Astrophysical Cosmology 4 2004/2005

Problem set 6

(1) Is it a good approximation to treat the matter in the early stages of the big bang as an ideal gas? Test this by comparing the typical kinetic energy to the electrostatic potential energy between a particle and a neighbour at a typical distance. Recall that the number density for relativistic particles in thermal equilibrium is $n \sim g\,(kT/\hbar c)^3$.

(2) The redshift of last scattering is approximately 1100, but this assumes that intergalactic matter is largely neutral for lower redshifts. Observationally, this is not true at low redshift, where the gas is ionized by ultraviolet light from stars and quasars. Assume that the baryonic material in the universe is re-ionized suddenly at a redshift z_c, and calculate the resulting optical depth due to Thomson scattering. How large does z_c have to be before this reaches unity, so that typical CMB photons would no longer be scattered at $z = 1100$? You may assume the distance-redshift relation

$$R_0 dr = \frac{c}{H_0}\left[(1 - \Omega_m - \Omega_v)(1+z)^2 + \Omega_v + \Omega_m(1+z)^3\right]^{-1/2} dz.$$

Assume that $z_c \gg 1$.

(3) The spherical collapse model represents the radius–time relation of a proto-object as

$$r = A(1 - \cos\theta)$$
$$t = B(\theta - \sin\theta).$$

Show that, if $A^3 = GMB^2$, these relations satisfy $\ddot{r} = -GM/r^2$. Expand these relations up to order θ^5 to show that, for small t:

$$r \simeq \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left[1 - \frac{1}{20}\left(\frac{6t}{B}\right)^{2/3}\right].$$

Hence show that the linear-theory perturbation to the density is

$$\delta_{\rm lin} = \frac{3}{20}\left(\frac{6t}{B}\right)^{2/3}.$$

What prediction does this expression make for the density inside the sphere if it is extrapolated to the point of collapse to a singularity?