The K2K shear estimator

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K2K

Kaiser (2000, ApJ, 537, 555)

Users so far:

- *N. Kaiser and D. Donovan (Hawaii)*
- *G. Wilson (Pasadena)*
- *H. Dahle (Oslo)*

Uses published so far:

- *Cosmic shear (Kaiser, Wilson & Luppino 2000)*
- *Galaxy-galaxy lensing (Wilson et al. 2001)*
- *Mass-light cross-correlation (Wilson et al. 2001)*
- *Galaxy Clusters (Dahle et al. 2002)*

Motivation

$$\delta e_\alpha = (P^\gamma_{\alpha\beta}(f_o) - (P^\gamma(g)/P^{sm}(g))P^{sm}_{\alpha\beta}(f_o))\gamma_\beta$$

- *KSB+ result is exact for Gaussian PSF*
- *More general expression includes a term which is formally divergent for a diffraction-limited PSF*
- *The divergence can be fixed with a reconvolution with a properly chosen kernel which also removes anisotropy*

$$f_o' = f_o - \gamma_\alpha M_{\alpha ij}(r_i \partial_j f_o - (r_i \partial_j h) \otimes f_o)$$

f_o and f_o' is the observed (post-seeing) surface brightness distribution in the absence and presence of weak shear, respectively.

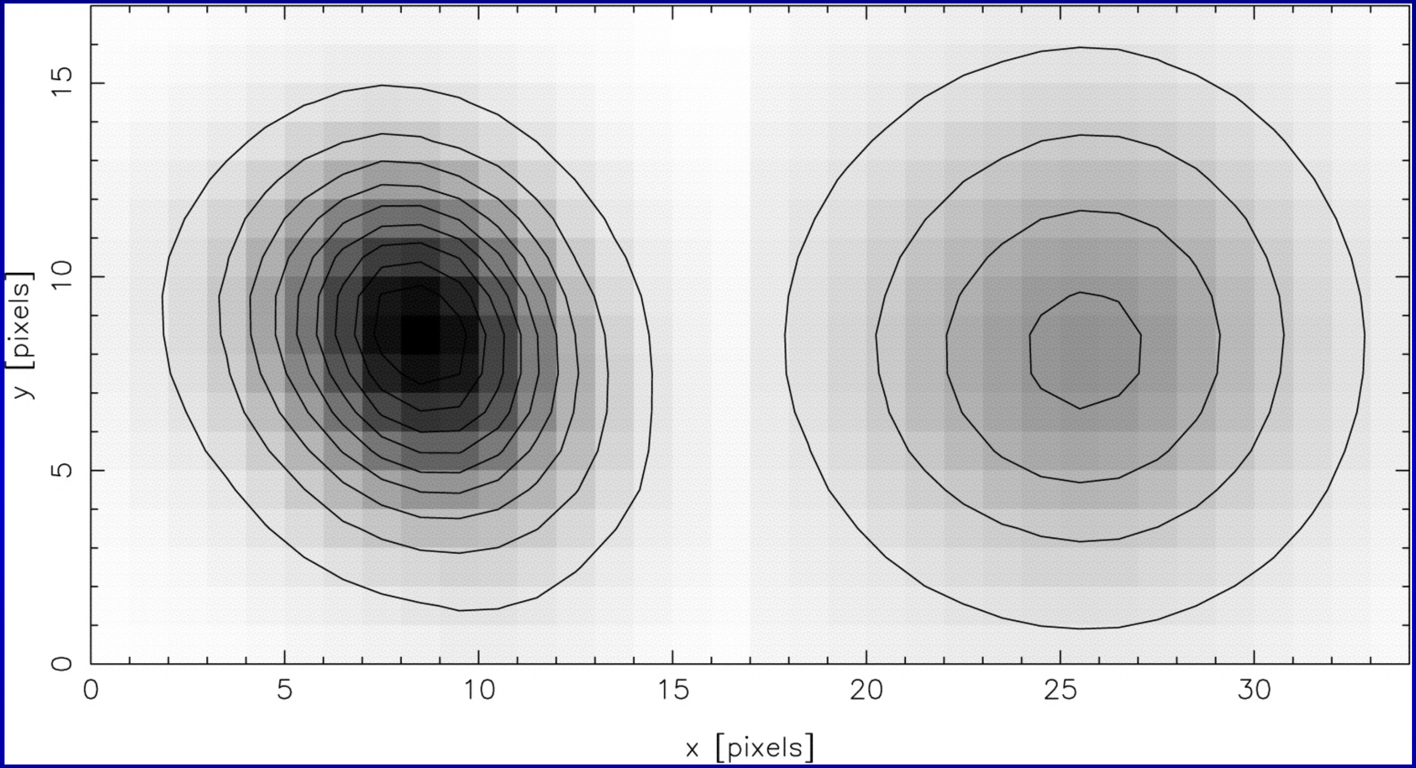
h is the inverse transform of the natural logarithm of the optical transfer function (OTF), i.e. $\tilde{h} \equiv \ln \tilde{g}$, where g is the PSF. In the case of diffraction limited seeing, the OTF will fall to zero at some finite spatial frequency $k_{\rm max}$. This divergence of $\tilde{h}$ can be removed by convolving the image with a filter $g^\dagger(r)$ to generate a smoothed image $f_s = g^\dagger \otimes f_o$.

One option is to use the stars in the field to generate a model for the PSF and let $g^\dagger = g$, in which case

$$f'_s = f_s + \gamma_\alpha M_{\alpha ij}(2(r_i \partial_j g) \otimes f_o - r_i(\partial_j g \otimes f_o))$$

Smoothing the original image with a 90 deg rotated model of the local PSF, $g^\dagger = R_{\pi/2}(g)$, has the added advantage of removing most of the PSF anisotropy

Convolution with rotated PSF



(Dahle et al. 2002)

PSF recircularization is done on each chip image from individual exposures before image combination. Solves problem with PSF discontinuities across chip borders for UH8K camera

A rotated PSF model was used for STEP1

Deriving recircularizing PSF

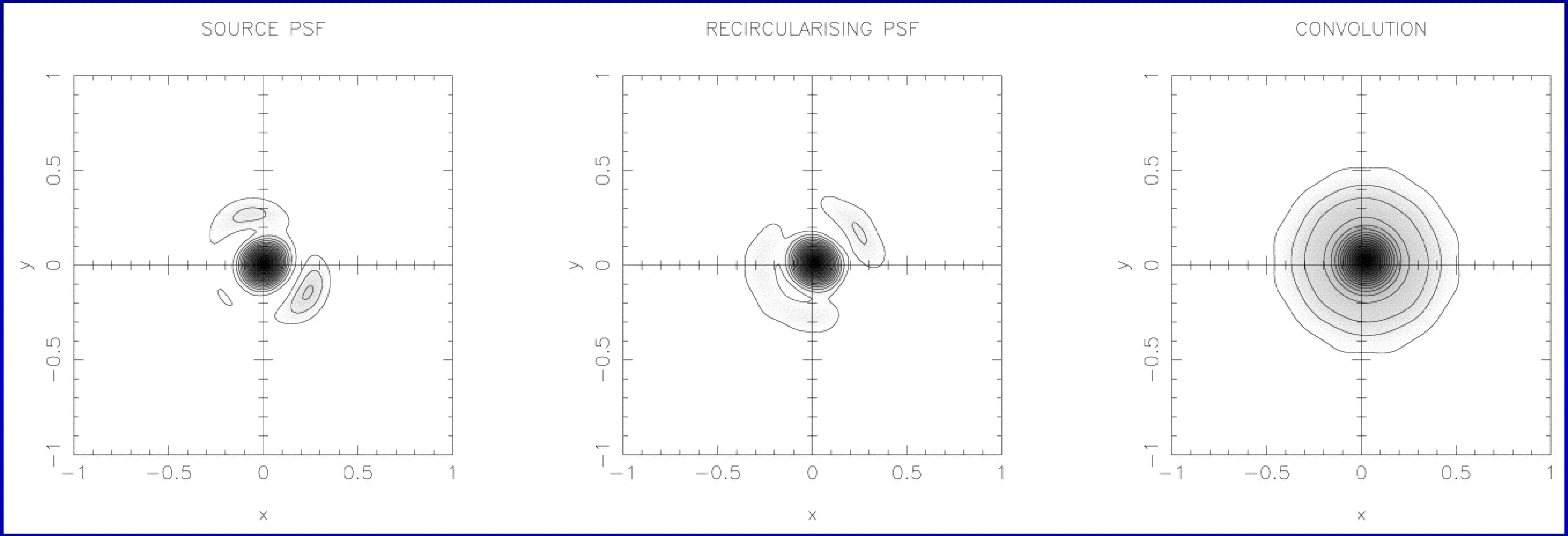


Illustration: Tiny Tim WFPC2 PSF

A simpler, 90-degree rotated PSF model works best for ACS data (D. Donovan)

$$q_{lm} = \int d^2 r w(\mathbf{r}) r_l r_m f_s(\mathbf{r})$$

$$q_A = \frac{1}{2} M_{Aij} q_{ij} = \left[\begin{array}{c} (q_{xx} + q_{yy})/2 \\ (q_{xx} - q_{yy})/2 \\ q_{xy} \end{array} \right]$$

q_0 measures the area of an object. q is a rotationally invariant shape parameter ($q_\alpha = q\hat{q}_\alpha$, where $\hat{q}_\alpha = \{\cos\phi, \sin\phi\}$ is the unit polarization vector). The weighted flux $F = \int d^2 r w(\mathbf{r}) f_s(\mathbf{r})$ is also measured.

The mapping of each parameter due to a shear γ_α is described by

$$flux : F' = F + R_\beta \gamma_\beta$$

$$size : q_0' = q_0 + P_{0\beta} \gamma_\beta$$

$$shape : q_\alpha' = q_\alpha + P_{\alpha\beta} \gamma_\beta$$

These quantities are calculated based on the "raw" image f_0, the smoothed image f_s, and the PSF g using the IMCAT routine `getshapes3`

q_0, q and the polarizabilities R_α, $P_{0\beta}$ and $P_{\alpha\beta}$ are rescaled by dividing by the flux F. The effective polarizability is given by

$$\bar{P}_{\alpha\beta} = \langle P_{\alpha\beta} \rangle - \frac{1}{n} \frac{\partial n \langle q_\eta P_{\eta\beta} \hat{q}_\alpha \rangle}{\partial q} - \frac{1}{n} \frac{\partial n \langle P_{0\beta} \hat{q}_\alpha \rangle}{\partial q_0}$$

$$+ \frac{q}{n} \left[1 - \frac{\partial}{\partial \ln F} + \frac{\partial}{\partial \ln q_0} + \frac{\partial}{\partial \ln q} \right] n \langle R_\beta \hat{q}_\alpha \rangle,$$

such that $\langle q_\alpha \rangle = \bar{P}_{\alpha\beta} \gamma_\beta$. This is determined using the IMCAT routine `makepeff`

The effective polarizability is **not** equal to the mean of the polarizabilities for individual objects. In the presence of a shear, the galaxies will systematically scatter in $F - q_0 - q$-space and bias the net polarization in a way which depends on the local gradients of the distribution function.

For an isotropic PSF, $\bar{P}_{\alpha\beta} = \bar{P}\delta_{\alpha\beta}$ where $\bar{P} \equiv \bar{P}_{\eta\eta}$, and the shear estimator for a cell in $F - q_0 - q^2$ space with occupation number N is given by

$$\hat{\gamma}_\beta = \frac{1}{N\bar{P}} \sum q_\beta.$$

The expectation value for the variance in $\hat{\gamma}$ is $q^2/N\bar{P}^2$ and the optimized total shear estimate when averaging over all cells is given by

$$\hat{\gamma}_\alpha^{\text{total}} = \frac{\sum_{\text{galaxies}} Q\hat{q}_\alpha}{\sum_{\text{galaxies}} Q^2},$$

where $Q \equiv \bar{P}/q$. The variance of the total shear estimator is $(\sum Q^2)^{-1}$, and this provides a useful 'figure of merit' $\sum Q^2/d\Omega$ by which the quality of different weak lensing data sets can be compared.

STEP1 results for K2K

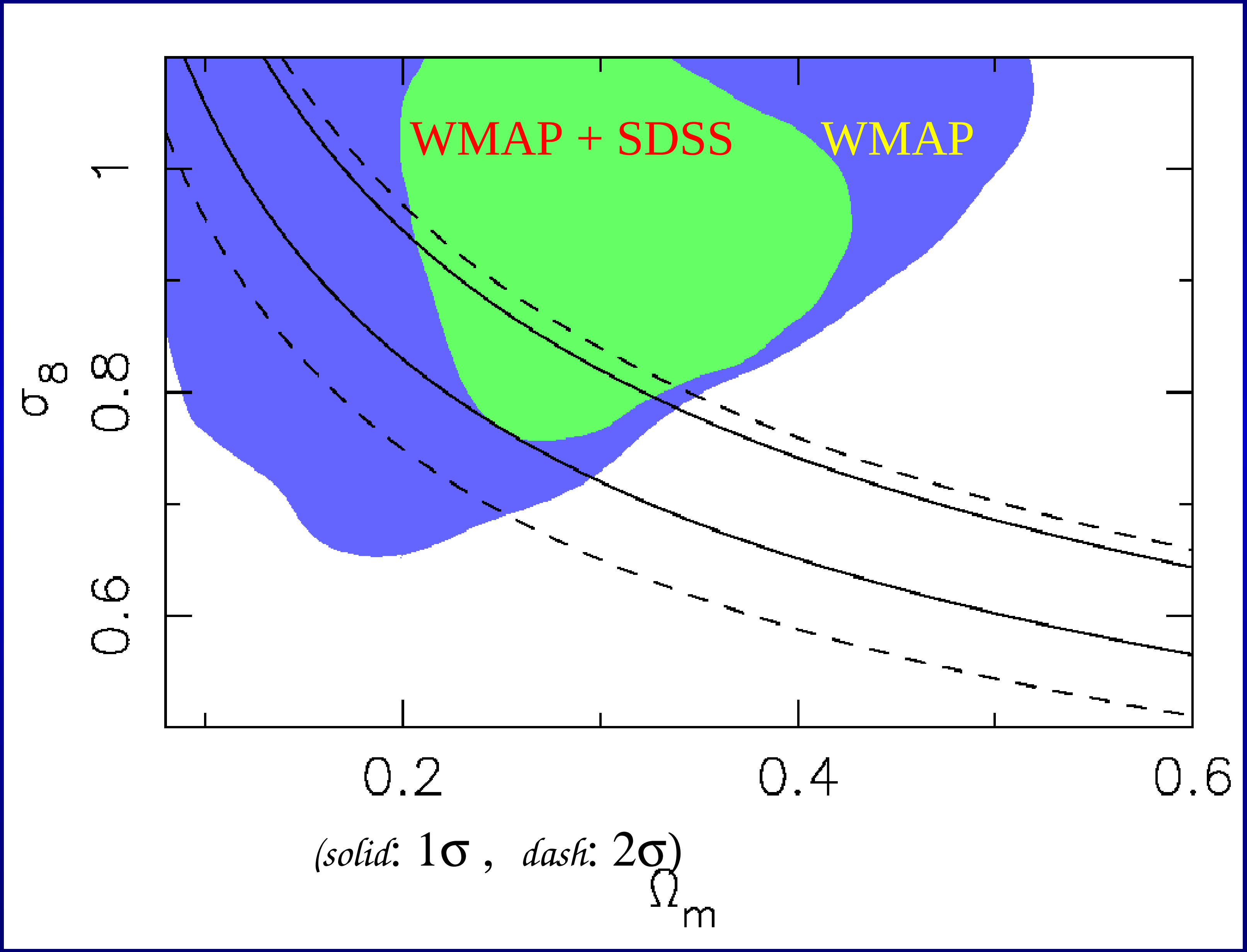
- *Strongest non-linearity in the shear response of any method:*

  γ = *0.05 is overestimated by ~10 %*

  γ = *0.1 is underestimated by ~3 %*

- *Also ~40% contamination by stars (oops!); adds noise, but demonstrates robustness of weighting scheme*
- *Now what ?*

Ω_m-σ_8 from lensing-based cluster mass function



From fit to mass function predicted from simulations by Warren et al. (2005)

Summary

- Assumptions behind KSB+ may be shaky
- K2K solves this in principle by reconvolution with re- circularizing PSF
- STEP1 results give non-linear shear response
- Further testing needed; space-based results will be interesting
- Help wanted