Astrophysical Cosmology 4 2004/2005

Problem set 7

(1) Solve the problem of matching an inflationary vacuum-dominated de Sitter expansion with $R \propto \exp(Ht)$ onto a radiation-dominated era via a sudden change in the equation of state. R and $\dot{R}$ must match at the join (why?). If this change is assumed to take place at a time t_c after the classical big bang, show that the condition $t_c = 1/2H$ is required, so that the two solutions can be written as

$$\frac{R(t)}{R(t=0)} = \begin{cases} \exp(t/2t_c) & (t < t_c) \\ \sqrt{e\,(t/t_c)} & (t > t_c) \end{cases} .$$

(2) The largest features measured in large-scale structure today are superclusters of size up to $100\,h^{-1}$ Mpc. Calculate the angle that such structures would subtend if placed on the last-scattering surface at $z = 1100$ (give the result as a function of Ω_m, and consider both models with no vacuum energy, and flat vacuum-dominated models). Given that we possess CMB data to a highest angular multipole of $\ell \simeq 1000$, discuss the observability of infant superclusters as a function of Ω_m.

A spherical perturbation exists in the present-day universe. Its radius is $50\,h^{-1}$ Mpc and its density is 1.1 times the mean value. Suppose we could observe the perturbation to the CMB temperature caused by the progenitor of such a structure located at $z = 1100$. What mechanisms dominate the temperature fluctuation, and what is the approximate magnitude of $\Delta T/T$? (assume that the universe is the matter-dominated $\Omega = 1$ Einstein-de Sitter model).

(3) [part of 2001 degree exam question] One hypothesis for the origin of dark matter is that it consists of weakly interacting relic particles. Explain the reason why it is normal to distinguish three types ('hot', 'warm', and 'cold') of such particles, and give the approximate mass scales associated with each case.

A relic dark-matter particle imposes a coherence scale on cosmic structure depending on the time at which it becomes non-relativistic. Given that the age of the universe is approximately $t = 1$ s when its temperature is 10^{10} K, calculate this coherence scale in comoving length units as a function of the mass (express your result in comoving Mpc, using mass units of eV). Hence summarize the astronomical reasons why it is considered unlikely that the universe is dominated by hot dark matter.