SHEARS & SHAPELETS

Konrad Kuijken
Leiden

SHAPELETS

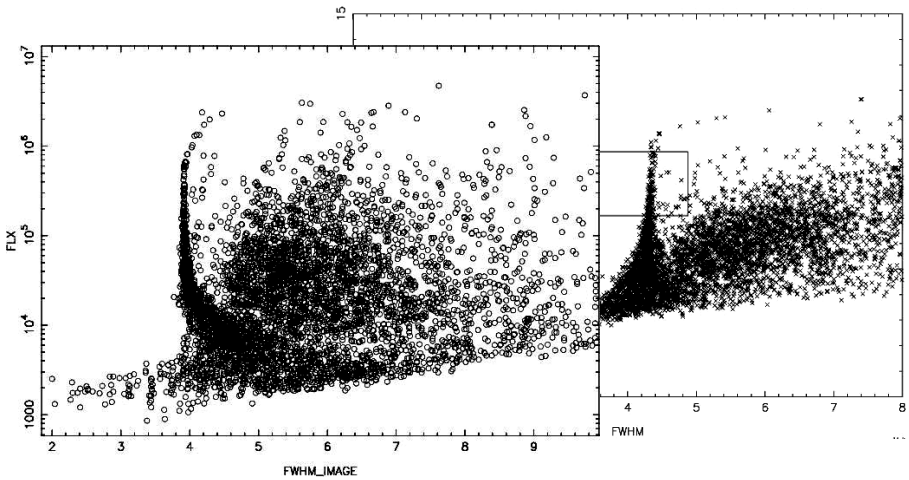
- Orthogonal basis functions in 2D
  - Gaussian x gauss-hermite polynomial
  - Nice transformation properties (little mixing) under
    - Translation
    - Rotation
    - Shear
    - Magnification
    - …
  - Reasonable approximations to PSF & galaxies (?)
- Will use cartesian shapelets - equivalent to polar shapelets provided keep terms to homogeneous order ($x^a y^b$ where a+b<=N)

SHAPELETS SHEAR PIPELINE

- Automatic from FITS image ⇒ individual shear estimators
- Fortran 77 code (sorry!)
- 4x4k image on 1.8Ghz w/s: 2min (order 8); 12 min (order 12)
  - # operations ~ N_{order}^4 (N^2 x N^2 matrices)

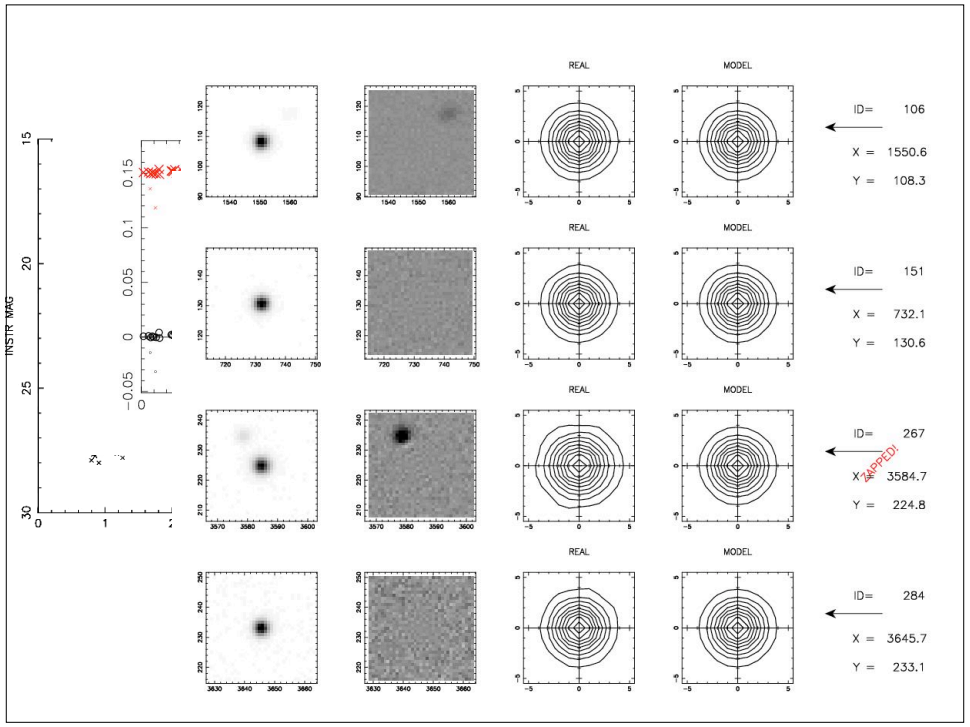
Source detection

- Sextractor
- Sizes measured with FLUX_RADIUS (not FWHM_IMAGE)

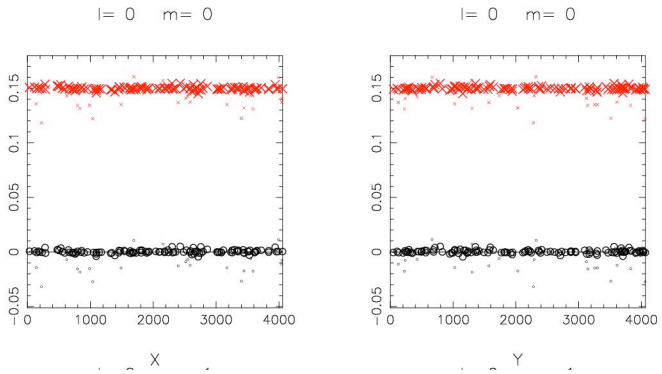


PSF mapping

- Select the stellar locus from MAG vs. FWHM plot
  - Heuristic algorithm:
    1. Find the 20th FWHM percentile of 10% brightest sources
    2. Everything in box within 0.5 pixel FWHM is candidate star. Strip off top magnitude, max magnitude range is 2.5.
    3. PSF FWHM is median FWHM of all these objects.
- Shapelet expansion to all candidate stars, using SAME β
- Diagnostic images of residuals after star models subtracted



PSF mapping

- 

  - Iteratively reject deviants:
    - Entire stars if S_{00} component is deviant
    - Single coefs otherwise

Shapelet encoding

- Each source gets a matched β
  - β=FWHM/2.3
  - Quantize β values to β_{PSF} x $2^{p/8}$
  - Shapelets coeffs are FITTED, not INTEGRATED
    - View pixel counts as regular samples of pixel-convolved images
    - Shapelet expansion describes the pixel-convolved image
    - Least-squares fit of linear combination of $S_{ij}(\mathbf{x}\text{-}\mathbf{x}_{SEx})$ to image
      - Fit all pixel values in a circle of 4β radius
        - » Need enough pixels: N=12 → 91 coefficients!
      - Propagate noise to shapelet coefs (~uncorrelated, equal)
    - Shift shapelet so that S_{01}=S_{10}=0 (few % of β)
- Fit all sources to same order N

Ellipticity determination

- Combination of PSF fit and ellipticity in one step (similar to Kuijken 1999)
- Find best-fit PSF-convolved, sheared circular source
  - *i.e., fit constant-ellipticity isophotes to pre-seeing source*
  - Requires evaluation of the matrix P that represents PSF convolution
    - Coefficients depend on β's of PSF, model galaxy and output (via C_{lmn} of Refregier)
    - Quantized β means that these coefs only need to be calculated a few times
    - Describe model (pre-shear, pre-PSF) galaxy with same β as observed image.

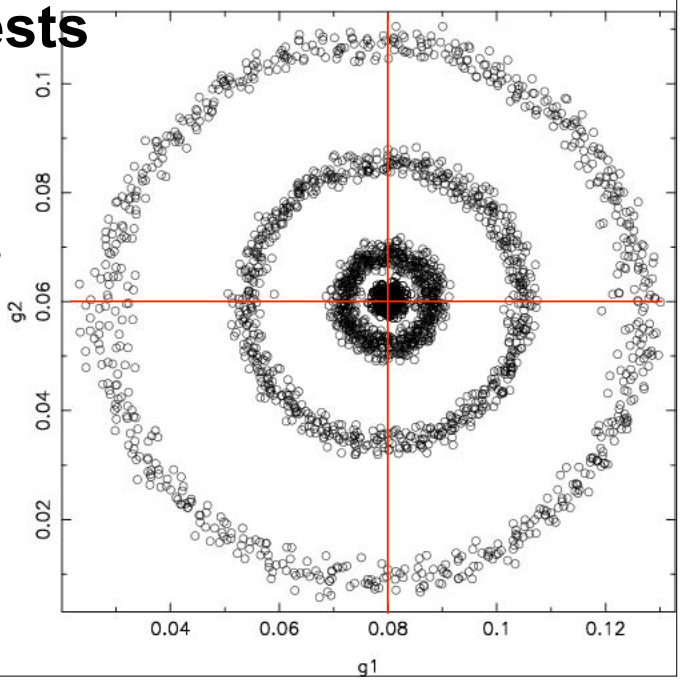
Ellipticity determination

PSF matrix **Shear + translation operator** **Arbitrary circular source**

- In shapelets formulation:
  - Model is $\mathbf{P} \times (\mathbf{1} + g_1 \mathbf{S_1} + g_2 \mathbf{S_2} + dx \mathbf{T_1} + dy \mathbf{T_2}) \times (c_0 C_0 + c_2 C_2 + c_4 C_4 + \ldots c_{N-2} C_{N-2})$
  - Marginalize over all parameters to get errors on g_1 and g_2
  - Do not weight all shapelet coefs the same in fit:
    - Do not fit order N (requires data from beyond truncation)
    - Downweight orders beyond N=2
    - Need higher orders as there is cross-talk due to PSF, but do not weight too high.
- Should work perfectly for
  - Small ellipticity (1st-order expansion in g_1 and g_2), **AND**
  - Constant ellipticity with radius for intrinsic galaxy shape
- Note: effect of pixelization is included in PSF matrix **P**

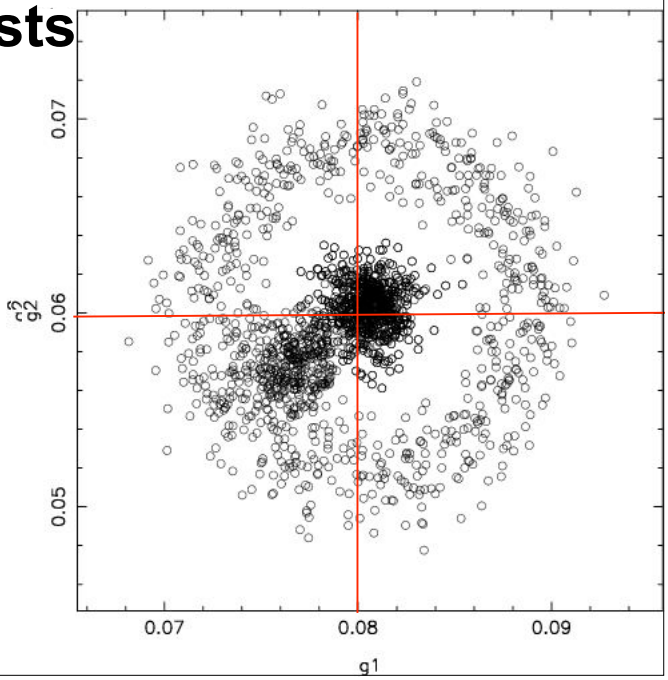
Simple Tests

- Exp disks + de Vauc', different flux and axis ratios
  - Input g_1=0.08, g_2=0.06
  - Expand to order N=12
  - **Well-resolved**



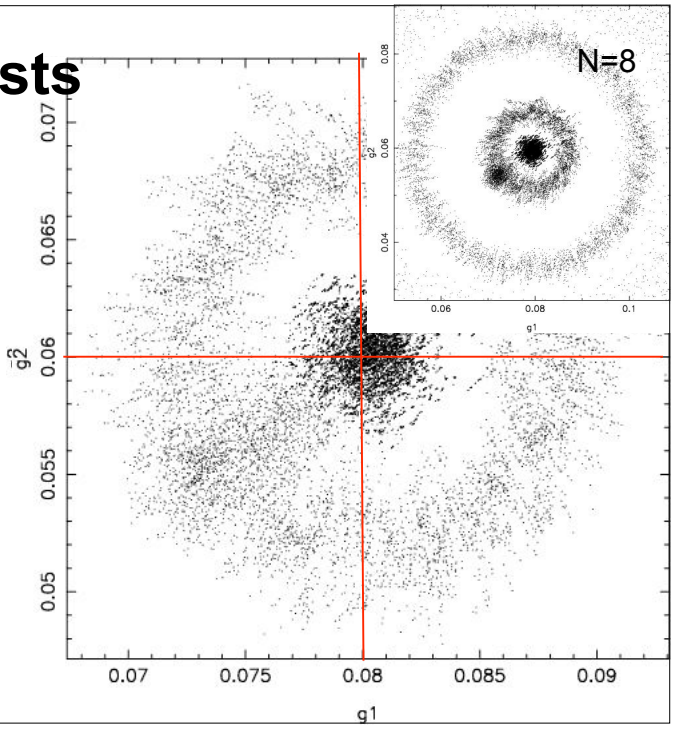
Simple Tests

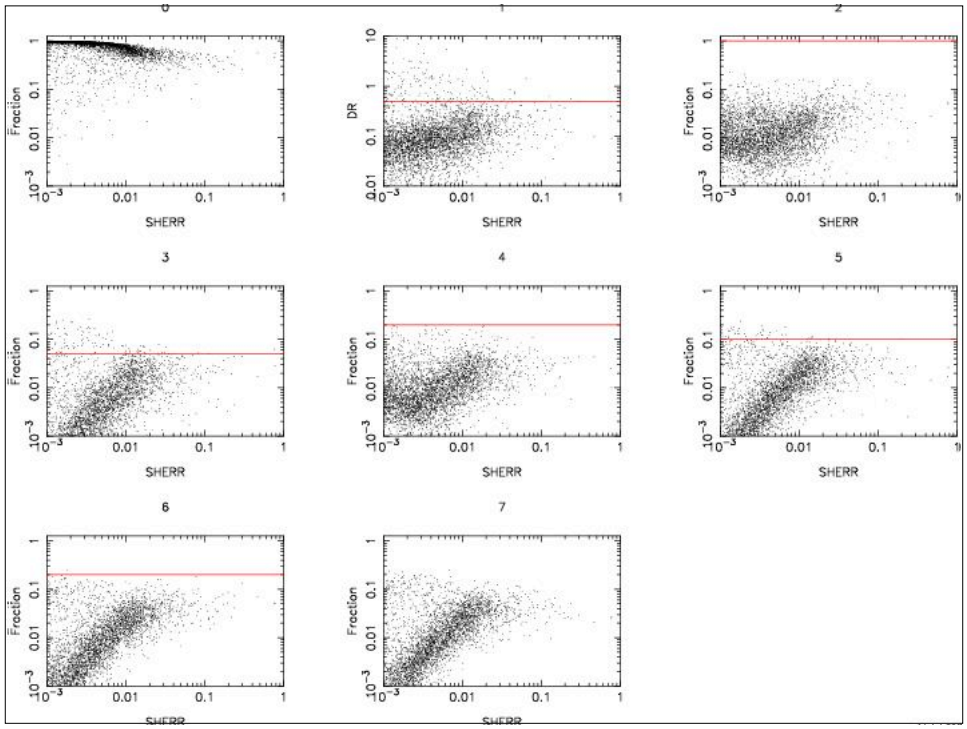
- Exp disks + de Vauc', different flux and axis ratios
  - Input g_1=0.08, g_2=0.06
  - Expand to order N=12
  - **Barely-resolved**



Simple Tests

- Exp disks + de Vauc', different flux and axis ratios
  - Input g_1=0.08, g_2=0.06
  - Expand to order N=12
  - **Ensemble**





Open issues/choices

- Which order to expand to?
  - Too low: bias shape
  - Too high: noise dominates / degenerate shapelet fits
- Which β to use for intrinsic (pre-PSF) image?
- Where does the limit of the source model (constant ellipticity with radius) kick in?
  - Use ensemble population to simulate and quantify effect
- Different basis functions (sech(r) $\times$ polynomials?)
- How to best estimate the centroid?

Other Applications of Shapelets

- PSF homogenization
  - Remove non-round component of PSF ("CLEAN")
  - Match PSFs for multi-colour or multi-epoch photometry
- Strengths:
  - Encode all useful info of sources in a linear way
  - Nice way to make a PSF map
  - Error propagation is straightforward

The ideal method ?

- Work with true 2nd moments
  - Prob: noise
- Fit galaxies with a single ellipticity
  - Prob: ellipticity and orientation changes with radius in galaxies
- Fitting of stacked galaxies (which are intrinsically round) as sheared, circular sources x PSF
  - Prob: centroiding accuracy
- Fitting of auto-correlation function of galaxies x ACF_{PSF}
  - Avoids centroid uncertainty
  - Form of stacking

Oort Workshop 2006, in Leiden

- July 31-Aug 4
- Lorentz Center
- Topic: lensing/cosmology
- STEP session?