PSF Interpolation Using Principal Components

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STEP collaboration meeting

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Overview

- Each analysis scheme measures something about stellar shapes in order to correct galaxy shapes:
  - KSB: Shear, smear polarizability matrices
  - Shapelet Deconvolution: Shapelet coefficients of PSF
  - BJ02 Reconvolution: Rounding kernel
  - RegLens: Non-Gaussian PSF residual
  - etc.

Overview

- The PSF shapes are generally not constant across an image.
- So correction needed at galaxy location requires interpolation from nearby stars.

Polynomial Interpolation

- The easiest interpolation method is to simply fit each component of the PSF measurement as a polynomial in (x,y).
- Typically somewhere from 2nd to 4th order polynomials are required.
  - Lower order gives poor fit.
  - Not enough stars to fit higher order well.

Rational Functions

- Henk and Ludo found that rational functions described their PSF variations better than simple polynomials for a given number of fitting coefficients.
- For example:

$$R(x,y) = \frac{P^{(3)}(x,y)}{P^{(1)}(x,y)}$$

Dense Stellar Fields

- Henk also tried observing dense stellar fields to fit a high order function, and use the stars in individual fields for a lower order correction.
- Only somewhat effective, since different exposures have variations in the PSF pattern of order unity.

Principal Components

- The variaion from exposure to exposure tends to follow the same rough patterns.
- A bunch will look one way, some others another way, etc.
- Potentially, we could use all the stars in similar looking patterns to fit the patterns more accurately and to higher order.

Principal Components

- Presumably, these variations are due to physical differences at the telescope for the different exposures
    - How far above/below focus
    - Degree of tracking error
    - Flex of telescope pointing off-zenith
    - Vibration or flex due to wind, etc.

Principal Components

- Describe PSF function as sum of several components:

$$F_i\left(x,y\right) = \sum_k \alpha_{ik} P_k^{(n)}\left(x,y\right)$$

- Exposures are i index
- Principal components are k index
- n is order of polynomial function

PCA Algorithm

- Start with lower (say 4th) order fits:

$$F_i(x,y) = P_i^{(4)}(x,y)$$

- Treat each polynomial as a vector of coefficients.
- Then all coefficients for all exposures are a matrix: A (N_{exp} x N_{coeff})

PCA Algorithm

- Perform Singular Value Decomposition on this matrix:

$$A = USV$$

- U is (tall) column-unitary
- S is diagonal
- V is square unitary.

PCA Algorithm

- Rows of SV are initial principal components (still 4th order functions).
- Elements of U are α coefficients.

$$F_i(x,y) = \sum_k \alpha_{ik} P_k^{(4)}(x,y)$$

PCA Algorithm

- Next, keep α's constant, and fit for higher (say 10th) order functions for P's.

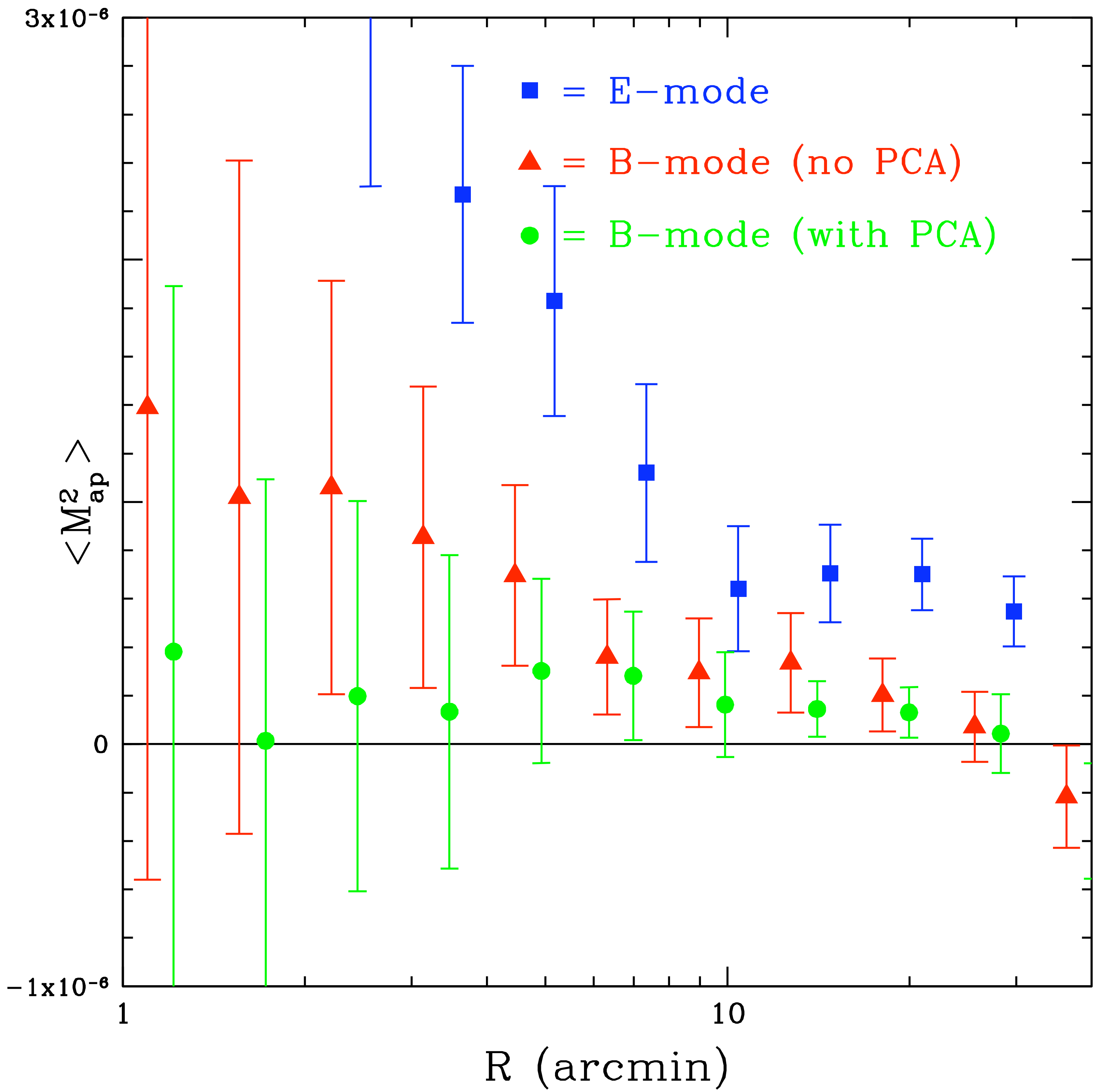
$$F_i\left(x,y\right)=\sum_k \alpha_{ik}P_k^{(10)}\left(x,y\right)$$

PCA Algorithm

- Optional improvements:
  - Find better α's given the new higher order principal components. Can iterate solving for P's and α's a few times.
  - Use rational functions instead of polynomials - will probably better describe the underlying function.
  - Can use dense stellar fields in addition to lensing data.

Results

- For CTIO survey, B mode now consistent with zero down to 1 arcminute.

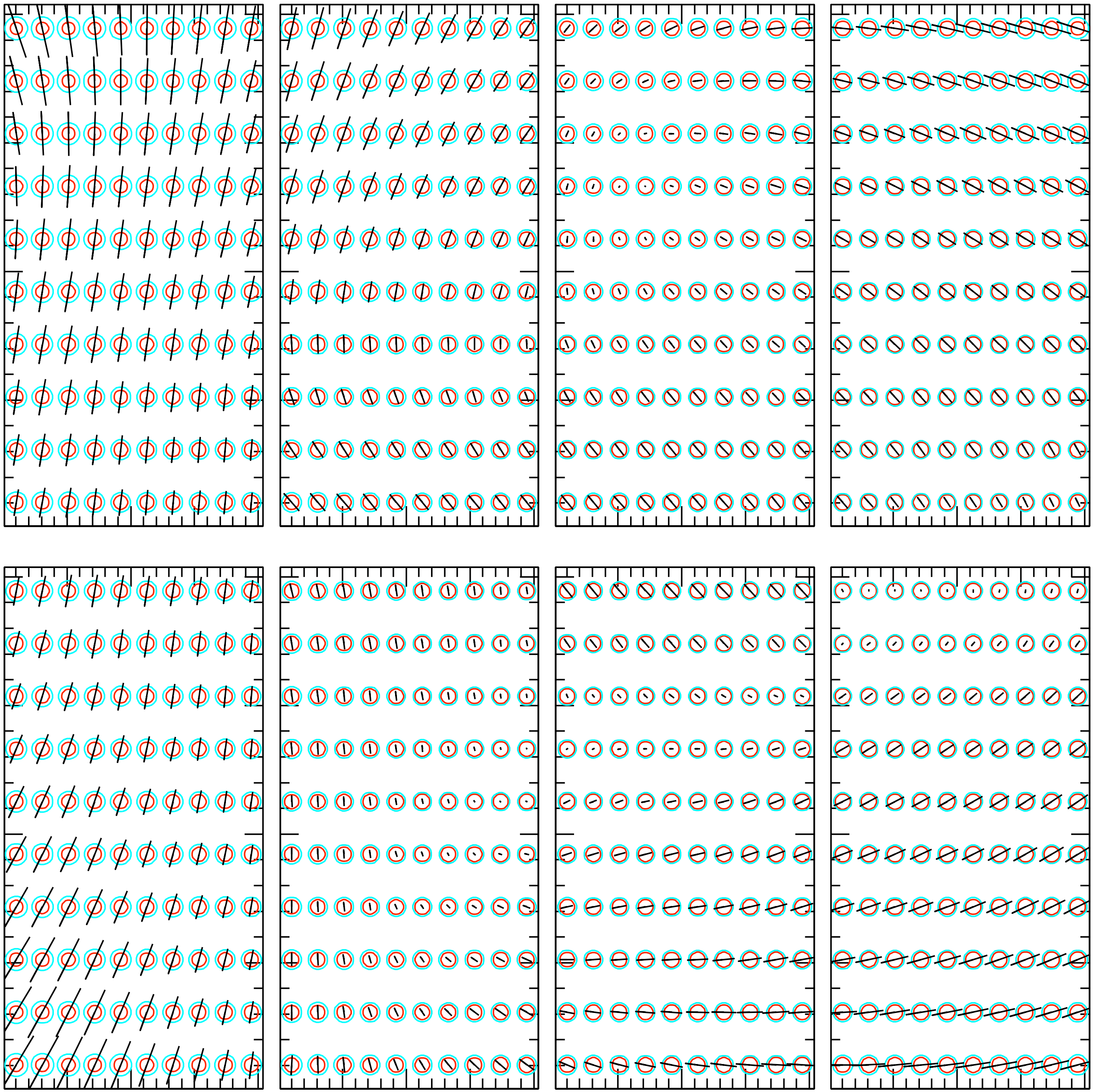


3x10⁻⁶
0
−1x10⁻⁶
1
10
R (arcmin)
<M²ap>
= E−mode
= B−mode (no PCA)
= B−mode (with PCA)

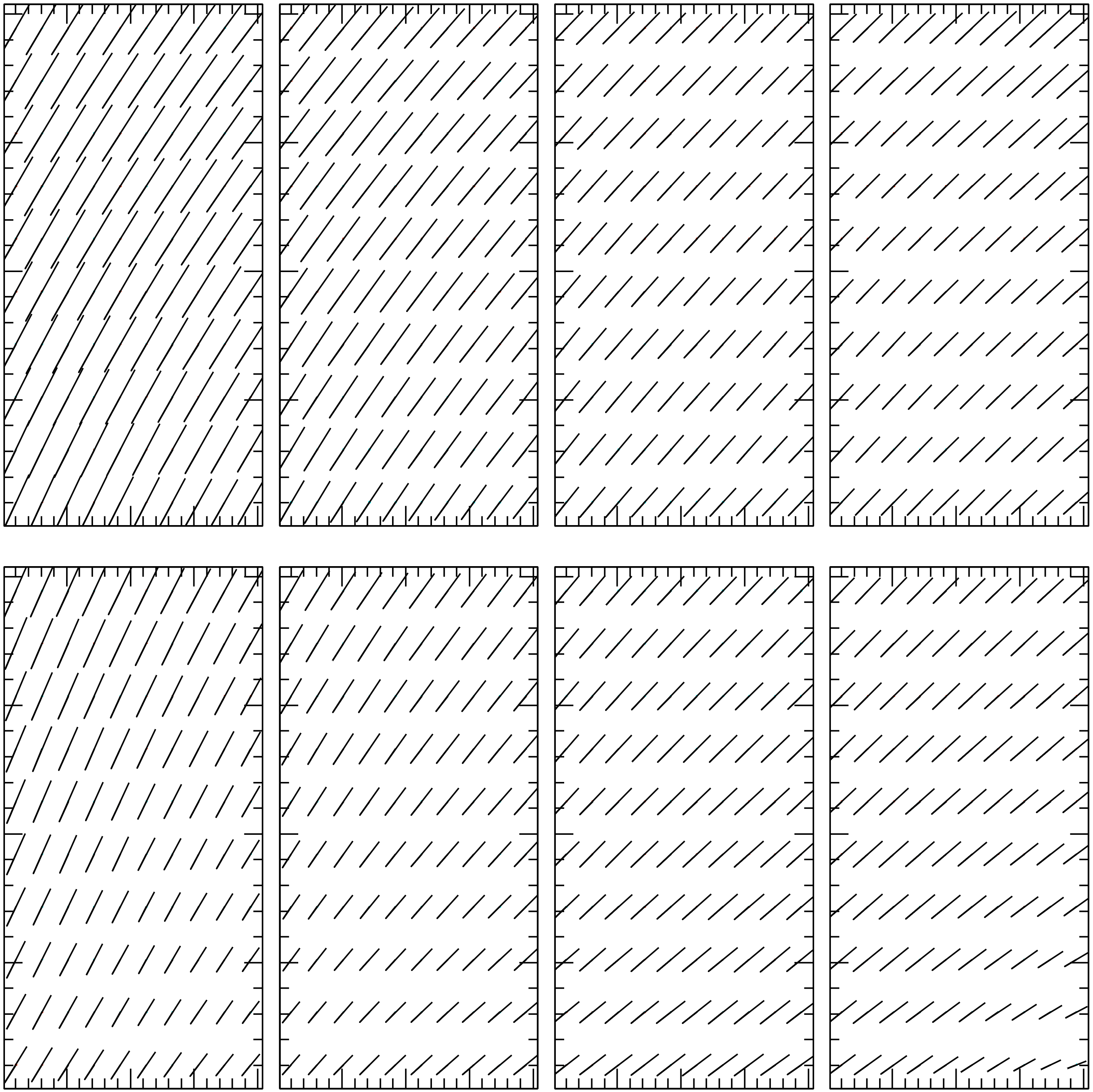
PCs for CTIO Mosaic

- PC 0: Static pattern.
- PC 1: Focus errors - primary mirror astigmatism, and off-axis camera.
- PC 2: Guiding errors.
- PC 3: Unknown.
- PC 4: Probably variable trefoil as relative pressure on “hard points” varies.
- PC 5: Unknown.
- PC 6: Probably mirror or support flex - correlated with hour angle.

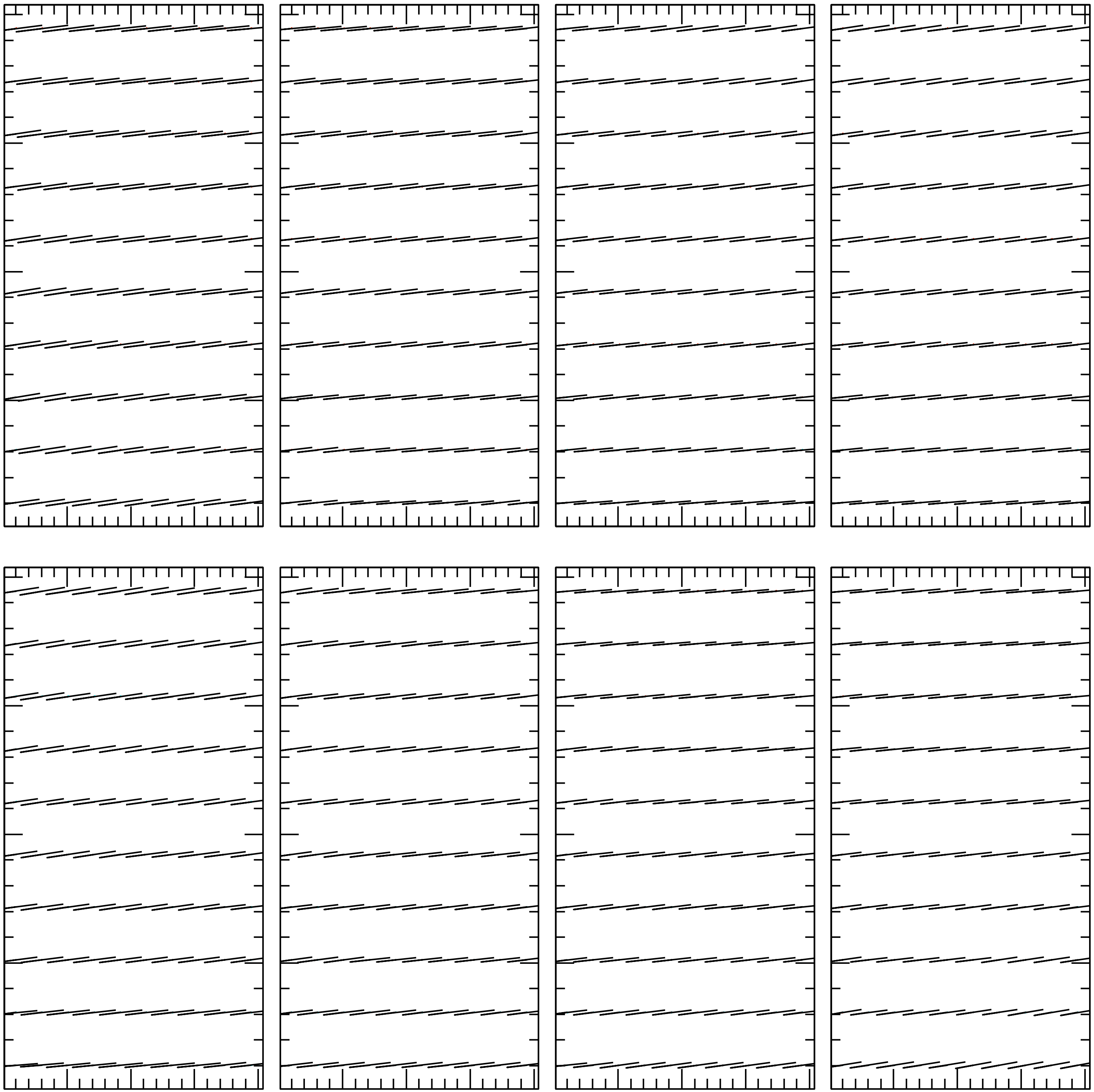
Principal Component # 0



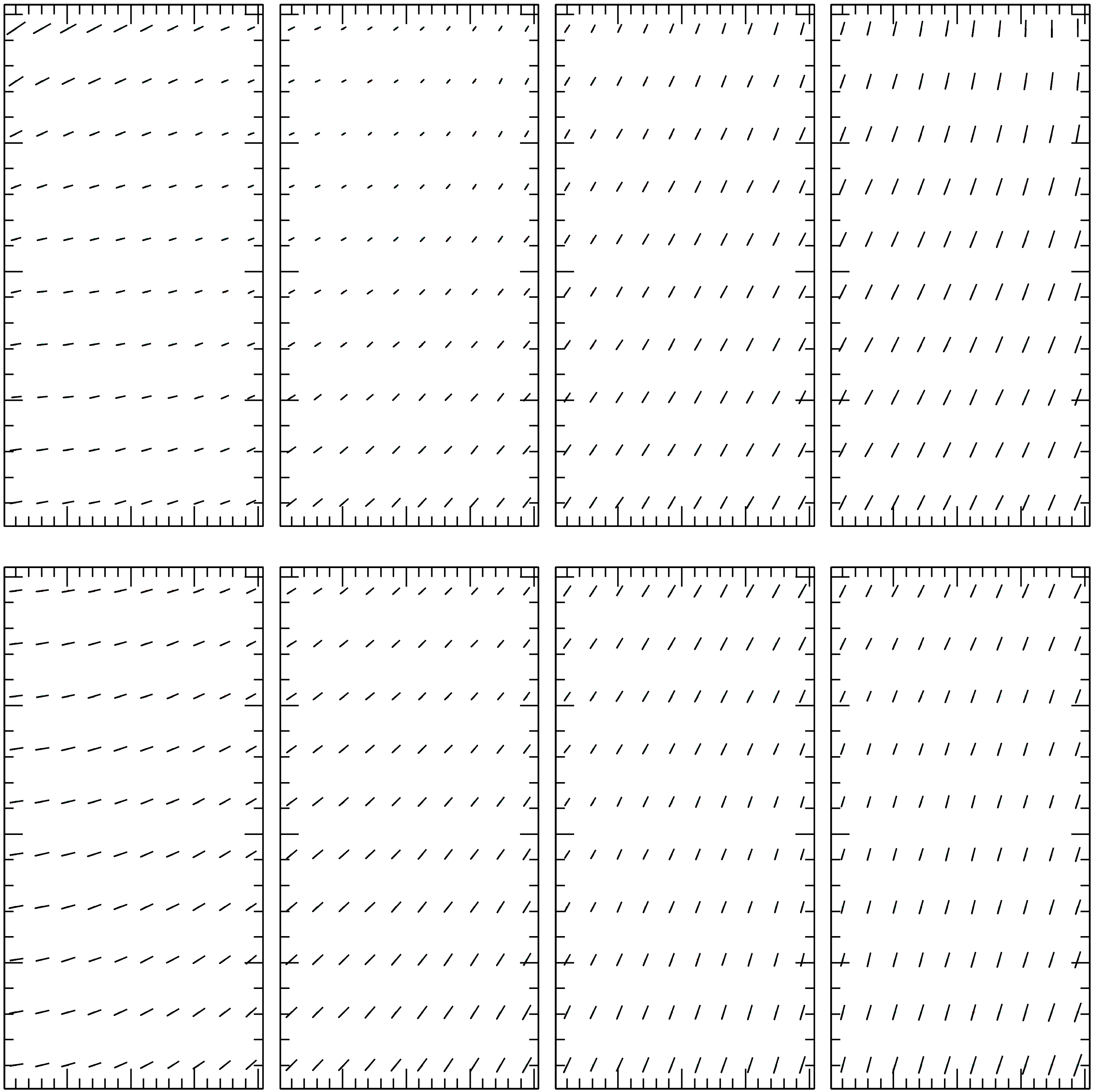
Principal Component # 1



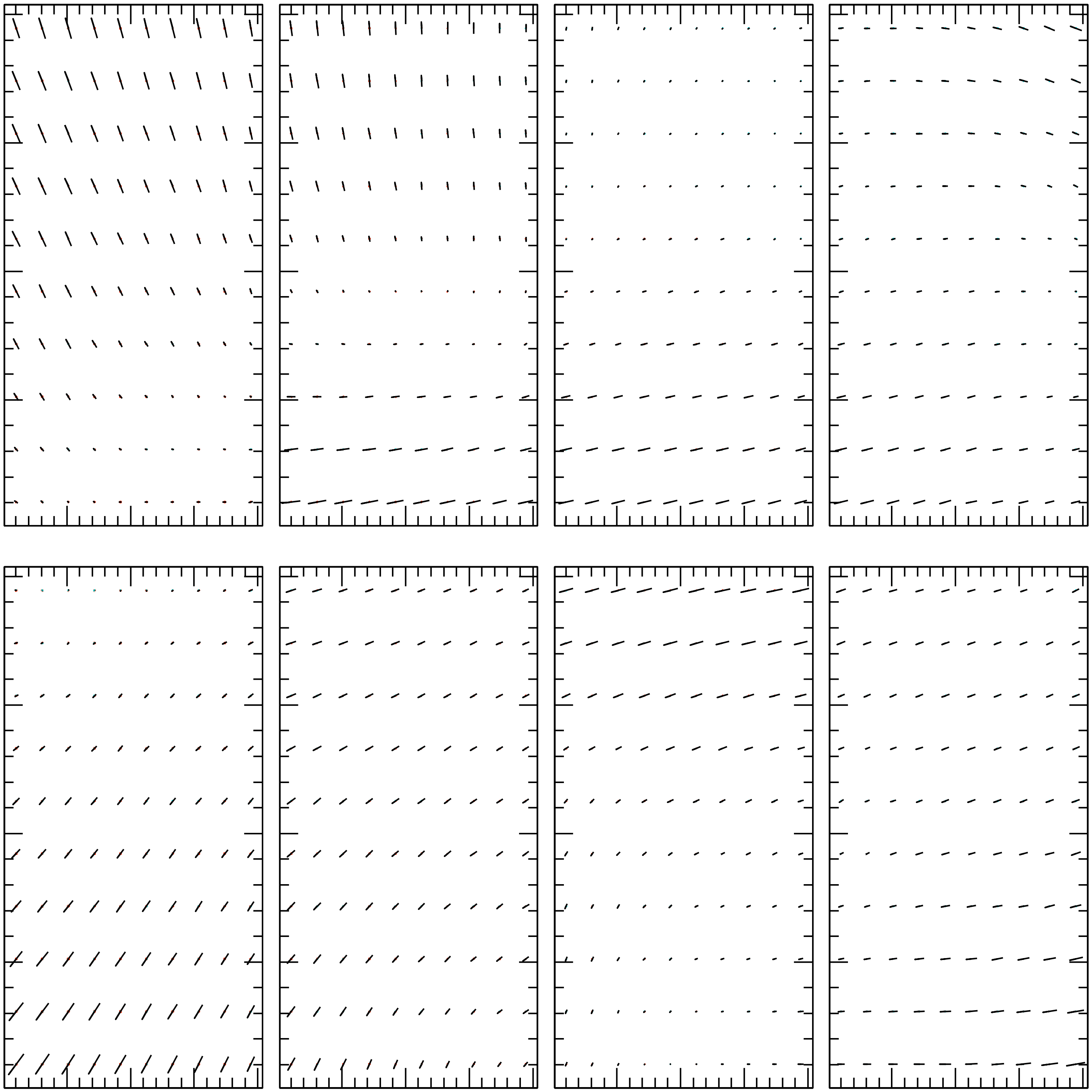
Principal Component # 2



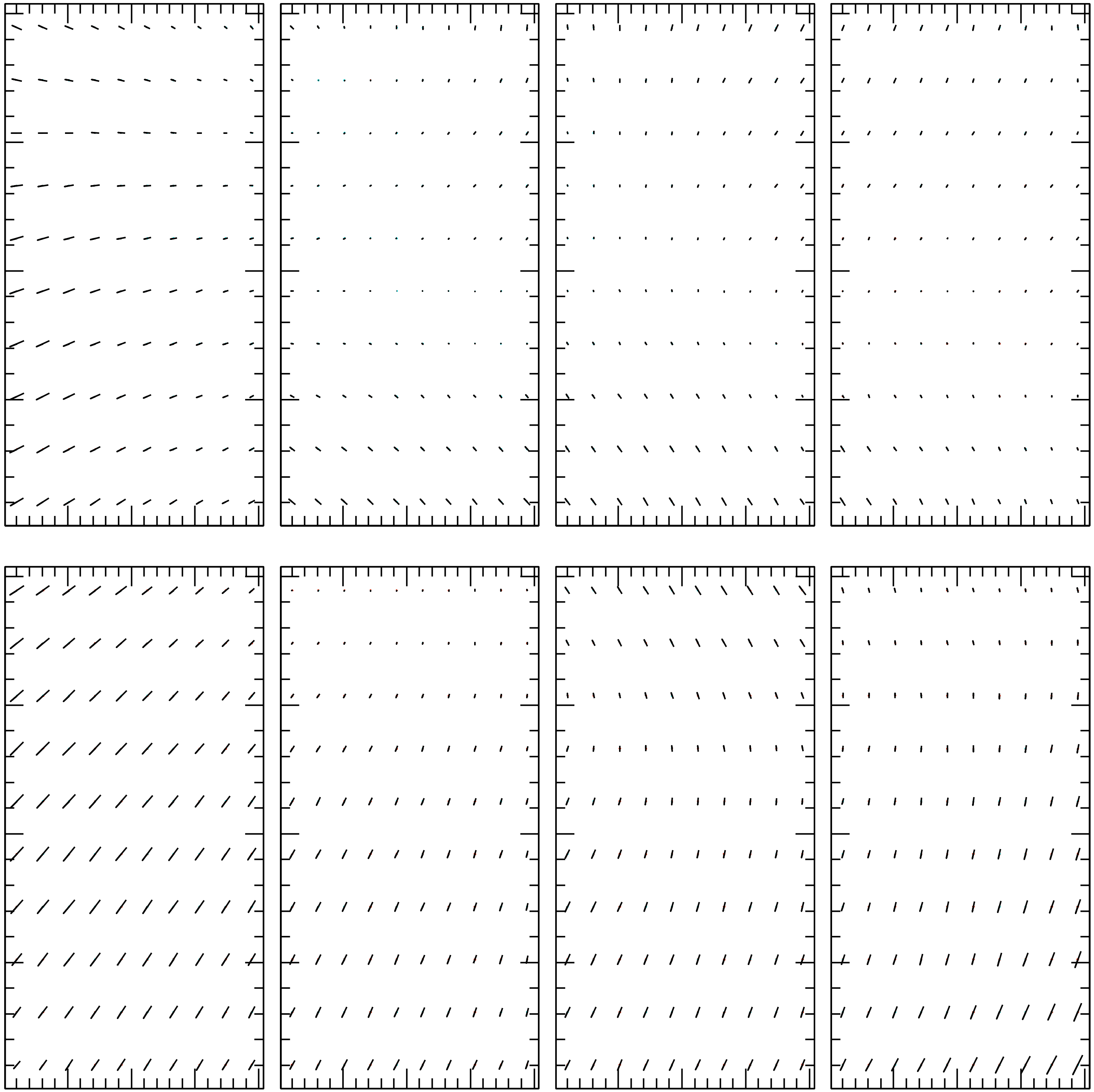
Principal Component # 3



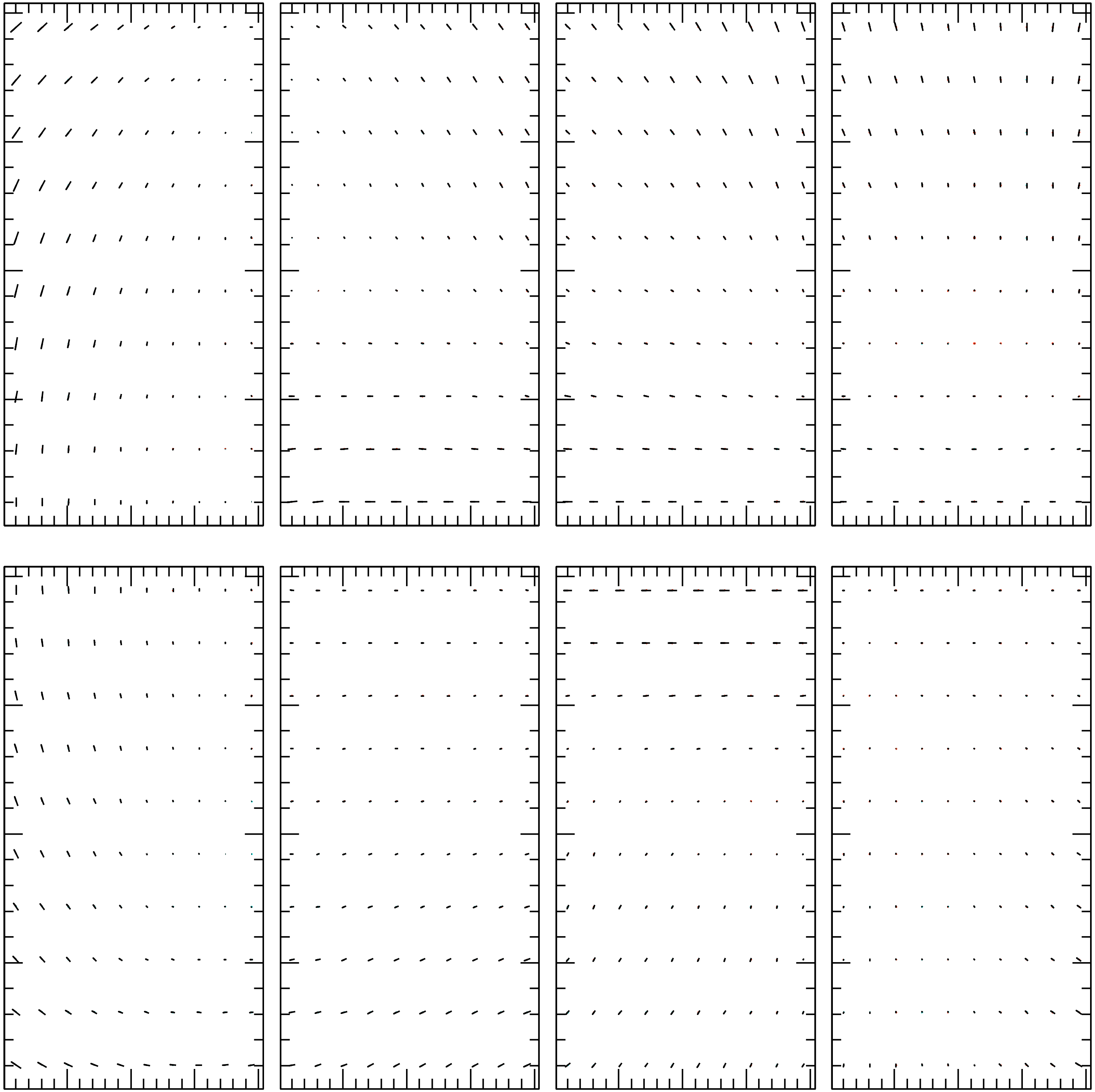
Principal Component # 4



Principal Component # 5



Principal Component # 6



Limits to PCA

- There are three places systematic errors can result using PCA:
  - Principal components can be estimated incorrectly.
  - One can use too few principal components to fully model patterns.
  - Coefficients (α) can be estimated incorrectly.

Limits to PCA

- Principal components can be estimated incorrectly:
  - The systematic errors on the principal components decrease as $1/\sqrt{N}$, where N is the total number of stars used in fits.
  - This systematic error decreases for larger surveys along with statistical errors.

Limits to PCA

- Too few principal components to fully model patterns.
  - We use PCs with singular values > 0.01 times largest singular value.
  - This gives about 25-30 PCs.
  - The optimal number to use will require some more investigating.
  - Trade-off: more PC's are each fit slightly less well.

Limits to PCA

- Coefficients (α) can be estimated incorrectly.
  - α is only fit from stars in a single exposure.
  - Need (minimum) as many stars as PCs.
  - Errors (roughly) inversely proportional to total S/N of stars in exposure.
  - This systematic does not decrease for larger surveys.

Limits to PCA

- Coefficients (α) can be estimated incorrectly.
  - Only affects correlation function when both galaxies are from same exposure.
  - Can completely remove it by only using pairs from different exposures.
  - Errors in α's are then statistical error, not systematic.