Shapelets
STEP3 and CFHTLS

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With

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Layout of the talk

- Overview : shapelets
- Updates since STEP2
- Shapelets on space-STEP
- Science with shapelets : CFHTLS

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What are shapelets ?

- Complete orthogonal basis functions
- Linear decomposition of localised objects
- 3 non-linear parameters : order of decomposition n_{max}, scale β, centroid x_{c}
- Capture all shape information of a localised object
- Simple and analytic form for (de)convolution and shear
- Adapted to galaxy shape modeling and cosmic shear
- We use two **equivalent** kinds of shapelets : *Cartesian* shapelets and *Polar* shapelets

Cartesian shapelets

Refregier 2003, Bacon & Refregier 2003

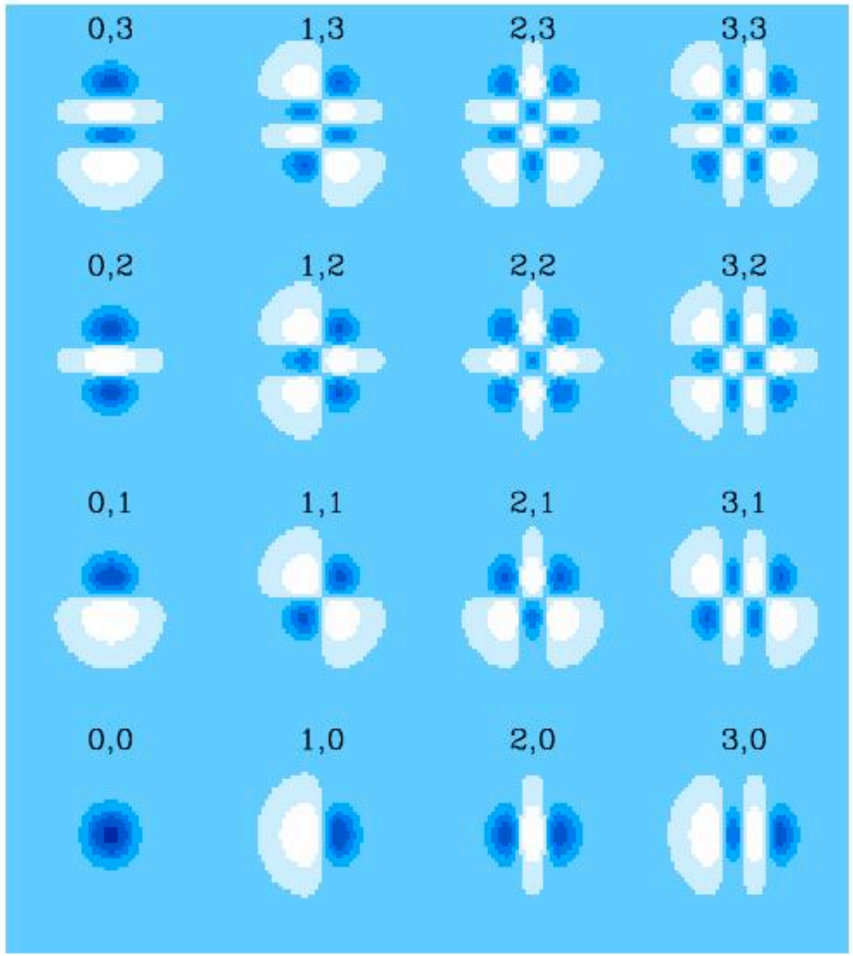
Gaussian-weighted Hermite polynomials

Basis functions

$$f(\vec{x}) = \sum_{n_1 n_2} \boxed{f_{n_1 n_2}} \boxed{B_{n_1 n_2}} (\vec{x}; \beta)$$

$$\boxed{f_{n_1 n_2}} = \int d^2x \; f(\vec{x}) \boxed{B_{n_1 n_2}} (\vec{x}; \beta)$$

Shapelet coefficients



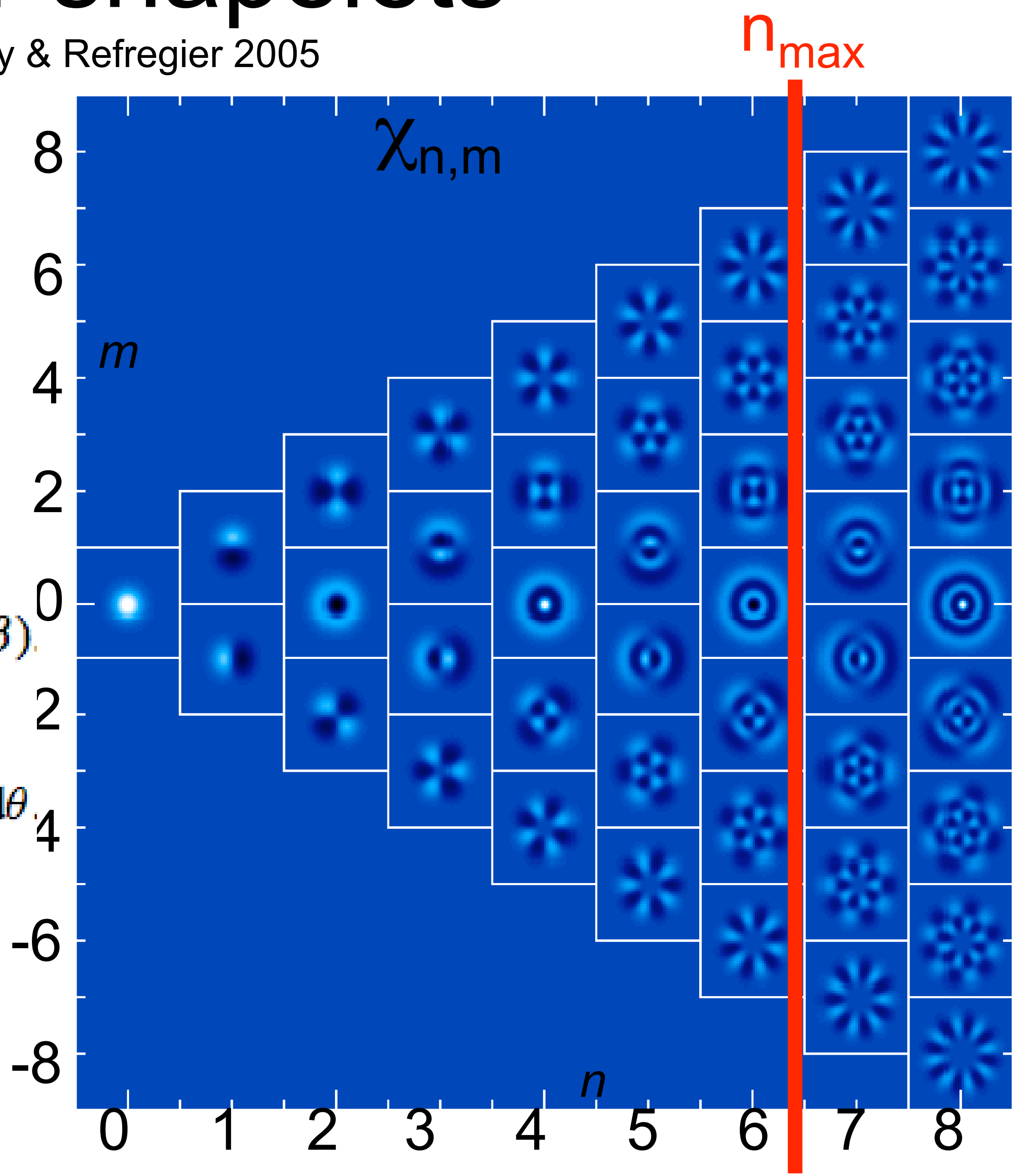
Polar shapelets

Refregier 2004, Massey & Refregier 2005

Gaussian-weighted Laguerre polynomials

$$f(r,\theta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n} f_{n,m}\chi_{n,m}(r,\theta;\beta)$$

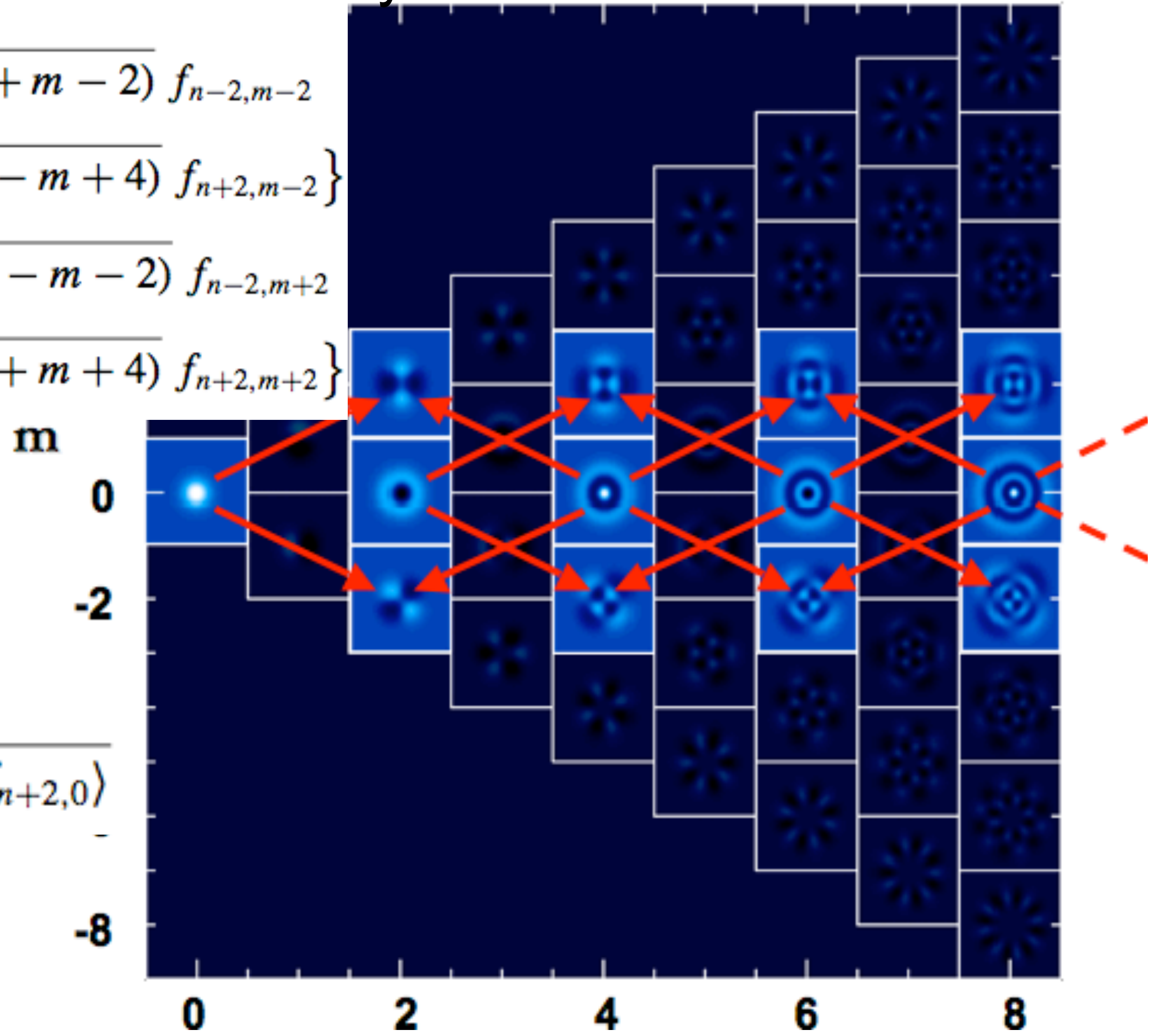
$$f_{n,m}=\iint_{\mathbb{R}} f(r,\theta)\,\chi_{n,m}(r,\theta;\beta)\,r\,\mathrm{d}r\,\mathrm{d}\theta$$



Shear estimation

Massey & Refregier 2005,
Massey et al. 2007

$$\widehat{S}: f_{n,m} \to f'_{n,m} = f_{n,m} + \frac{\gamma}{4}\left\{\sqrt{(n+m)(n+m-2)}\, f_{n-2,m-2} - \sqrt{(n-m+2)(n-m+4)}\, f_{n+2,m-2}\right\} + \frac{\gamma^*}{4}\left\{\sqrt{(n-m)(n-m-2)}\, f_{n-2,m+2} - \sqrt{(n+m+2)(n+m+4)}\, f_{n+2,m+2}\right\}$$

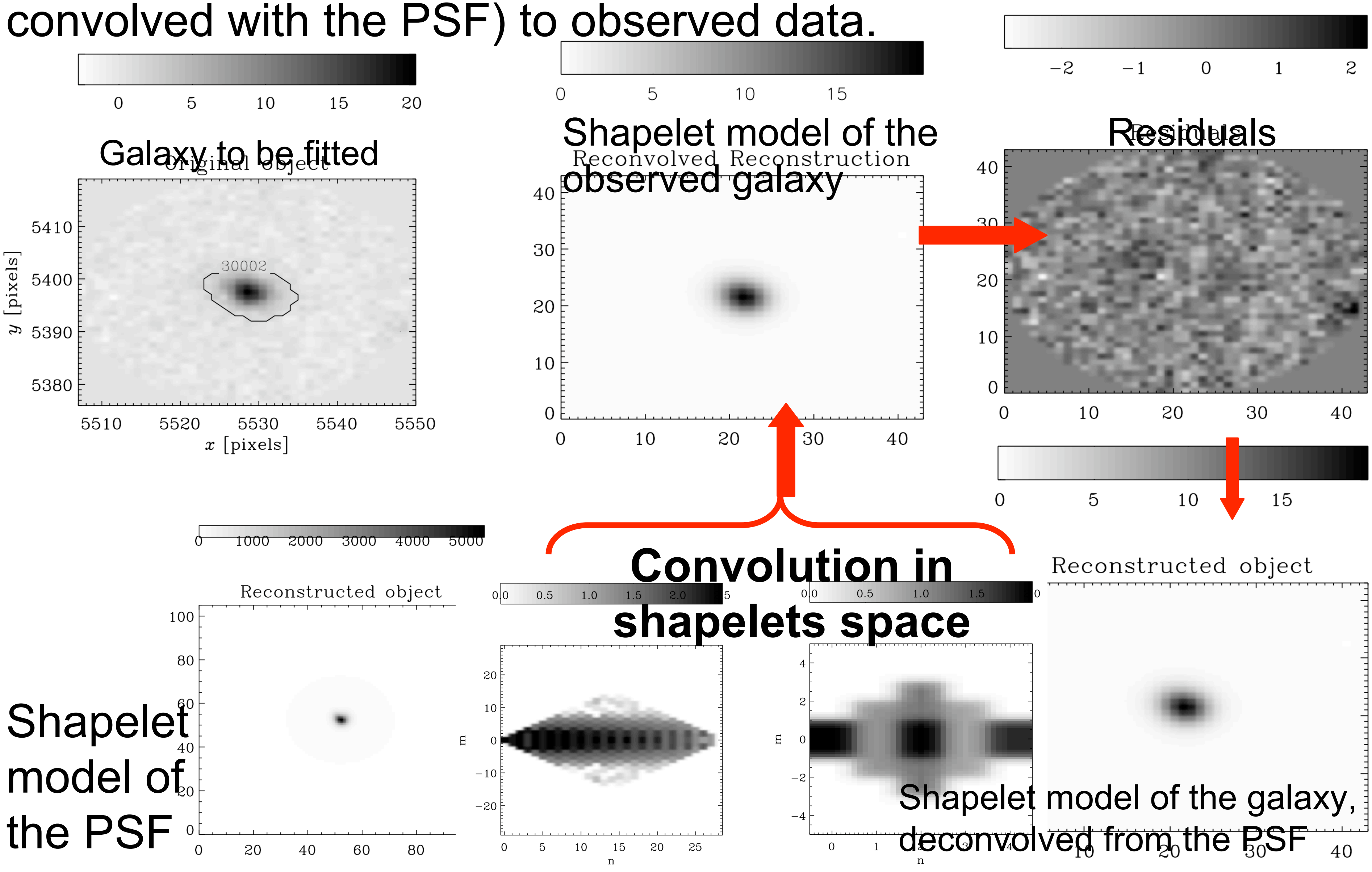


Shear estimator(s)

$$\tilde{\gamma}_n = \frac{4}{\sqrt{n(n+2)}} \frac{f'_{n,2}}{\langle f_{n-2,0} - f_{n+2,0} \rangle}$$

Shapelets pipeline

Least-square fitting of an analytical model (pixellised and convolved with the PSF) to observed data.



Shapelets pipeline

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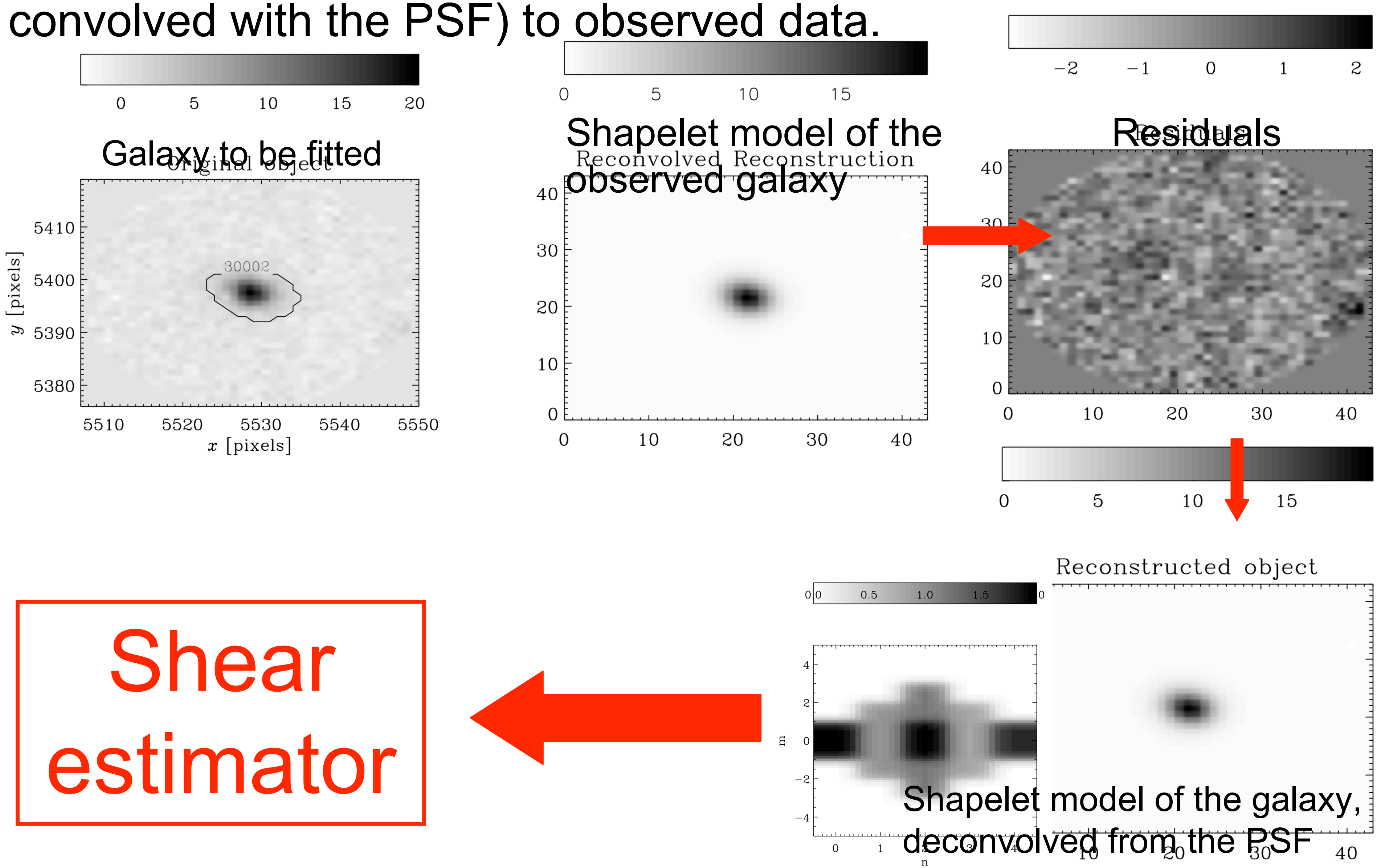


PSF modelling

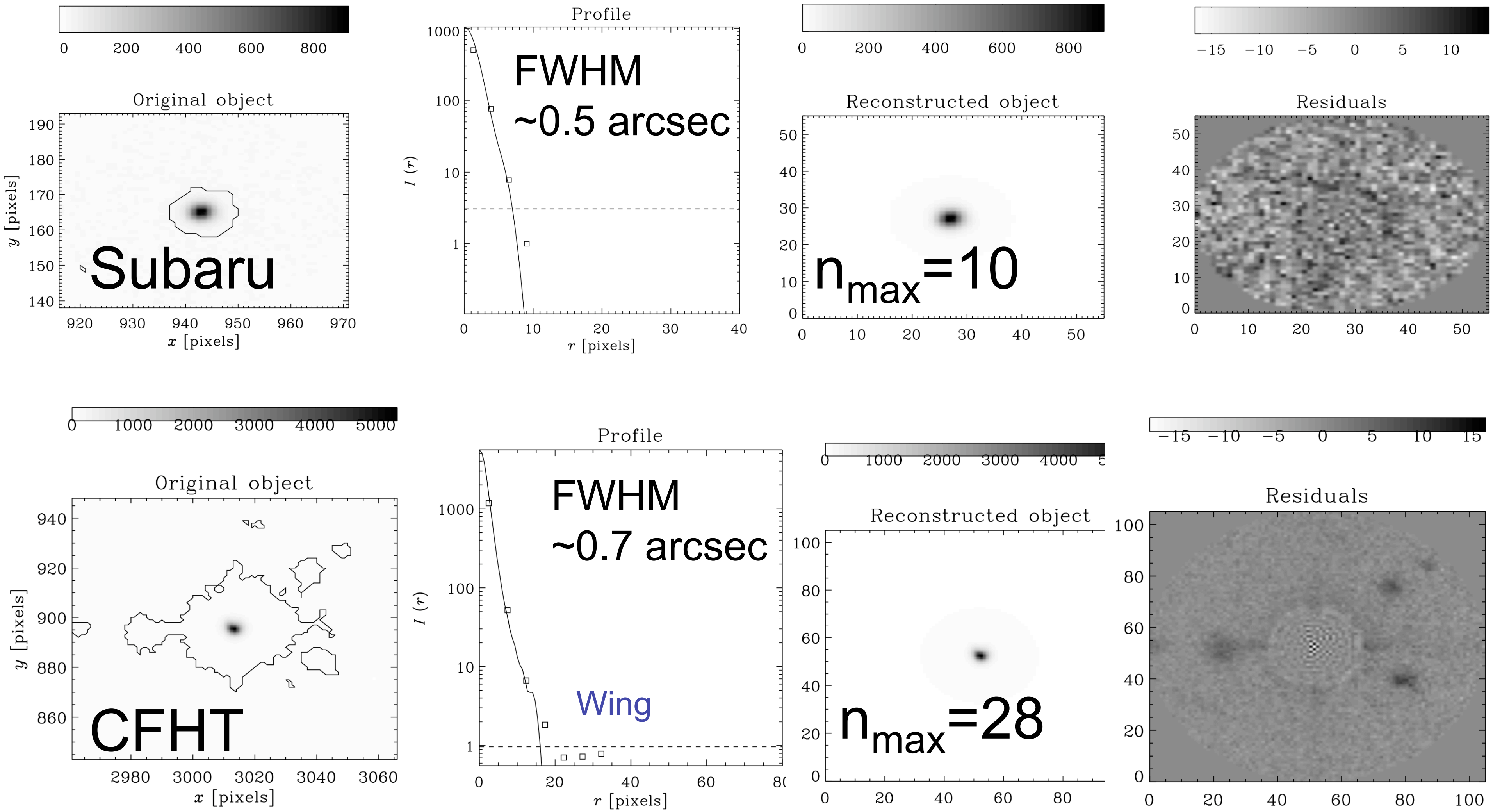
- Selection of useful stars
- 2-step shapelet modelling of stars
  - 1 : full focus (search for $n_{\max}$, β and centroid) on each star
  - 2 : β fixed to the same value for each star
- Polynomial fitting of shapelet coefficients

- Full PSF shape information captured

$\Rightarrow$ PSF eventually fully corrected for

PSF model

Difficulty : must account for all PSF shape, even wings



PSF characterization

- Possibility to characterize spatial variations of PSF shape information

Coefficients f_{nm}

Flux $$F \equiv \iint_{\mathbb{R}} f(\boldsymbol{x})\,\mathrm{d}^2x = (4\pi)^{1/2}\beta \sum_n^{\text{even}} f_{n0}$$

Size $$R^2 = \frac{(16\pi)^{1/2}\beta^3}{F} \sum_n^{\text{even}} (n+1)\, f_{n0}$$

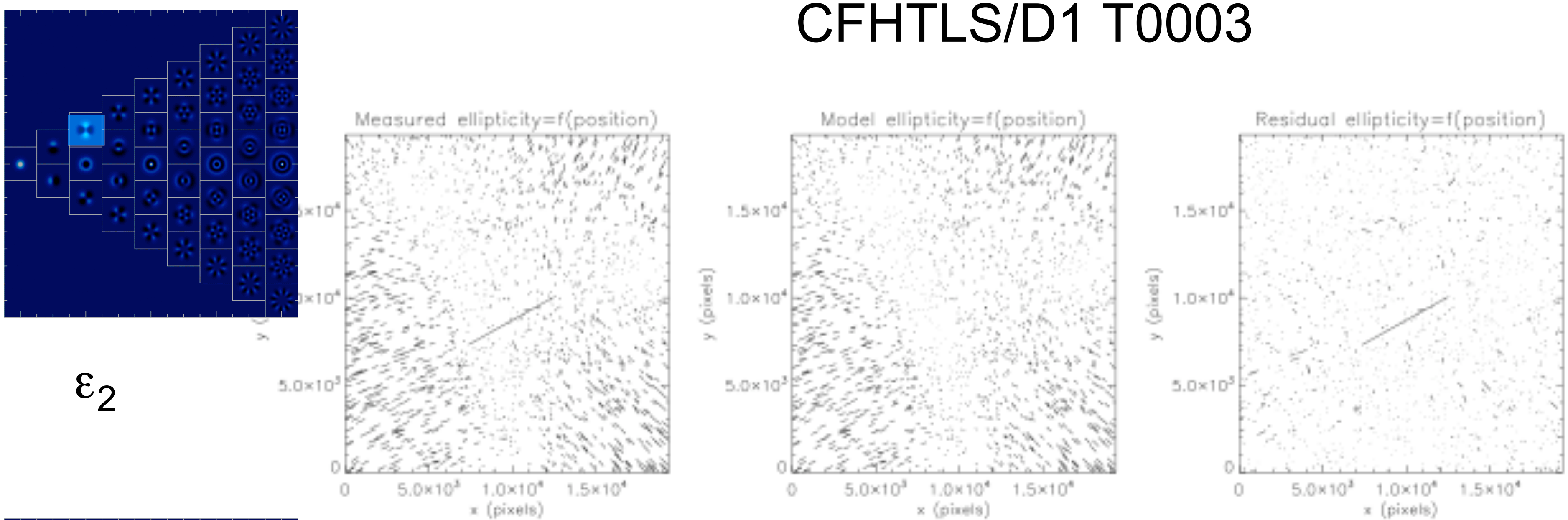
Ellipticity, order by order

$$\varepsilon = \frac{F_{11} - F_{22} + 2\mathrm{i}F_{12}}{F_{11} + F_{22}} = \sum_n^{\text{even}} \varepsilon_n$$

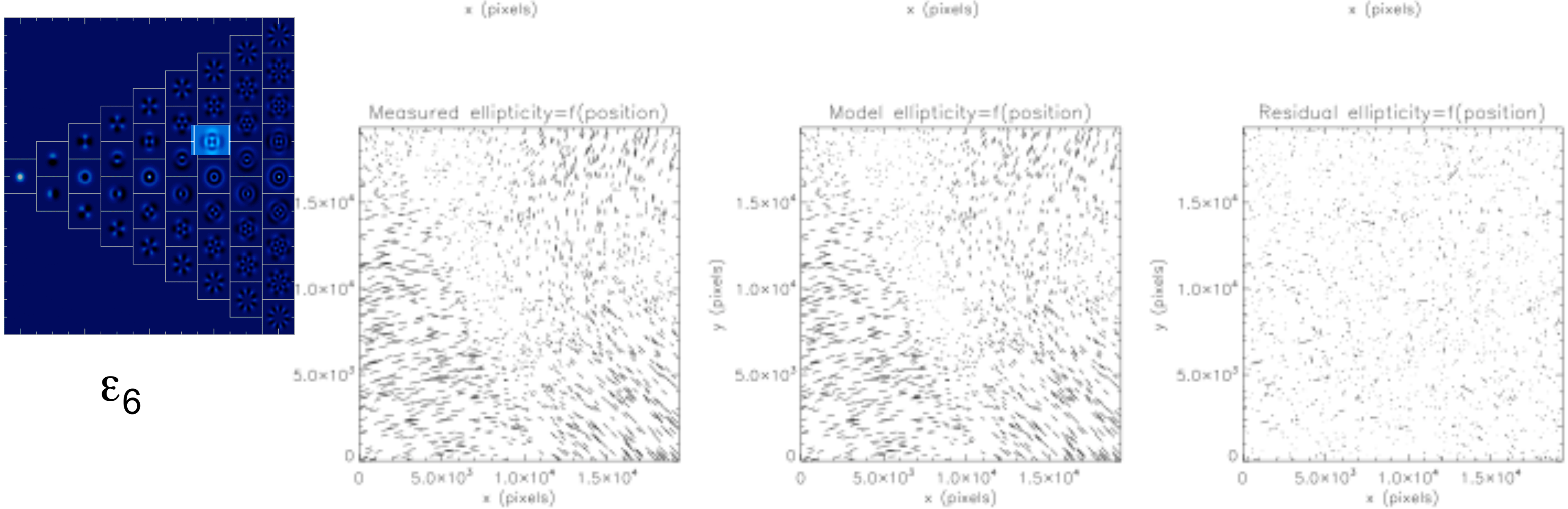
$$\varepsilon_n = \frac{(16\pi)^{1/2}\beta^3}{FR^2} \left[n(n+2)\right]^{1/2} f_{n2}$$

PSF spatial variations

CFHTLS/D1 T0003



ε_2



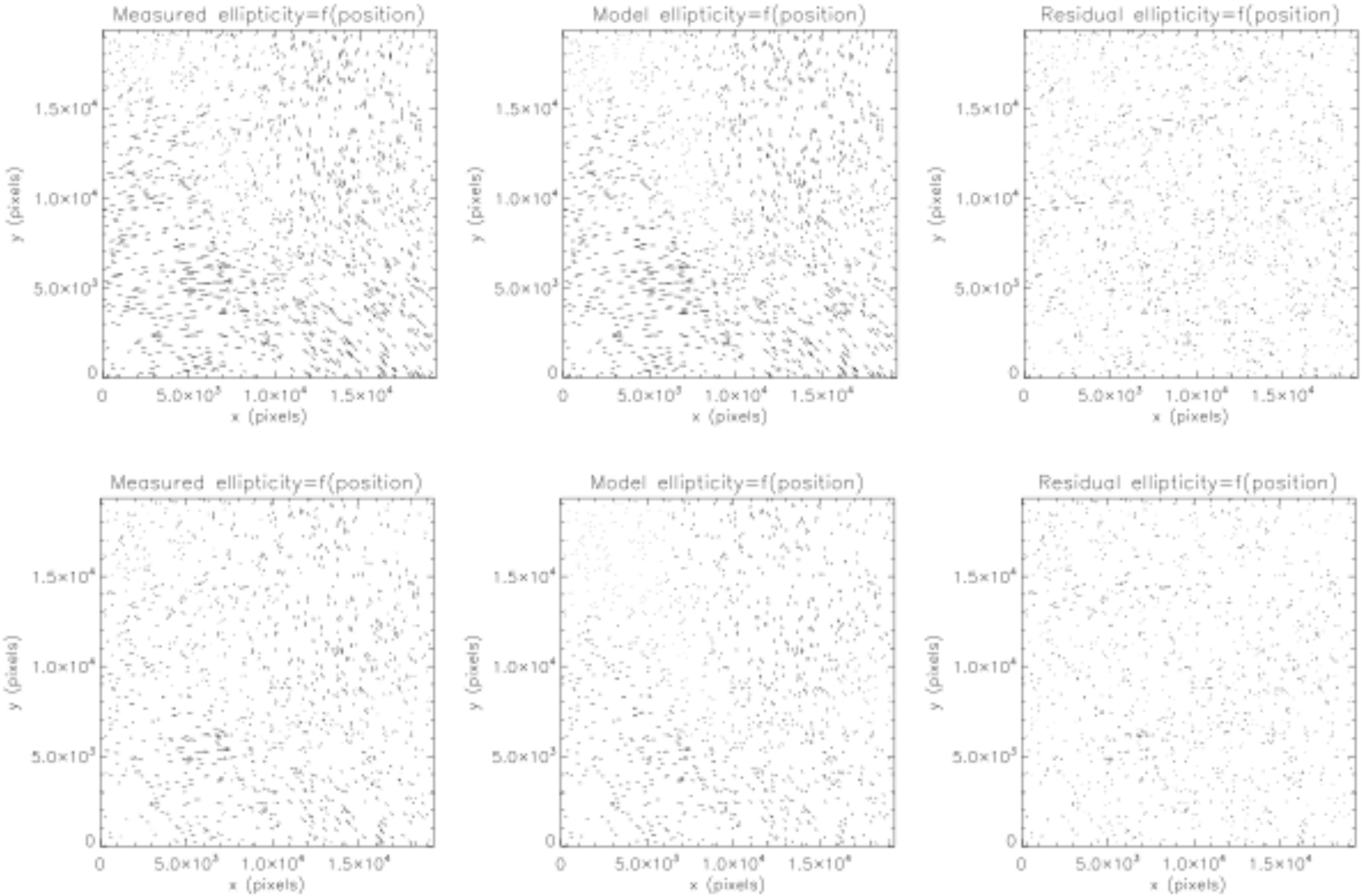
ε_6

PSF spatial variations

CFHTLS/D1 T0003

ε_{14}

ε_{18}



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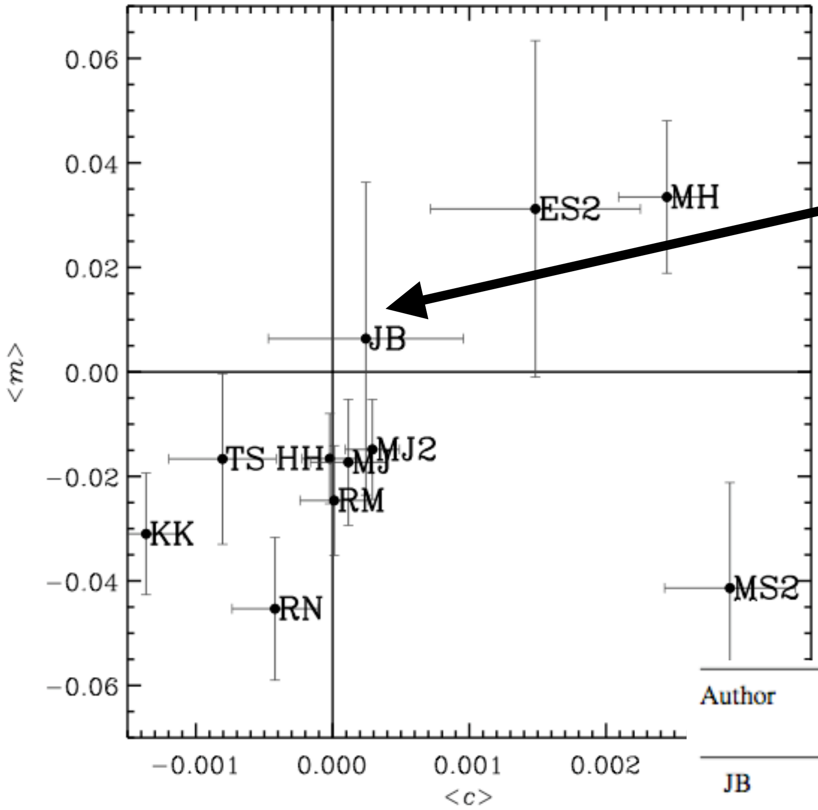
Shapelets in STEP2

shapelets



Author	Image set	n_{gals} Original/	Matched	Mean magnitude (original)	Per cent mag Decrease
JB	A	37 (0)	25	24.04	1.2
	C	28 (1)	21	23.50	1.0

Main concern : error bars

Pipeline updates

- Better decomposition success rate (70-80% => ~ 95%)
- Fit P_γ as a function of size and magnitude
- Use γ_2 shear estimator instead of estimator based on unweighted ellipticity
- Galaxy weighting scheme

$$w_g = (\underbrace{\sigma^2_{\varepsilon,g}}_{\text{Shape measurement error}} + \underbrace{\sigma^2_{P_\gamma,g}}_{P_\gamma \text{ measurement error}} + \underbrace{\sigma^2_{\text{int}}}_{\text{Intrinsic shape dispersion}})^{-1}$$

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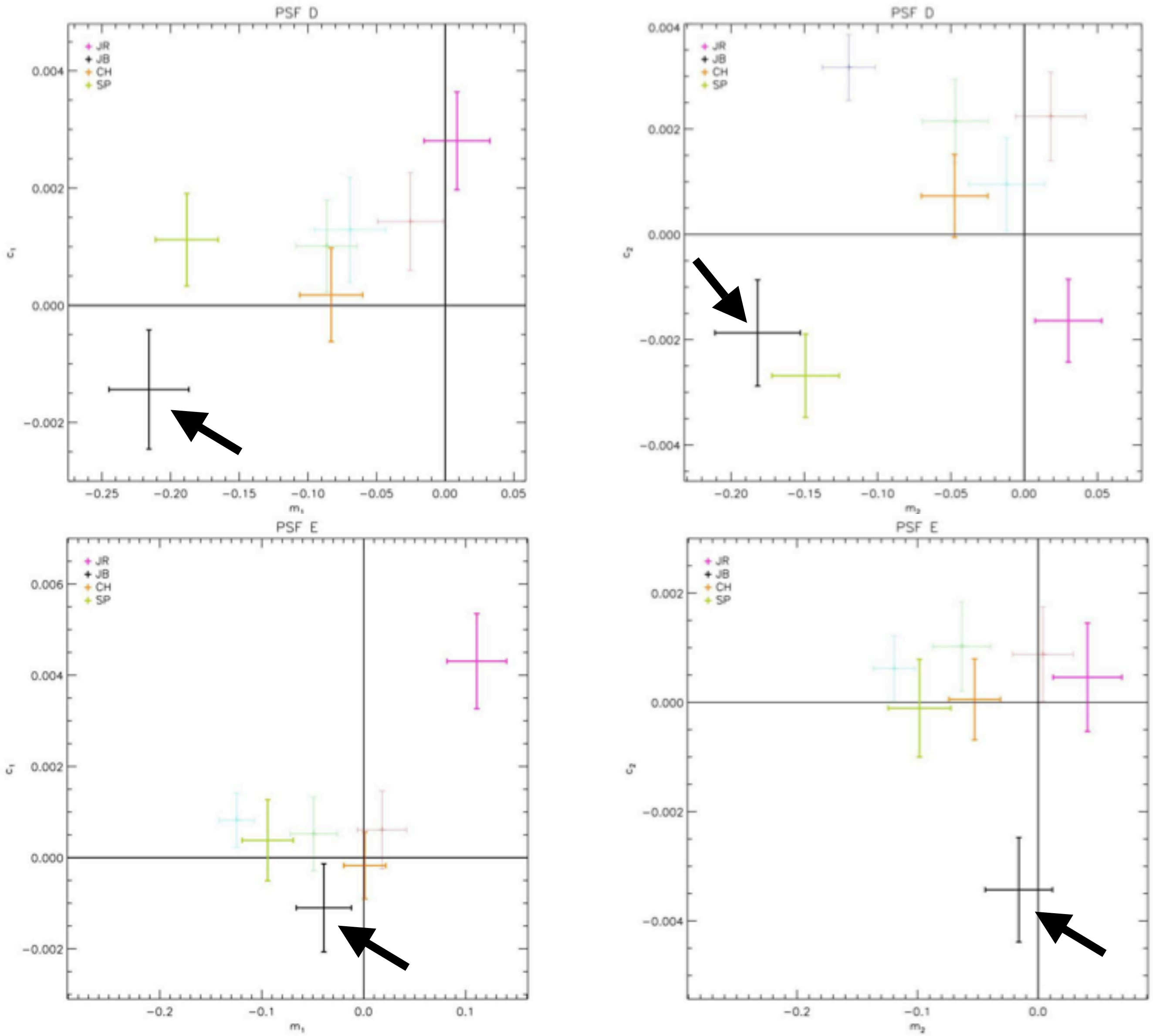
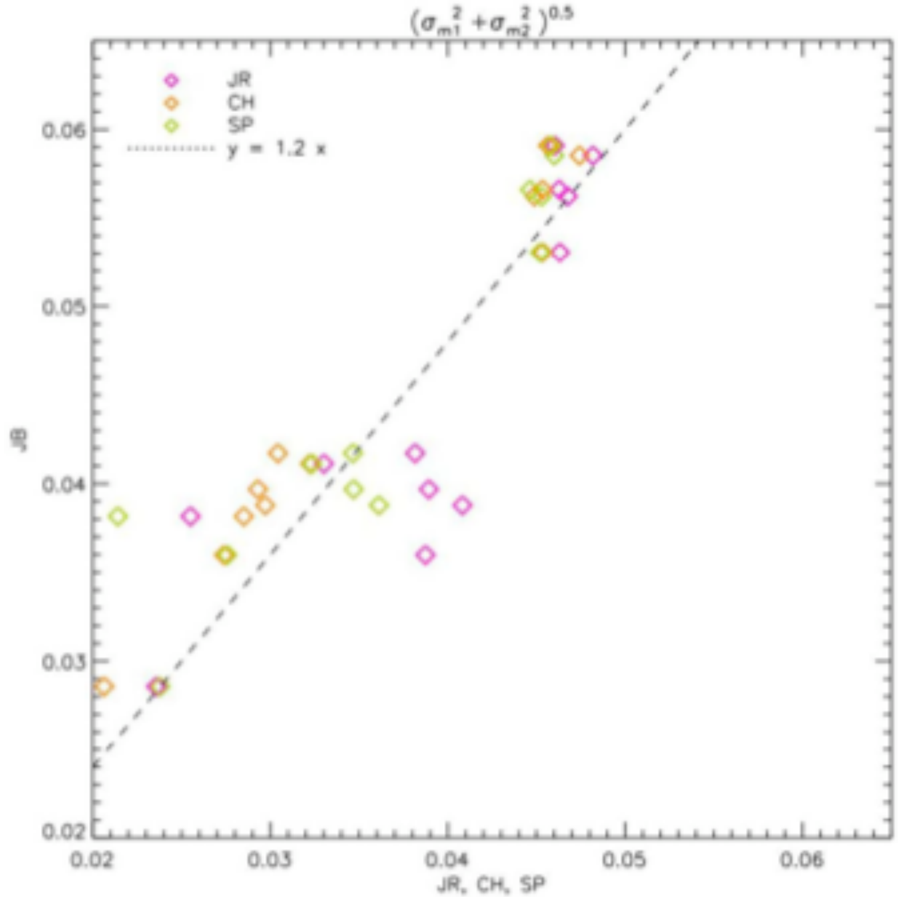
Compared to others



Error bars

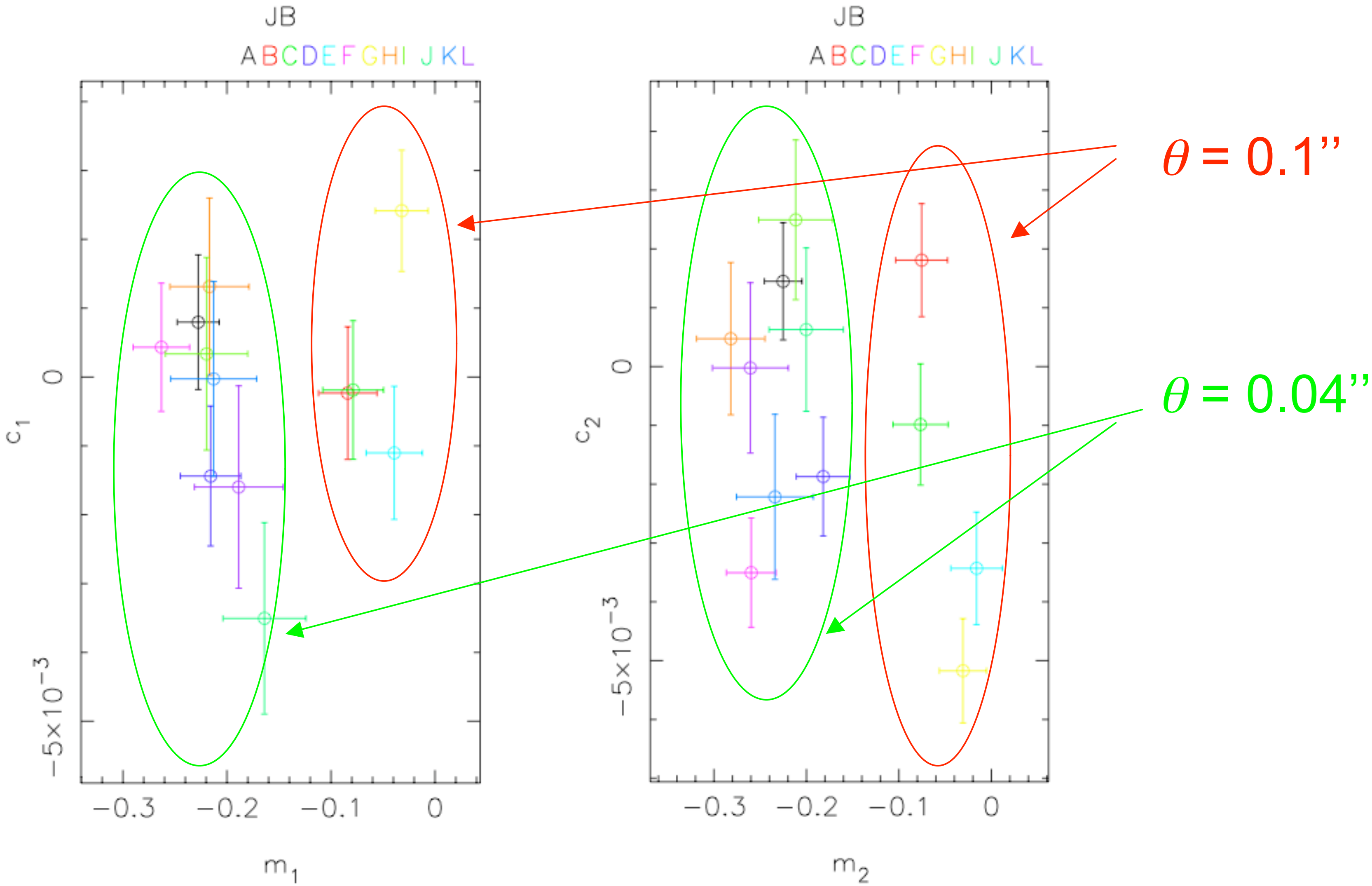
Courtesy Catherine Heymans, Stéphane Paulin-Henriksson, Jason Rhodes



Shapelets error bars slightly higher than considered methods' (~20%)

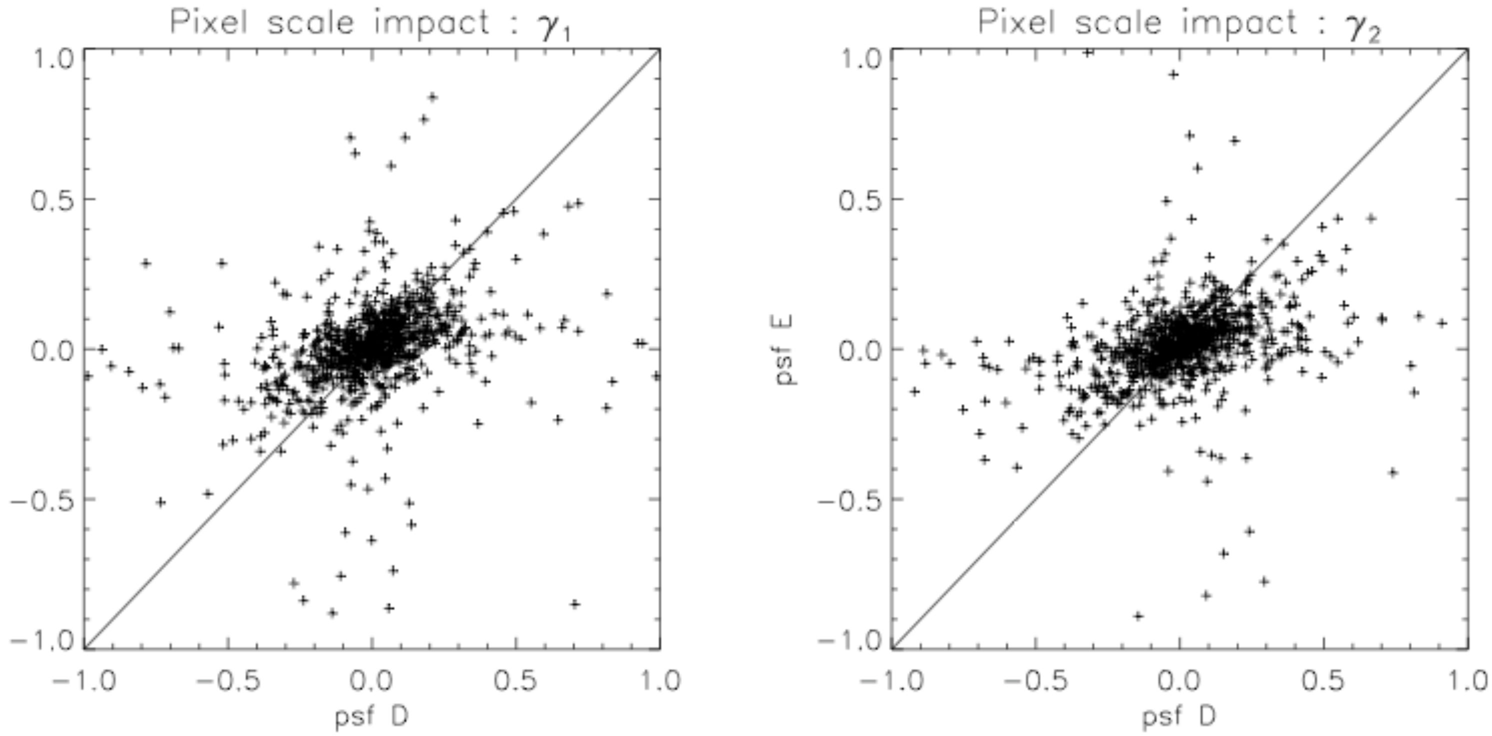
Non-optimal galaxy selection can explain bigger error bars in spite of our weighting scheme.

Pixel scale impact



Pixel scale impact

Comparison of shear galaxy by galaxy between PSF D and PSF E (same patch of sky, different θ)



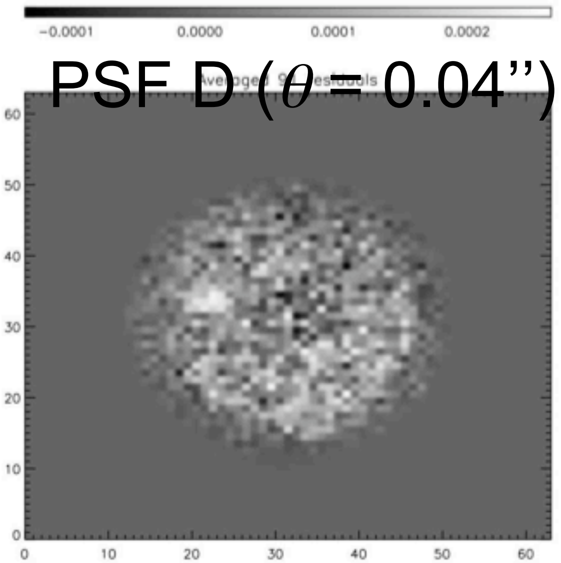
θ = 0.1’’ θ = 0.1’’

θ = 0.04’’ θ = 0.04’’

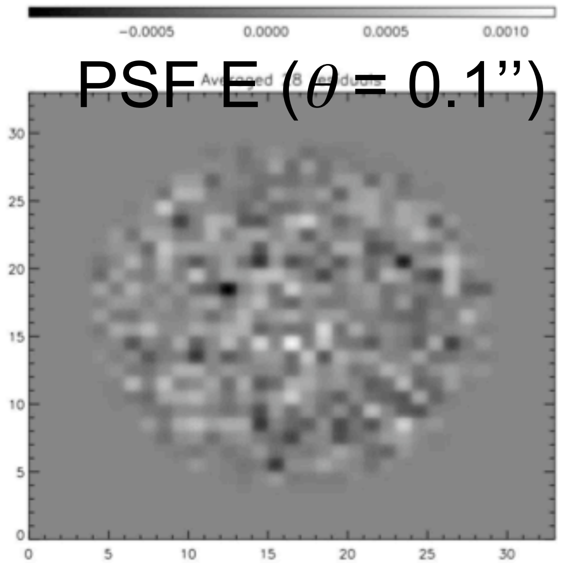
What's happening ?

Stack residuals of similar galaxies

PSF D (θ = 0.04'')



PSF E (θ = 0.1'')



Wings not caught by shapelet model
=> model too circular
=> underestimated shear

Wings better caught.

$n_{\max}$ doesn't increase enough for small pixel scales, unable to catch outer regions of galaxies

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CFHTLS/D1 shapelet κ map

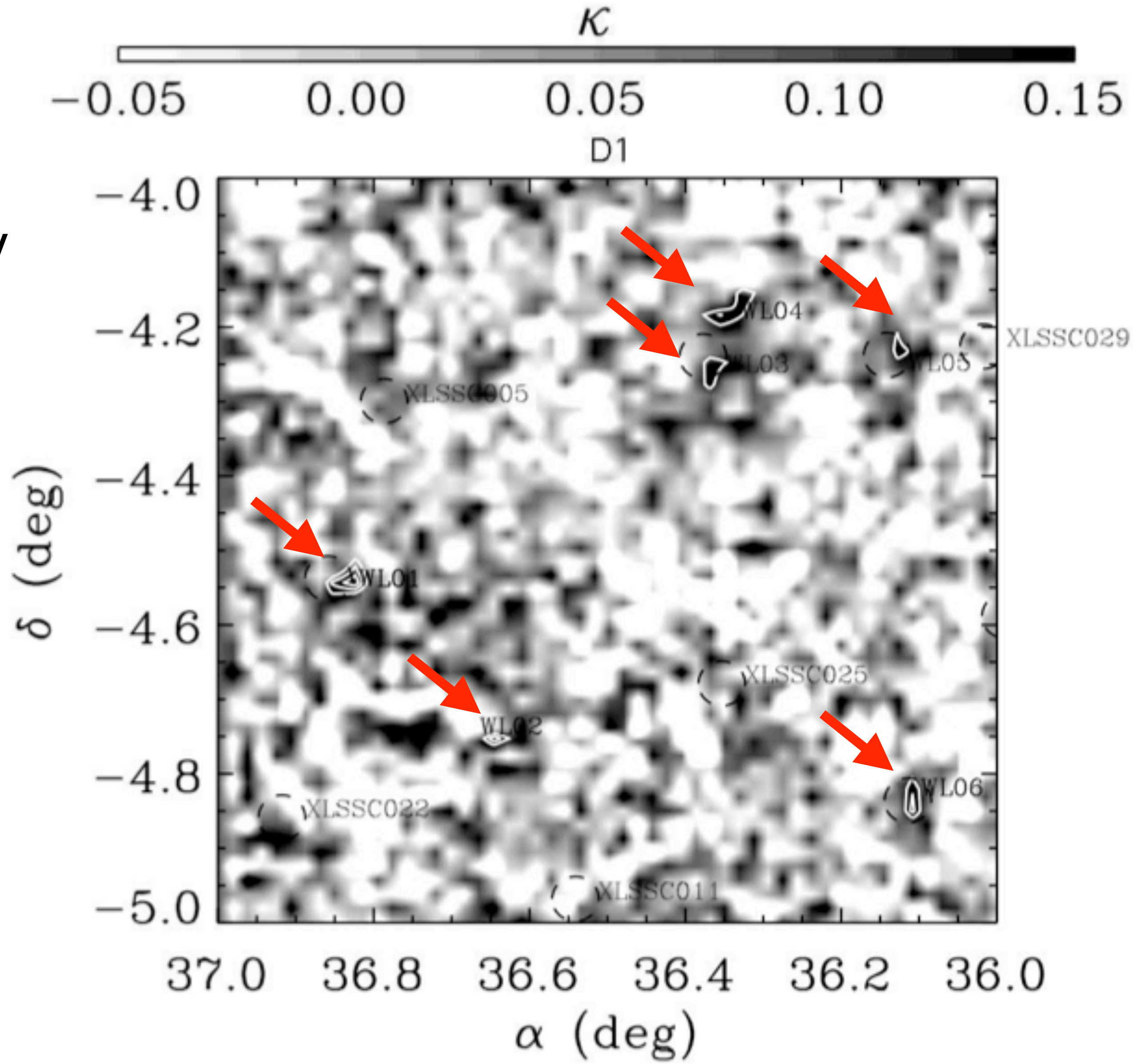
Bergé et al. 2007

1 deg^2, deep

Region observed by XMM-LSS

Complex PSF

We find 6 clusters
5 have X-ray and/or KSB counterpart



Weak lensing selection function

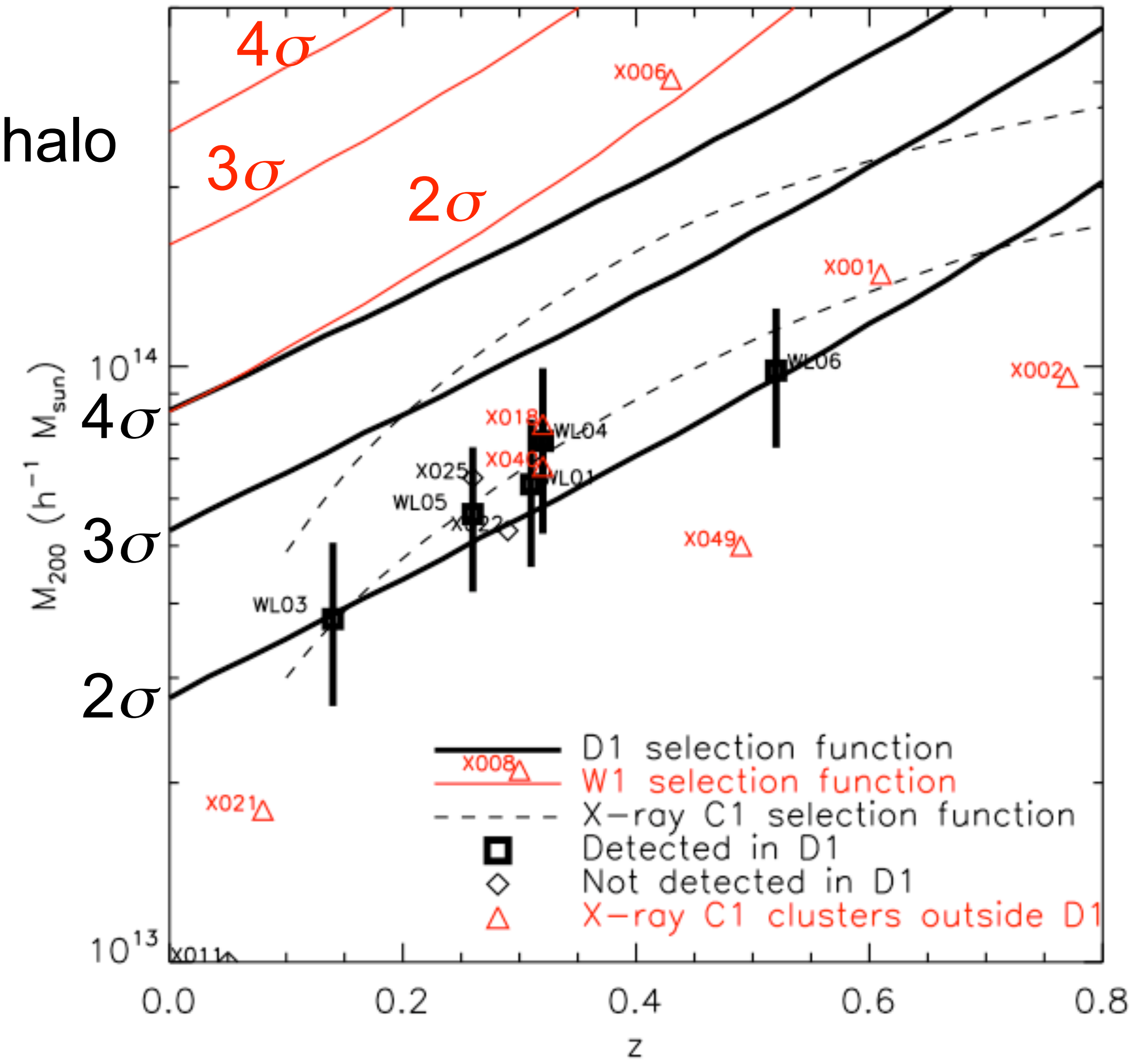
For clusters in D1 and its surrounding W1 area

S/N ratio by an NFW halo

$$\nu = \frac{\sqrt{n_g}}{\sigma_\gamma}\sqrt{\int \mathrm{d}^2x\,\kappa^2(x)}$$

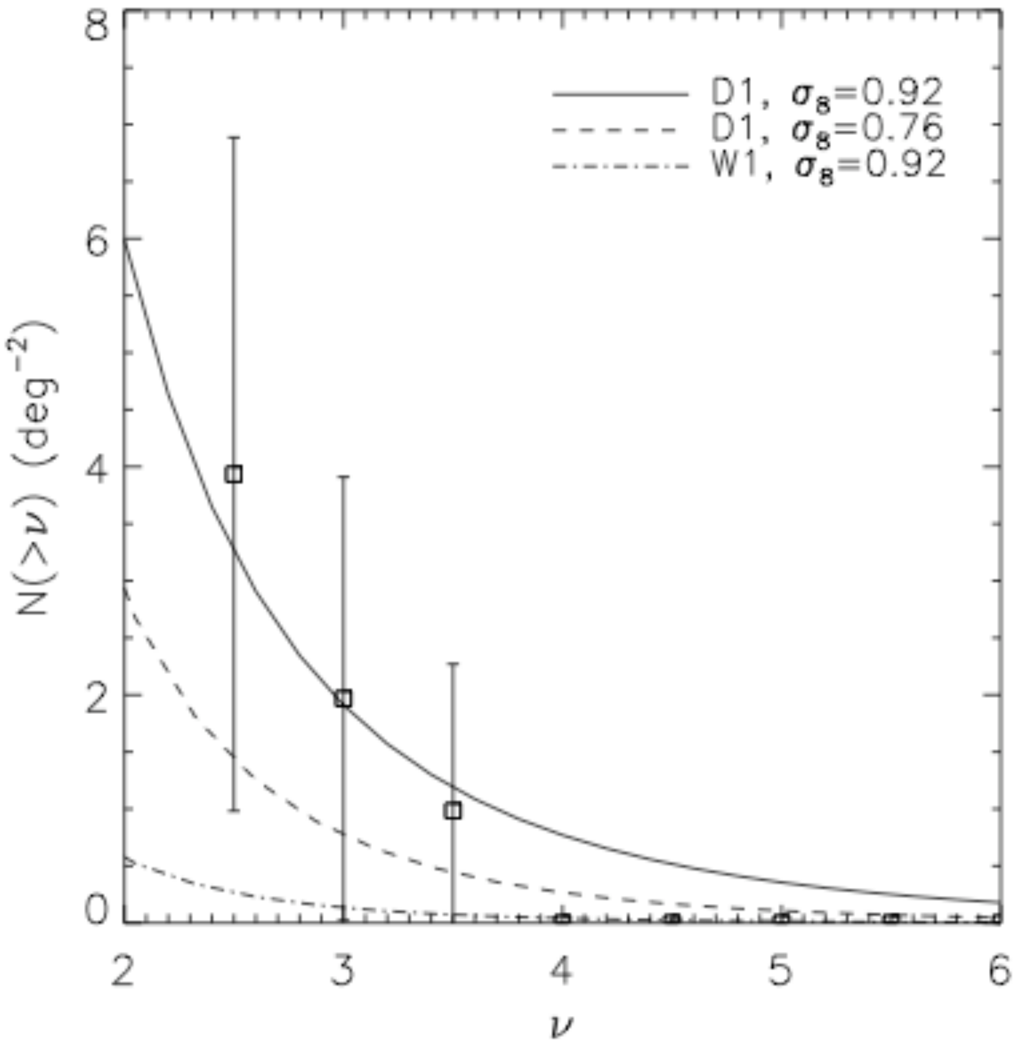
WL-selected clusters : WL mass

Others : X-ray mass



Cluster counts

Press-Schechter approach => expected cluster counts

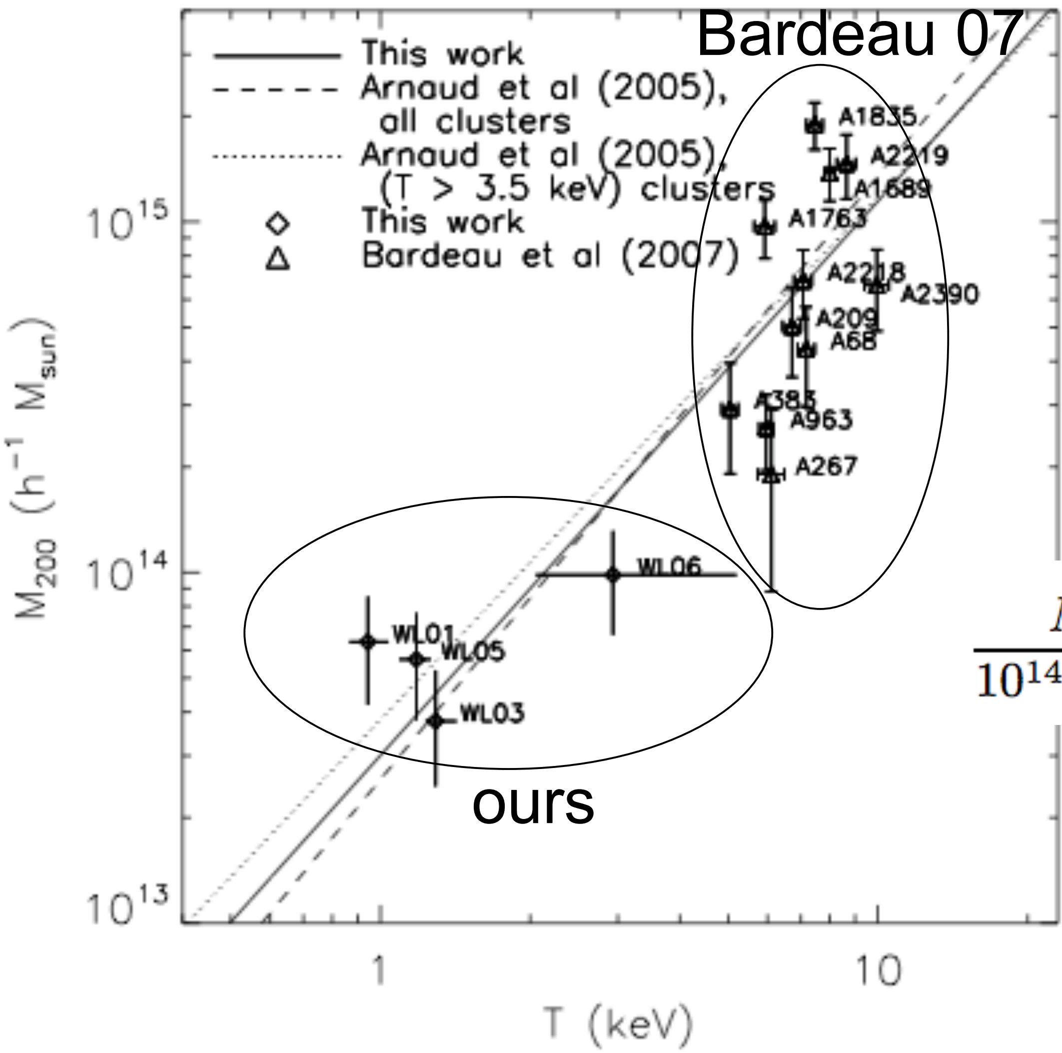


Fit to our data => σ_8

$$\sigma_8 = 0.92^{+0.14}_{-0.16}$$

with $\Omega_m = 0.24$

M-T relation



Large mass range

2-parameters fit : slope and normalization

$$\frac{M_{200}}{10^{14}h^{-1}M_\odot} = 2.69^{+0.68}_{-0.55}\left(\frac{T}{4\ \mathrm{keV}}\right)^{1.58\pm0.38}$$

Conclusion

- Shapelets : fitting method aiming to be as linear as possible, well suited to weak lensing analysis
- Provide full characterization of an object's shape (in particular, PSF)
- STEP2 has allowed us to update and improve our pipeline
- Space-STEP has emphasized a pixel scale dependent modelling problem, we're working on it
- Shapelets have been successfully used on real CFHTLS data : we found galaxy groups, with a striking consistency with X-ray detections, and measured σ_8 and M-T relation slope and normalization.

Convergence of non-linear parameters