Astrophysical Cosmology 4 2004/2005

Problem set 4

(1) The flux density of an object at redshift z, observed at frequency ν_0, is

$$f_\nu(\nu_0) = \frac{L_\nu([1+z]\nu_0)}{(1+z)\,[R_0 S_k(r)]^2}.$$

Consider a source that emits a thermal spectrum of temperature T. If observations are made at low frequencies ($h\nu_0 \ll kT$), show that the flux density *increases* with redshift for $z \gtrsim 1$, until a critical redshift is reached. Give an order-of-magnitude estimate for this redshift in terms of ν_0 and T.

(2) The 'deceleration parameter' is defined as a dimensionless form of the second time derivative of the scale factor: $q \equiv -\ddot{R}R/\dot{R}^2$. Use the acceleration form of Friedmann's equation to obtain an expression for the current value of q, for a universe containing a mixture of vacuum energy and nonrelativistic matter. Show that the expansion decelerates only if $\Omega_m > 2\Omega_v$. Explain qualitatively why objects at a given redshift appear brighter in a decelerating universe and fainter in an accelerating universe.

(3) The equation for a radial null geodesic in the Robertson-Walker metric is $dr = c\,dt/R(t)$, which can be cast in the observational form

$$R_0\,dr = \frac{c}{H_0}\ \left[(1-\Omega_m-\Omega_v)(1+z)^2 + \Omega_v + \Omega_m(1+z)^3\right]^{-1/2}\ dz,$$

for $z \lesssim 1000$. Expand this relation as a series in z to obtain an approximation for the luminosity distance $D_{\rm L}(z) = (1+z)R_0 S_k(r)$ that is valid to second order in z. Show that the second-order correction depends only on the combination $\Omega_m/2 - \Omega_v$. Hence explain the sense of the near-degeneracy between Ω_v and Ω_m as determined from the supernova Hubble diagram.

(4) An object is observed at redshift z in a matter-dominated universe with density parameter Ω. Calculate the observed rate of change of redshift for the object (hint: remember $1+z = R_0/R_{\rm emit}$, where both R_0 and $R_{\rm emit}$ change with time, and that time intervals in high-redshift objects are observed to be time-dilated). What fractional precision in observed frequency would be needed to detect cosmological deceleration in a decade?