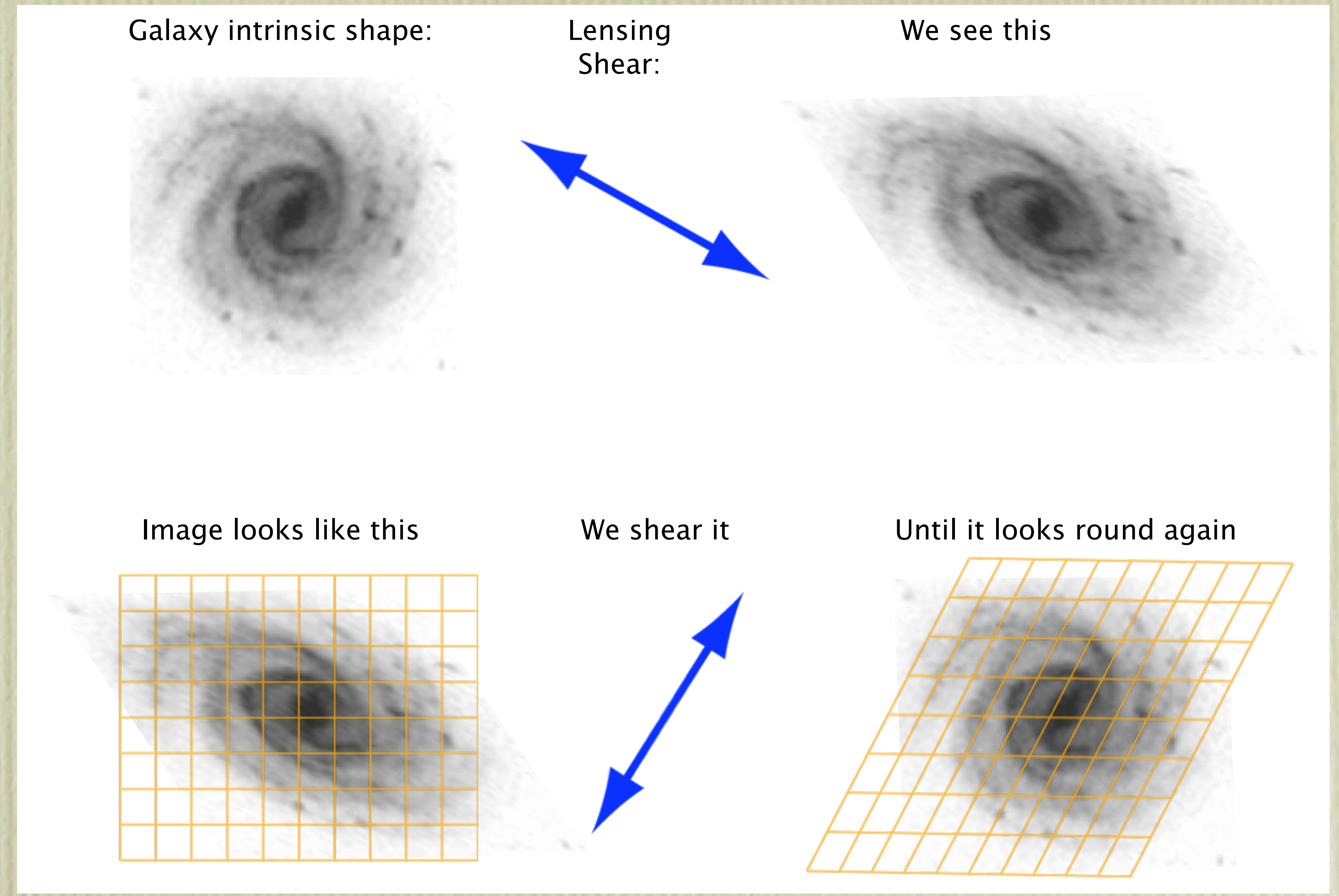
Galaxies in Pennsylvania

Bernstein, Jarvis, Nakajima, & Rusin:
Implementation of the BJ02 methods

Bases of our methods:

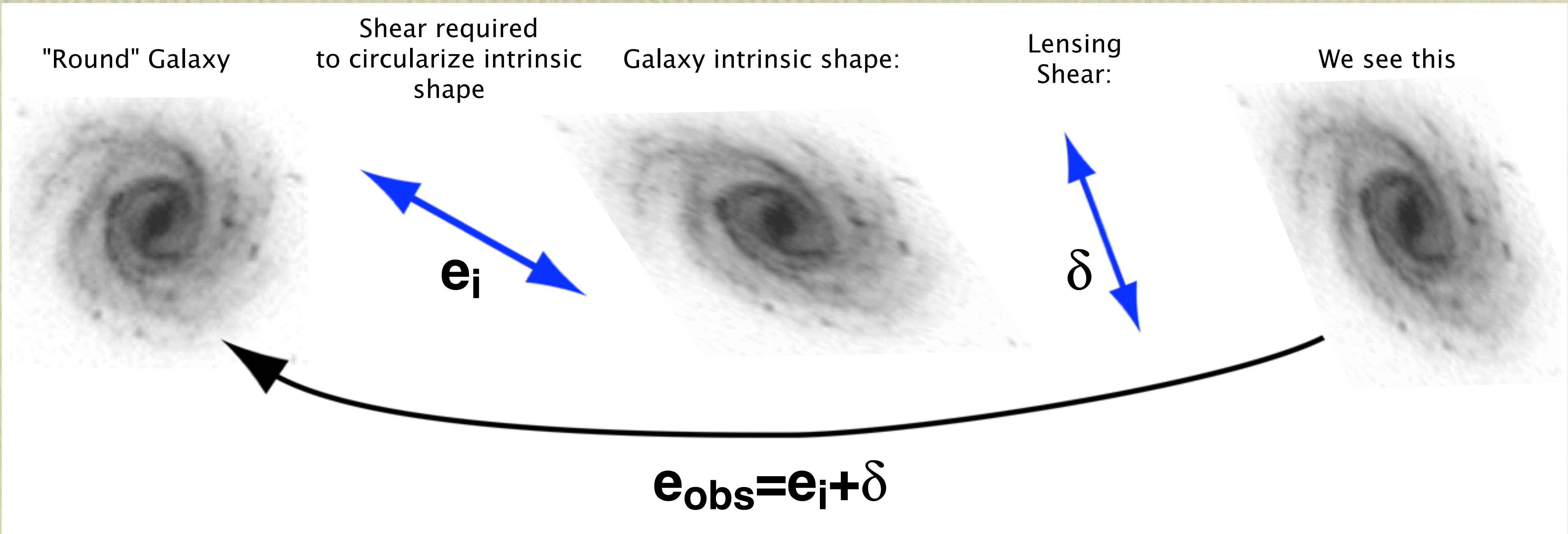
- **Shapes with geometric meaning:**
  - No empirical polarizabilities; simple optimizations of weighting.
  - As in **ellipto** (D. Smith '98), and SDSS methods of Chicago & Princeton groups
- **Model the pre-seeing galaxy: convolve model & fit to the pixel values.**
  - PSF correction of arbitrary accuracy; proper handling of sampling; propagation of errors
  - As in Kuijken, Phil Fischer methods
- **Orthonormal expansion as galaxy model - Gauss-Laguerre expansion**
  - Adjustable accuracy through order of expansion; rapid numerics
  - As in "polar shapelets" but geometric shapes require elliptical basis

Geometric Shapes



Applied shear to circularize is opposite of the lensing shear, independent of galaxy details

What if galaxy is not round?



Applied shear to circularize is now (opposite of) the sum of intrinsic shape and applied shear.

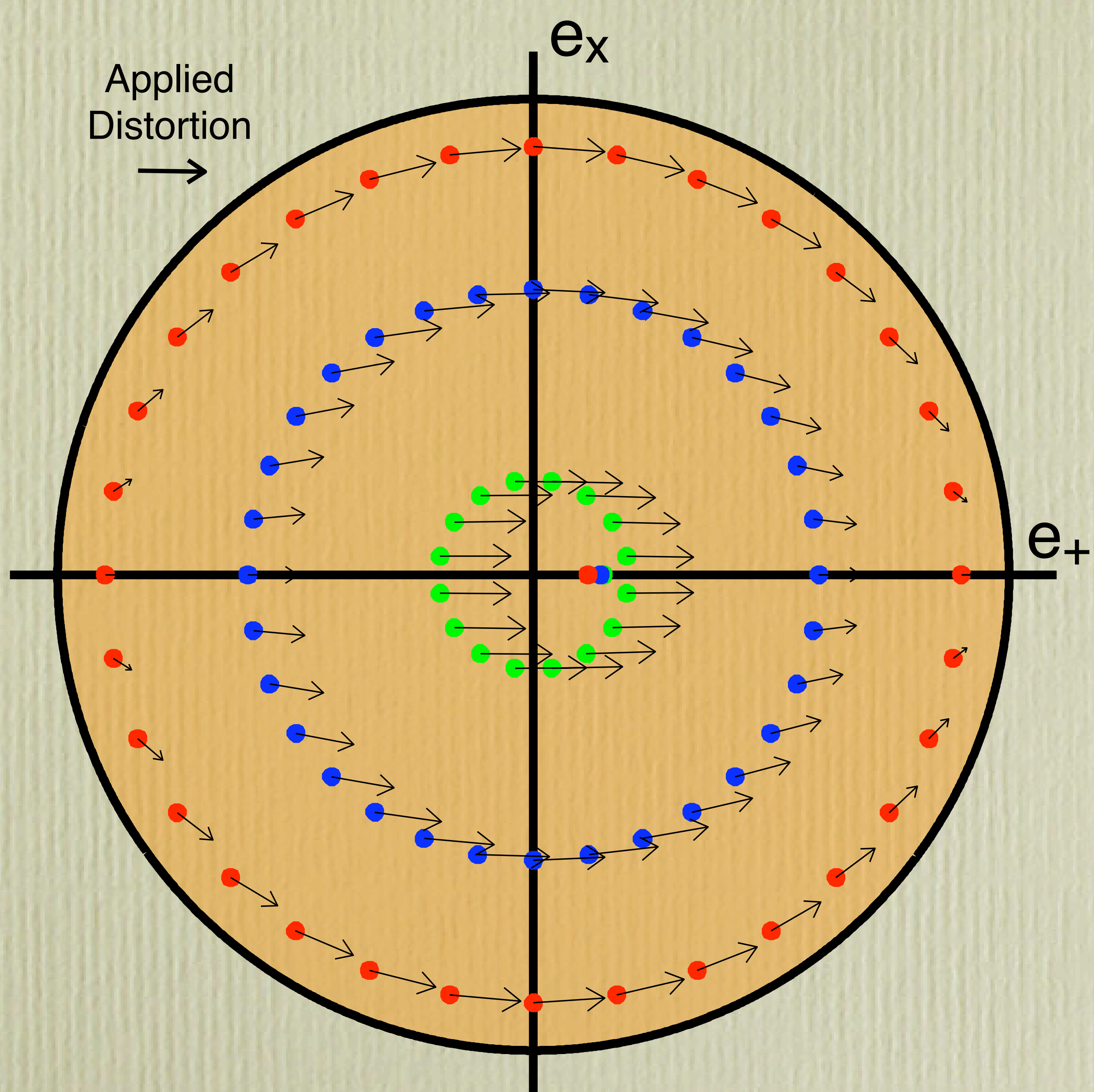
All measured shapes transform under shear according to Miralda-Escude formulae:

$$e_+^{\mathrm{obs}} = \frac{e_+^{\mathrm{i}} + \delta_+ + (\delta_\times/\delta^2)[1-\sqrt{1-\delta^2}](e_\times^{\mathrm{i}}\delta_+ - e_+^{\mathrm{i}}\delta_\times)}{1+\mathbf{e}^{\mathrm{i}}\cdot\boldsymbol{\delta}},$$

$$e_\times^{\mathrm{obs}} = \frac{e_\times^{\mathrm{i}} + \delta_\times + (\delta_+/\delta^2)[1-\sqrt{1-\delta^2}](e_+^{\mathrm{i}}\delta_\times - e_\times^{\mathrm{i}}\delta_+)}{1+\mathbf{e}^{\mathrm{i}}\cdot\boldsymbol{\delta}}.$$

Need a definition of “round” for galaxies without elliptical isophotes: but can work with many possible definitions.

Response of Geometric Shapes to Shear



For galaxies with isotropic intrinsic shapes e, the average shape after some small distortion is simply:

$$\langle \mathbf{e}_{\mathrm{obs}} \rangle = \delta \left(1 - \frac{e^2}{2}\right) = 2\gamma \left(1 - \frac{e^2}{2}\right)$$

Response to shear depends only on shape!
No polarizabilities

Weighting Geometric Shapes

- Our intuition that high-e galaxies should be deweighted is correct. Make a shear estimator:

$$\hat{\delta} = \langle w(e)\, e_{\rm obs} \rangle$$

- With geometric shapes, response to shear is very simple:

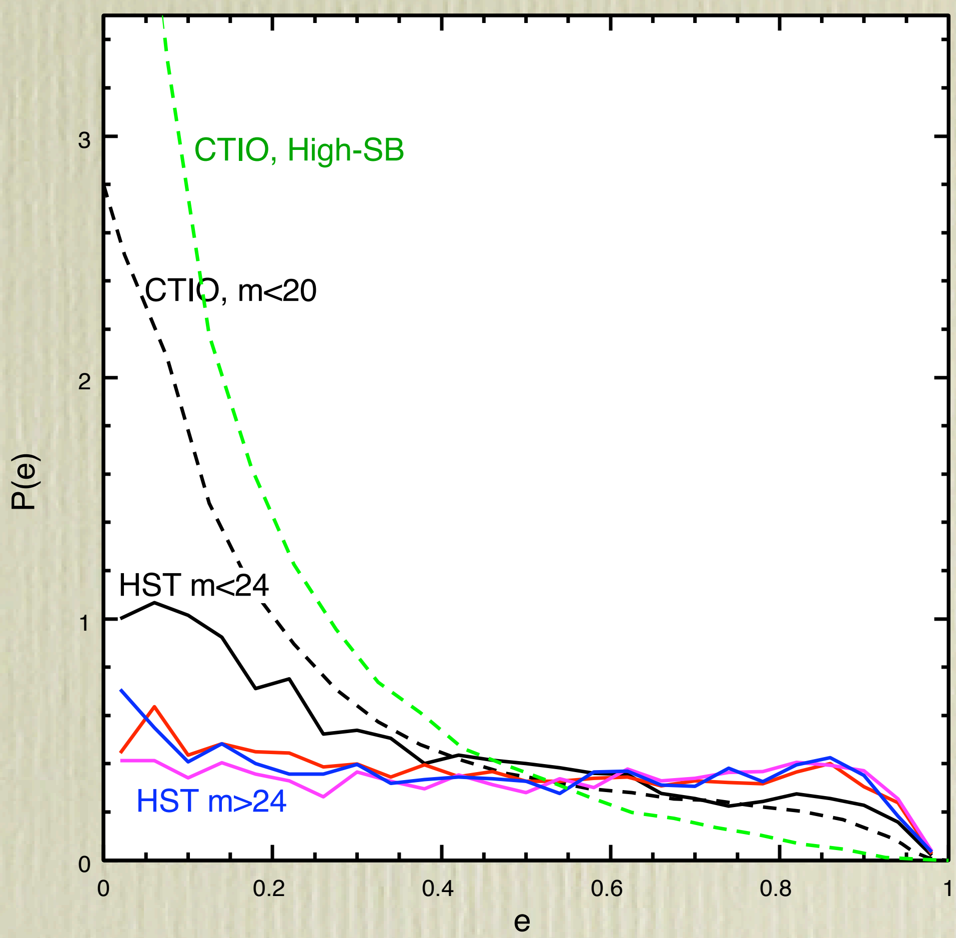
$$\mathcal{R} = \frac{d\hat{\delta}}{d\delta} = \langle w(e)\,(1 - e^2/2) \rangle$$

- Noise is shot noise from intrinsic galaxy shapes. Simple to find optimal weight:

$$w_{\rm opt}(e) \propto 3 - \frac{1-e^2}{e}\frac{d\log P}{de}$$

- Quite substantial gain from weighting if the P(e) has a strong slope, as for bright galaxies/ellipticals. Note that a steep weight function will induce a less-linear response to applied shear - but this nonlinearity is well predicted.

- More complicated if there is measurement noise on shapes.



Testing Geometric Shapes:

- If galaxy has all isophotes same ellipse:
  - It suffices to show that code recovers the input ellipticity, since we now exactly how any shear will transform this object's ellipticity
- If galaxy has non-elliptical structure:
  - Show that the mean measured e of an isotropic population of galaxies scales properly with applied shear.
- This does not address selection biases, crowding issues, or errors in PSF estimation; just the shape/shear recovery.

Fitting pixel data with convolved models

- Construct the model of the *pre-convolution* image. With our geometric method, this means that we guess at some coord transformation E that makes the galaxy appear round, then we assume

$$\hat{J}(\mathbf{u}) = \sum_i b_i \psi_i^{\mathbf{E}}(\mathbf{u}) = \sum_i b_i \psi_i(\mathbf{E}^{-1}\mathbf{u})$$

- Next convolve each basis function with the PSF, perform any other distortions, etc., that happen before focal plane, to get *observed* basis functions.

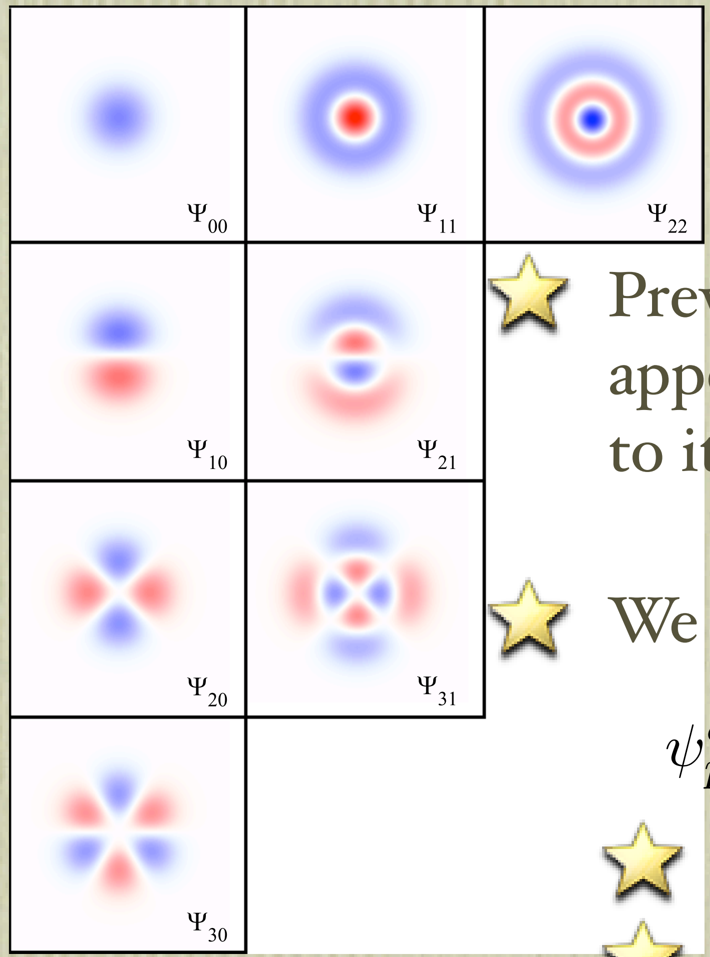
$$\hat{I}(\mathbf{x}) = C * \hat{J}(\mathbf{u}) = \sum_i b_i \phi_i^{\mathbf{E}}(\mathbf{x}), \qquad \phi_i^{\mathbf{E}} \equiv C * \psi_i^{\mathbf{E}}$$

- Fit to samples at pixels, minimizing chi-squared.

$$\chi^2 = \sum_{k \in \text{pixels}} \sigma_k^{-2} \left[I_k - \sum_i b_i \phi_i^{\mathrm{E}}(\mathbf{x}_k) \right]^2$$

  - Note that we can easily use multiple images of a galaxy, with different PSFs, etc.
  - Exclude bad pixels
  - Ambiguities & covariances from finite sampling are apparent from SVD of the fitting matrix.
- If galaxy is not round/centered, adjust E and iterate fit.

Gauss-Laguerre Decomposition



- Previous ideas work for any parameterization of the galaxy's intrinsic appearance, as long as we can define criteria for centering and roundness to iterate our E.

- We use Gauss-Laguerre decomposition:

$$\psi^{\sigma}_{pq}(r,\theta) = \frac{(-1)^q}{\sqrt{\pi}\sigma^2}\sqrt{\frac{q!}{p!}}\left(\frac{r}{\sigma}\right)^m e^{im\theta} e^{-r^2/2\sigma^2} L_q^{(m)}(r^2/\sigma^2) \qquad (p \geq q)$$

  - (a.k.a. *polar shapelets*, but note our use of elliptical basis)
  - Rapid recursive formulae for shear, translation, dilation (in the E matrix) and for convolution.
  - Simple, nearly optimal "roundness" criteria are:

    Centroid: $b_{10} = 0$

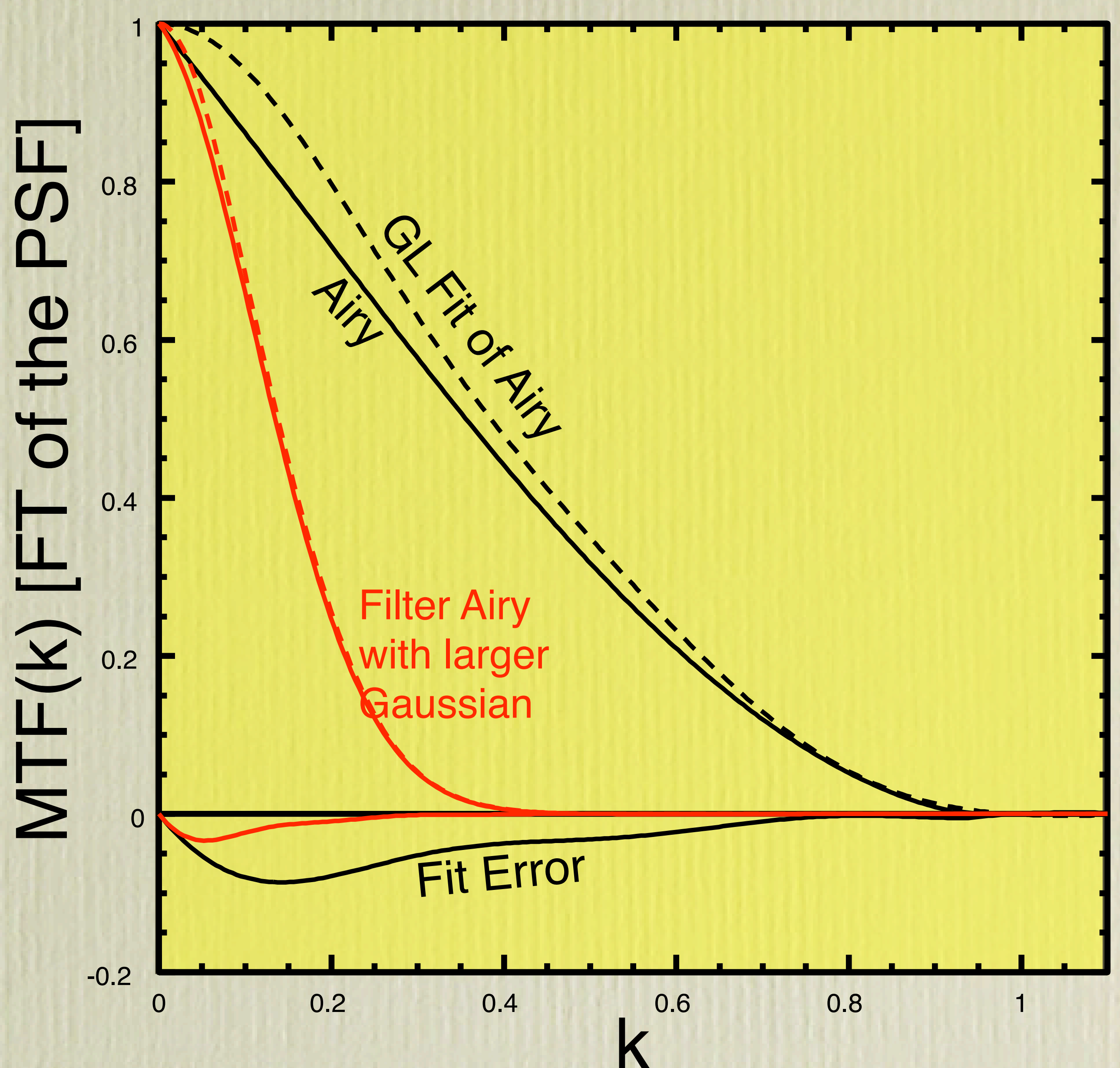
    Roundness: $b_{20} = 0$

    Size: $b_{11} = 0$

  - Good approximation for exponential disk galaxies.
  - A **poor** approximation to the large-scale (low-k) behavior of the Airy function, however.

Airy Failure

- Gauss-Laguerre decomposition cannot capture the k=0 cusp in the Airy function, i.e. misses the long tails for the real-space PSF
- Look for symptoms of this in Reiko's talk!
- Solution: filter away the high-k parts of Airy, concentrate on getting good GL fit near k=0.
- Or don't use GL decomposition for convolutions with Airy functions!



Summary

- Shape measurement method designed to have no built-in assumptions about the nature of the galaxy or PSF
- No empirical information needed for calibration beyond the (pre-seeing) distribution P(e) of the shapes.
- All effects of convolution & sampling are modelled properly.
- Easily extended to multiple-exposure fits
- Errors are propagated, degeneracies recognized.
- It exists! It works!
- Future: selection biases; low-S/N behavior