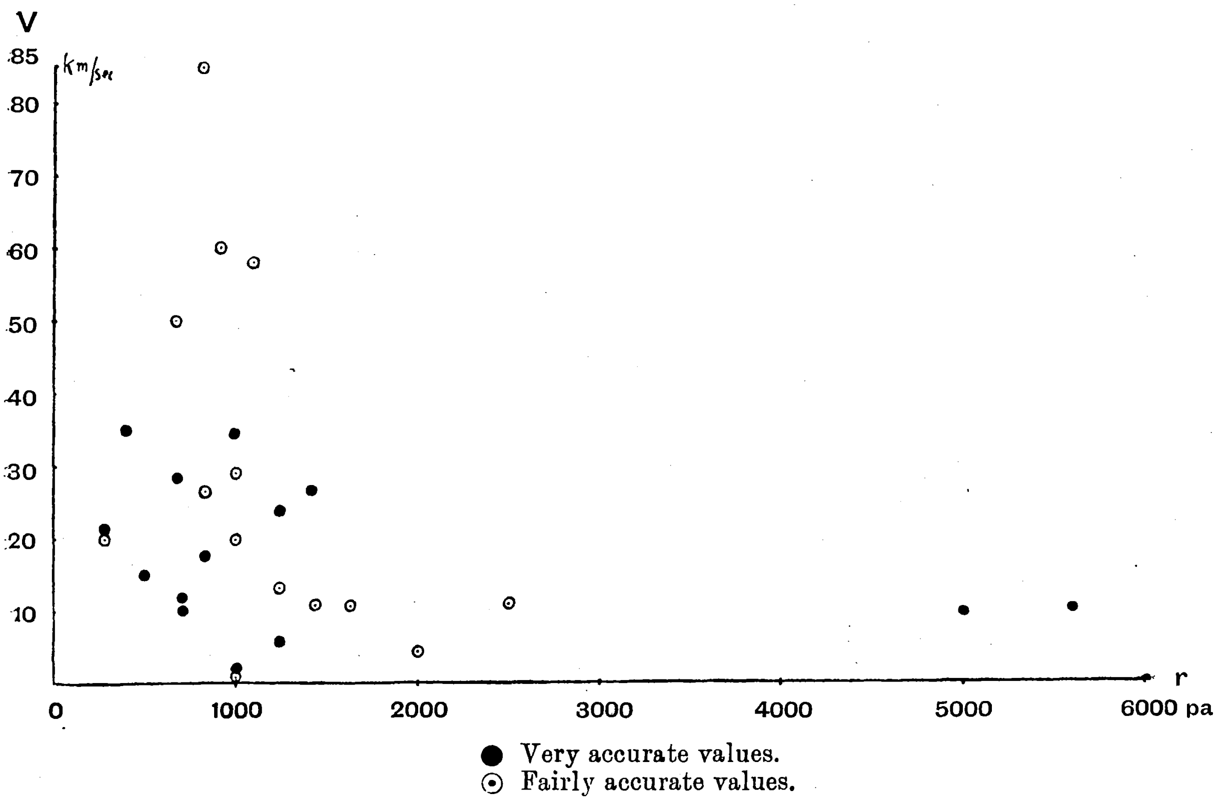

FIG. 3.—Radial velocities of O stars plotted against their distances.

tacitly assumed that the stars in question are practically at the same distance, it has been found:

Abs. mag. for O stars – Abs. mag. for Bo stars = $-1\cdot74$.

Adopting for the absolute magnitude of Bo stars $-2\cdot6$, in agreement with determinations of Charlier, Kapteyn, Adams, and others, we find the corresponding value for the O stars to be $-4\cdot34$. Combining the two values for the absolute magnitude, we see that the parallaxes of the O stars can be computed from the formula

$$\pi = 10^{-(1\cdot88+0\cdot2m)},$$

where m is the apparent visual magnitude.

No clear progression is indicated when grouping the radial velocities according to distance (fig. 3). As the individual values for the parallaxes may be uncertain on account of the assumption of a small dispersion in the absolute magnitude,* we use, in order to determine R, the mean of the distances, or 1560 parsecs, compared with the mean radial velocity 24·2 km./sec. We find

$$R = 4{\cdot}0 \,.\, 10^{12} \text{ km.}$$

This is fairly close to the value computed by Silberstein, but it cannot have much significance, as the values of R computed from individual velocities will show a very large dispersion, and, besides, the most distant O stars will give a value for R around 200 . 10^{12} km.

It will be pointed out that the radial velocities are in many cases very accurate, as being determined from the orbits of spectroscopic binaries. The scattering in the R values mentioned above can by no means be explained either by the uncertainty in the measured Doppler

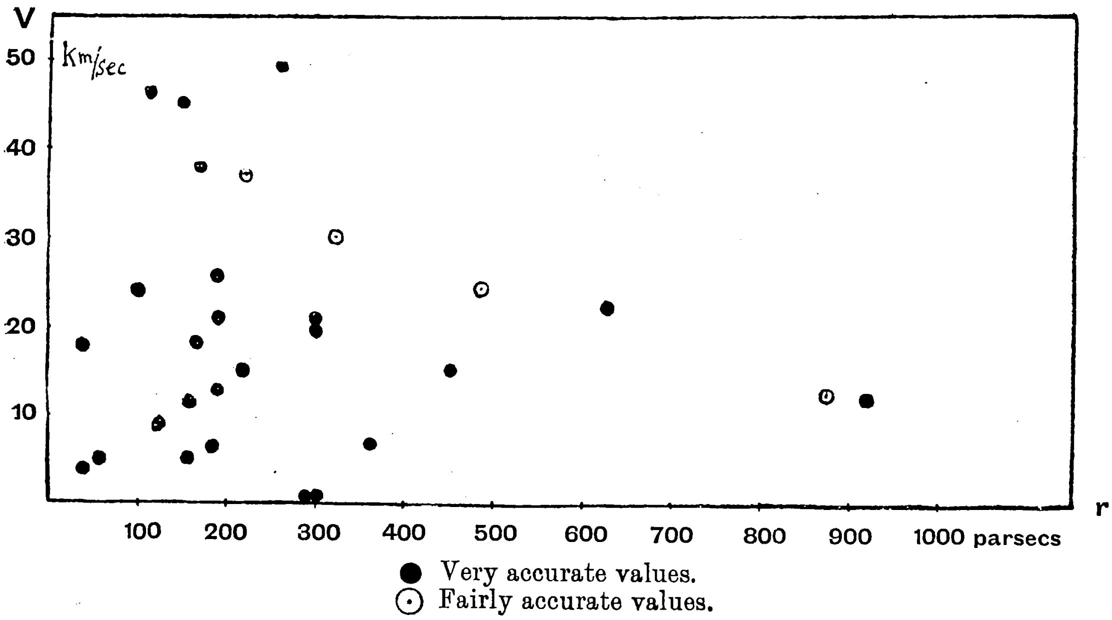

FIG. 4.—Radial velocities of eclipsing variables plotted against their distances.

shifts or by the uncertainty in the parallaxes derived from our assumption of negligible dispersion in the absolute magnitudes.

(*d*) *The Eclipsing Variables.*—The distribution in space of these stars has been obtained by Russell and Shapley † by using the surface intensity of the different spectral classes and certain elements from orbital determinations. The radial velocities are nearly all obtained from spectroscopic investigations of the orbit, and thus are very accurate.

There does not seem for these stars to be any progression in the radial velocities when plotted according to the corresponding distances (fig. 4). The mean value of 31 radial velocities is 18·2 km./sec., and the mean distance of the known variables is 800 parsecs.

Thus $$R = 2{\cdot}7 \,.\, 10^{12} \text{ km.}$$

* The dispersion in the absolute magnitudes seems to be fairly close to ±1·0 magnitude, corresponding to a dispersion of about 45 per cent. in the distances.

† *Astrophys. Journ.*, 40, 422, 1914.

As the distances of the eclipsing variables are comparatively small (all smaller than 1000 parsecs), it is probable that R cannot very well be determined from these stars, where the whole displacement may represent real motions. The fact that no progression is indicated in the radial velocities suggests that there is no effect of the curvature of space-time even in the distant stars of this group.

(*e*) *The R Stars.*—Accurate radial velocities for this interesting group of stars are due to the spectrographic work of Rufus * and of Sanford.† Using the known proper motions and radial velocities, Sanford found that the R stars are giants, as their mean absolute magnitude is $-1{\cdot}5$.

Using this value, distances for 29 stars have been correlated with Sanford's radial velocities according to the following condensed table:—

$\bar{r}$.	$\bar{V}$.	Number of Objects.	R.
parsecs.	km./sec.		km.
600	35·5	4	$1{\cdot}0 \,.\, 10^{12}$
810	16·2	5	3·2
1130	114·5	6	0·6
1360	62·5	5	1·3
1720	34·1	5	3·1
2580	34·2	4	6·7

This case is really rather favourable for Dr. Silberstein's theory, but against the result stands the testimony from several other cases. Besides, the radial velocity for the R stars is seen to decrease with increasing distance, which suggests that if still fainter (or more remote) stars could have been investigated the R value would have been larger than $6{\cdot}7 \,.\, 10^{12}$. Of course much weight cannot be attached to the first four values in the table, as the effect of the curvature may not show up for distances smaller than, say, 1000 parsecs.

(*f*) *The N Stars.*—There exist accurate radial velocities for 25 bright N stars, which have been determined by Moore,‡ who found that the absolute magnitudes for these stars must be high. The mean value was determined by him to be $-1{\cdot}1$ for the Na and $-2{\cdot}4$ for the Nb stars. The mean radial velocity is 18·8 km./sec., and as there does not seem to be any progression with distance, we adopt this value for the whole group. From Wilson's revision of the mean parallax it follows that the mean distance for the 92 N stars will be 710 parsecs, which gives the R value

$$2{\cdot}3 \,.\, 10^{12} \text{ km.}$$

The dispersion in the radial velocities is rather large. Besides, Wilson § is of opinion that the range in mean luminosity both in classes R and N makes estimations of distances of individual stars upon the assumption of equal luminosity within the class "decidedly uncertain."

* *Publications of the Observatory, University of Michigan*, **2**, 140, 1916.
† *Astrophys. Journ.*, **59**, 339, 1924; *Mount Wilson Contr.*, No. 276, 1924.
‡ *Lick Observatory Bulletin*, **10**, 160, 1922; *Publ. A.S.P.*, **35**, 124, 1922.
§ *Astr. Journ.*, **35**, 125, 1923.

As to other classes of red stars, extensive measures of the radial velocities for the M and S stars have been made by Merrill.* The mean parallaxes for the various subdivisions have been determined by Wilson,† Doig,‡ Strömberg and Merrill,§ and others. As the stars in question are comparatively near, the material has not been used for a determination of R. Furthermore, an uncertainty is involved by deriving parallaxes from the mean magnitudes, as the maximum brightness for the same long-period variable star does not seem to be a constant quantity.

Another group of stars that may be useful for testing Silberstein's theory are the so-called *c*-stars. These have not been used, as it is uncertain if all stars in the New Draper Catalogue said to have narrow sharp lines really can be included among the *bona-fide c*-stars.

(*g*) *The Spiral Nebulæ.*—The distances of these very interesting objects have been discussed during recent years from several points of view. Dr. Silberstein's statement that distances of 200,000 and 2800 parsecs have been derived by us for the Andromeda Nebula does not perhaps give an adequate expression to the views held in our papers of 1919 and 1920.‖ The different values given there for the Andromeda Nebula were deduced in order to show the state of the subject when approached in different ways. Besides, it was stated that the smaller distances were thought to be either limiting values or very uncertain, and that the largest one of 200,000 parsecs is probably the best. This value for the distance of the Andromeda Nebula was derived from the assumption that the absolute magnitude of *Novæ* at maximum brightness *in the mean* is the same for *Novæ* in the Galaxy and in the Andromeda Nebula. Later on it was adopted or confirmed by Curtis,¶ Luplau-Janssen and Haarch,** Doig,†† and others. From another method Öpik ‡‡ has derived a distance of 500,000 parsecs for the nebula. Later investigations by the writer suggest, on the basis of improved values for the absolute maximum magnitude of *Novæ*, a distance of the order of magnitude of half a million parsecs.§§

Since our first papers on the subject appeared it has been pointed out on several occasions ‖‖ that the advances during later years in this branch have practically excluded the possibility of spiral nebulæ being as close to us as suggested by the parallax work of Bohlin ¶¶ or the theory of Jeans.*** The mean distance of spirals has at least to be

* *Mount Wilson Contr.*, Nos. 264, 265, 1923; *Astrophys. Journ.*, **58**, 215, 1923.

† *Astr. Journ.*, **34**, 183, 1923.

‡ *Journ. B.A.A.*, **33**, 326, 192, 1923.

§ *Mount Wilson Contr.*, No. 267; *Astrophys. Journ.*, **59**, 97, 1924.

‖ *Astr. Nachr.*, **209**, 369, 1919; *Kungl. Sv. Vet. Akad. Handl.*, Bd. 60.

¶ Discussion with Shapley published as the pamphlet, *The Scale of the Universe*, also *Rivista di Scienza*, **35**, No. 1, 1924.

** *Astr. Nachr.*, **215**, 285, 1922.

†† *Journ. of the B.A.A.*, **32**, 138, 1922.

‡‡ *Astroph. Journ.*, **56**, 406, 1922.

§§ *Publ. A.S.P.*, **35**, 95, 1923.

‖‖ See, for instance, *Publ. A.S.P.*, **34**, 108, 1922.

¶¶ Bohlin measured a value for the parallax of the Andromeda Nebula of $0''{\cdot}171 \pm 0''{\cdot}051$, which later has been reduced to $0''{\cdot}051 \pm 0''{\cdot}079$. Other observers have got much smaller values (for instance, Barnard and van Maanen).

*** *Problems of Cosmogony and Stellar Dynamics*, Cambridge, 1919.

counted in several ten-thousands of light-years, and thus it is of quite another order of magnitude than the mean distance of the stars in the regions where most of the spiral nebulæ have been observed. On the other hand, the closest spirals cannot very well be situated at distances of many millions of light-years as originally suggested by Curtis,* inasmuch as several of the largest spirals are in their outer regions resolved into stars (*e.g.* Messier 33), and in accordance with our present knowledge ordinary stars as a rule very seldom have an absolute magnitude higher than -7 or -8. Besides, *Novæ* have been observed in seven spiral nebulæ, variable stars in one, and bright diffuse nebulosity in a few, and our knowledge concerning the physical properties of these objects prevents us placing the ones observed in spirals as far away as millions of light-years. From the measures of the effective wavelength of some 40 spirals, Lundmark and Lindblad † concluded that there was no sensible selective absorption in space, and at the same time Shapley ‡ arrived at the same conclusion from his cluster work.

8. Jeans § has recently computed distances corresponding to parallaxes of $0''\cdot0007$, $0''\cdot0011$, and $0''\cdot0066$ for the spiral nebulæ M32, M101, and M51 respectively. In the last case the parallax is really so large that it ought to be possible to measure it by trigonometric methods. Although Dr. van Maanen ‖ has by direct measurement found a parallax of $0''\cdot005$ for this object, I do not think he considers this circumstance as actually proving the nebula to be so close.¶ Besides, the proper motion of the object is small, and the internal motions suggest a larger distance than this. The method of Jeans for finding the parallaxes of nebulæ is very ingenious, but it does not give very reliable information about the real distances. The method is based upon a calculation of the mean distance between adjacent nuclei in the spiral arms. The comparison of these calculated distances with the mean apparent distances as estimated on photographs then gives the parallax. Taking the theory for granted, one can still question if it is possible to get a fair estimate of the apparent distance of nuclei. For instance, Jeans finds the nuclei in M101 to be $5''$ apart. But a much larger instrument than the 100-inch at Mount Wilson would undoubtedly reveal many more nuclei and show them to be apparently much closer than is the case on the photographs of to-day.** The so-called nebulous background in the spirals is as a rule certainly not nebulosity, but the light from numerous minute nuclei apparently gathered together on account of the vast distance from which they are seen. Jeans' estimates show us that the objects probably *are not closer* than estimated by him, but that they might very well be farther away. His method will be of value in the future, when

* *Modern Theories of the Spiral Nebulæ*, Washington, 1919; *Publ. A.S.P.*, **29**, 206, 1917; *Lick Obs. Bull.*, No. 300, 1917.

† *Astroph. Journ.*, **46**, 206, 1917, and also **50**, 376, 1919.

‡ *Mount Wilson Contr.*, 156, 1918.

§ *The Nebular Hypothesis and Modern Cosmogony*, Halley Lecture, 1922, Oxford, 1923.

‖ *Mount Wilson Contr.*, 158, 1918. ¶ van Maanen, *op. cit.*, p. 7.

** We may compare the photographs of Isaac Roberts and Ritchey of the same spirals. Where the former show a nebulous appearance the latter in many cases show numerous nuclei.

we know for certain that we have succeeded in completely resolving the spirals into stars.

One of the methods most free from objection is to derive the distances from comparisons between the proper motions and radial velocities. From available measures of the position of the central nuclei proper motions have been derived in Upsala for 82 spiral nebulæ.* The mean annual proper motion is found to be $0''\cdot042$. The motions thus derived are certainly not real. Computing the apex by using Airy's method, we find:

$$A = 19^h\cdot2$$
$$D = -69^\circ$$
$$q = 0''\cdot0135.$$

The deviation of this apex-value from the one found from radial velocities, and earlier referred to in this paper, suggests that our proper motions are strongly influenced by systematic errors. A detailed comparison with the proper motions derived by Curtis for 66 spirals from measures of photographs has made us believe that our motions are illusory in consequence of the influence of accidental errors in the measures, and only of use for giving an upper limit to the motion, as the real mean motion will be equal to or smaller than the mean error of the mean motion.

Using the value of 793 ± 88 km./sec. for Campbell's K term, we find the following values for the mean parallax of spiral nebulæ:—

π_m.	Material.	Number of Objects.
0·00010	Curtis	66
0·00011	Lundmark	82
0·000043	van Maanen	7
0·000054	,, (Smart's values)	7

If we had assumed $K = 0$ the mean parallaxes would not have been materially changed.

As stated before, the values of van Maanen are certainly the most accurate of the existing proper motions. Still it is quite possible that the actual motions are smaller, and that even the last value in the table represents an upper limit for the mean parallax of the spiral nebulæ investigated.

The mean radial velocity of the five of van Maanen's nebulæ for which this quantity has been measured is 220 km./sec., and the mean radial velocity for the nebulæ of Curtis and Lundmark will be around 600 km./sec. Computing R according to Silberstein's formula, we have:

R. km.	Material.
$1\cdot0 . 10^{12}$	Curtis
$0\cdot9 . 10^{12}$	Lundmark
$6\cdot6 . 10^{12}$	van Maanen
$5\cdot2 . 10^{12}$	,, (Smart's values)

* *Pop. Astr.*, **30**, 623, 1922; also read at Twenty-eighth meeting of the Amer. Astr. Soc., Williams Bay, 1922.

The very close agreement between the two last values and the one adopted by Silberstein is at first very striking, and it is tempting to take the result as confirming Silberstein's theory. But such a comparison would be premature, for the following two reasons. First, there are, as mentioned above, reasons to believe that the mean distance derived by this method is too small; and, second, in deriving the parallax we have used the Doppler shifts measured in spectrograms of spiral nebulæ as indicating true motions. But as the effect of the curvature of space-time will certainly be present in the spectra of spiral nebulæ in still higher degree than in the globular clusters, our procedure, which involves the supposition that the spectral shifts represent radial motions only, may well lead to wrong ideas about the distances.

If we want to investigate the relation between V and r for the spiral nebulæ measured for Doppler shift, we can make use of the hypothetical parallaxes given in Table III. The values have been founded

Table III.

Hypothetical Parallaxes, Relative Distances, Radial Velocities, and Relative R Values for Spiral Nebulæ. a is a Scale Factor.

N.G.C.	Parallax. From Total Magnitude.	Parallax. From Apparent Diameter.	Distance. The Unit is the Dist. of Andromeda Neb.	Radial Velocity.	Relative Values of R.
221 }	0″·00000510	0″·00000510	1·0	− 300	1·00a
224 }	510	510	1·0	− 316	0·95
278	3	6	130	+ 650	60·0
404	7	5	88	− 25	1056·0
584	8	7	68	+1800	11·3
598 }	225	230	2·2	− 70	9·4
604 }	225	230	..	− 270	..
936	6	13	62	+1300	14·3
1023	14	25	28	+ 200	42·0
1068	70	25	14	+1020	4·1
1700	2	3	212	+ 800	79·5
2681	6	4	106	+ 700	45·4
2683	16	29	25	+ 800	9·4
2841	35	29	16	+ 600	8·0
3031	105	75	5·8	− 30	58·0
3034	50	35	12	+ 290	12·4
3115	31	22	20	+ 600	10·0
3368	16	14	34	+ 940	10·8
3379	35	8	39	+ 825	14·2
3489	6	10	68	+ 600	34·0
3521	15	25	27	+ 730	11·1

TABLE III.—*continued.*

N.G.C.	Parallax. From Total Magnitude.	Parallax. From Apparent Diameter.	Distance. The Unit is the Dist. of Andromeda Neb.	Radial Velocity.	Relative Values of R.
3623	0″·00000017	0″·00000033	23	+ 800	8·6
3627	55	33	12	+ 650	5·6
4111	11	10	49	+ 800	18·3
4151	4	13	83	+ 940	26·5
4214	4	29	72	+ 300	72·0
4258	90	75	6·2	+ 500	3·7
4382	16	16	32	+ 500	19·2
4449	31	19	22	+ 200	33·0
4472	16	12	37	+ 850	13·1
4486	24	12	32	+ 800	12·0
4526	6	19	56	+ 580	29·0
4565	7	62	40	+1100	10·9
4594	50	29	14	+1140	3·7
4649	29	8	40	+1090	11·0
4736	95	23	14	+ 290	14·5
4826	45	33	13	+ 150	26·0
5055	29	34	16	+ 450	10·8
5005	4	21	76	+ 900	25·3
5194 }	106	50	7·5	+ 270	8·3
5195 }	106	50	7·5	+ 230	(9·8)
5236	31	42	14	+ 500	8·4
5866	4	11	87	+ 650	40·2
7331	0″·00000011	0″·00000038	30	+ 500	18·0

on the above-mentioned value of 200,000 parsecs for the Andromeda Nebula and derived from the supposition that the apparent angular dimensions and the total magnitudes of the spiral nebulæ are only dependent on the distance. Of course this is a rather rough hypothesis, but probably the best we can use for the present. The parallaxes derived from the total magnitudes have been reduced to the scale of those derived from the apparent diameters. The total magnitudes used have generally been estimated by Holetschek and revised by Hopman. The dimensions of the objects have been measured on plates from the Crossley collection of the Lick Observatory.

An inspection of the Table III. will show that a computation of R from the individual values of V will give inconsistent values for the radius of curvature.

From the parallaxes the mean distances have been calculated, the distance of the Andromeda Nebula being taken as the unit. Plotting the radial velocities against these relative distances (fig. 5), we find that there may be a relation between the two quantities, although not a very definite one. If this phenomenon were due to the curvature of space-time, we could derive the mean linear distance or determine the scale of our relative distances in the following way:—

Assuming that the average $\overline{v_0^2}$ for a group of objects is the same as

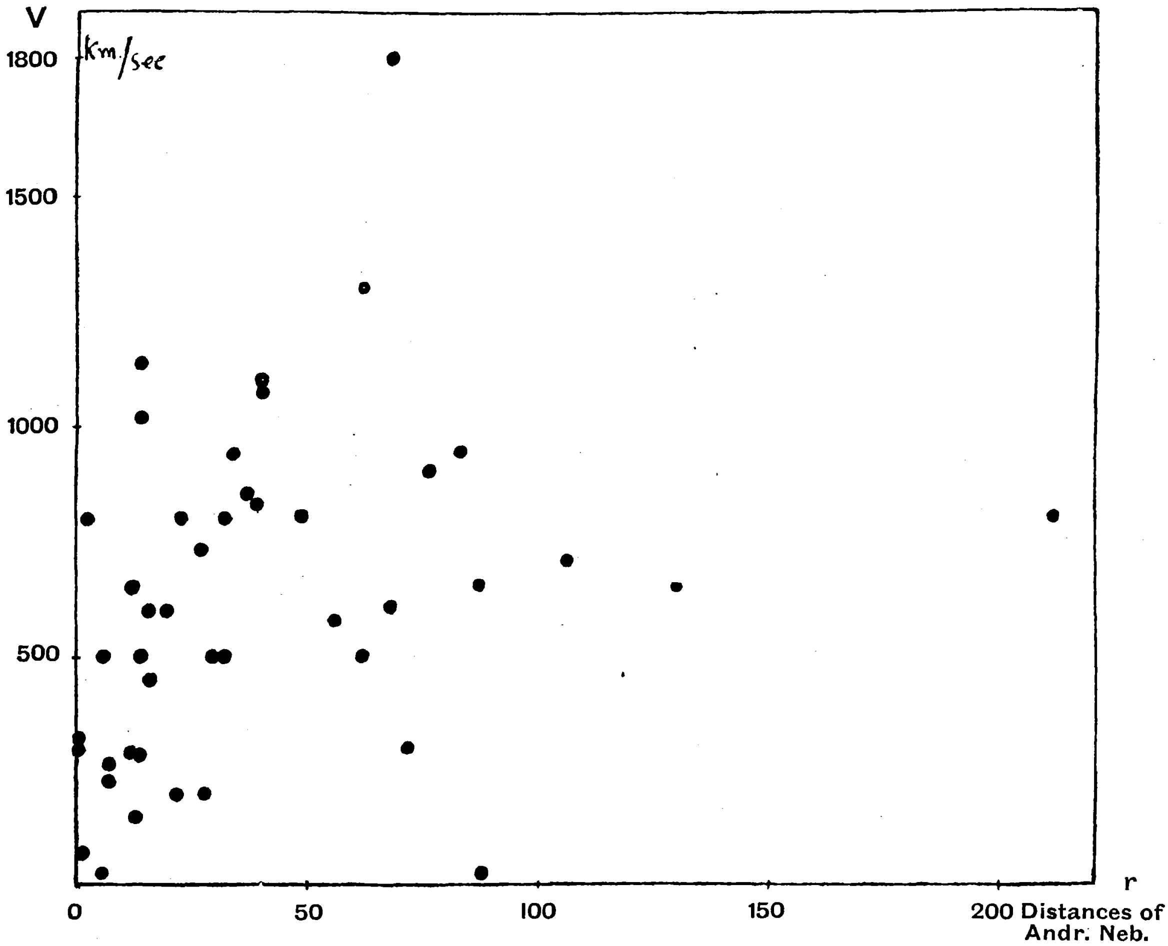

FIG. 5.—Relation between the relative distances (the unit is the distance of the Andromeda nebula) and the measured radial velocities of spiral nebulæ.

for another group, placed at another distance, Silberstein has from the reduced Doppler effect deduced the formula

$$\overline{D_1^2} - \overline{D_2^2} = \frac{1}{R^2}(\overline{r_1^2} - \overline{r_2^2}),$$

where D are the Doppler shifts and r the distances. [The bars denote the average values and the suffixes two different groups of objects.]

Taking two groups of spiral nebulæ we could compare them, for instance, with a group of globular clusters. Denoting the quantities of spirals and globulars with suffixes s and g, and eliminating R, we obtain

$$\frac{\overline{r_s^2} - \overline{r_g^2}}{\overline{D_s^2} - \overline{D_g^2}} = \frac{\overline{r_{s1}^2} - \overline{r_{s2}^2}}{\overline{D_{s1}^2} - \overline{D_{s2}^2}}.$$

In our table we have not r_s but the relative values ρ_s of the spiral distances. Putting

$$r_s = \rho_s \epsilon,$$

where ϵ is the scale-factor to be determined, we find the following equation for ϵ:

$$\epsilon^2 = \frac{\overline{r_g^2}(\overline{D_{s1}^2} - \overline{D_{s2}^2})}{\overline{\rho_s^2}(\overline{D_{s1}^2} - \overline{D_{s2}^2}) - (\overline{\rho_{s1}^2} - \overline{\rho_{s2}^2})(\overline{D_s^2} - \overline{D_g^2})}.$$

If the dispersion of the individual values of r or ρ within a group is small compared with the mean distance itself, then the formula above can be written

$$(\overline{D}_1)^2 - (\overline{D}_2)^2 = \frac{1}{R^2}\left(\overline{r}_1^2 - \overline{r}_2^2\right).$$

Grouping our material in the table according to the size of radial velocity, we find

$\bar{V}$.	$\bar{r}$.	No. of Objects.
km./sec.		
386	$30{\cdot}8 \pm 32{\cdot}0$	23
963	$54{\cdot}5 \pm 47{\cdot}0$	18

As the dispersion in r is of the same order of magnitude as $\bar{r}$ itself, the application of the simplified formula is not justified in this case.

Computing ϵ according to the rigorous formula, we find that its value or the linear distance of the Andromeda Nebula is

36,000 parsecs.

The value of R is $2{\cdot}4 \,.\, 10^{12}$ km., and if we use the Andromeda Nebula alone we find it to be $6{\cdot}6 \,.\, 10^{12}$ km.

If this distance of the nebula is correct, the parallaxes in Table III. should be multiplied by a factor close to 6. On account of the rather obscure correlation between V and r, not much weight can be attached to this determination of the distance of the Andromeda Nebula. Besides, the presumption that $\overline{v_0^2}$ has the same value for the globular clusters and the spiral nebulæ certainly is open to objection.

Dr. Silberstein's above-quoted formula, in which $\overline{v_0^2}$ is eliminated, will certainly be practicable for determining R if rather near objects are used. The best objects for such an application ought to be the moving clusters and some of the open clusters, because in several cases their parallaxes and radial velocities are known with very high accuracy. Besides, such groups are in much the same physical state, and the assumption of $\overline{v_0^2}$ being equal is probably fairly justified.

As the dispersion in the distances and in the Doppler shifts of the individual stars in a cluster is small, we can use the simplified formula for the computation.

The following list of objects has been selected :—

Object.	ϖ.	$\bar{V}$.
	"	km./sec.
Taurus group	0·0275	+36·4
Coma Berenices	0·012	− 2·0
Pleiades	0·0080	+10·0
Præsepe	0·0068	+33
Orion cluster	0·0036	+21·4
Messier 67	0·0013	+13
h and χ Persei	0·0007	−43·2
N.G.C. 1647	0·0022	+66
N.G.C. 6709	0·0012	+15

If we first compare the *Taurus* group with the more distant clusters, we see that R in several cases turns out *imaginary*, since the formula demands if $r_1 > r_2$ that $D_1 > D_2$. The same is the case, with two exceptions, if we compare *Præsepe* with more distant clusters, and, taking together the combinations where we get real values for R, we see that R cannot be determined at least from our present material with any accuracy.

Greenwich:
1924 *August.*

Naked-Eye Observations of Venus.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

Recent work has assigned a new importance to the ancient observations of the planet Venus, and modern observations of the times when Venus is first seen and last seen near the Sun may be of value in interpreting the ancient ones, even though the climatic conditions may be seriously different. The following observations have been put into my hands, and I take full responsibility for publishing them, to which the observers are somewhat reluctant, as perhaps implying too great an importance. But it seems to me that they may be useful to others as a guide at any rate, and perhaps a stimulus: and with the above disavowal plainly stated, publication cannot do much harm.

All observations were made with the naked eye, except that J. K. F. wore ordinary eye-glasses. They were all made at Oxford except the last. No places of Venus were computed in advance. All altitudes are true, refraction being disregarded.

First Appearance in Evening.

1923 Nov. 4. J. H. J. watched from $4^h\ 35^m$ to $4^h\ 55^m$ G.M.T. Clear but slight mist. Venus not seen.

5 and 6. Cloudy.

7. Venus picked up at $4^h\ 33^m$; very difficult; became quite easy by $4^h\ 40^m$; disappeared in mist between $4^h\ 45^m$ and $4^h\ 50^m$, say $4^h\ 47^m{\cdot}5$.