Astrophysical Cosmology 4 2004/2005

Problem set 3

(1) The equation for a radial null geodesic in the Robertson-Walker metric is $dr = c\,dt/R(t)$. Using the relation between redshift and scale factor, $1 + z \propto 1/R(t)$, plus Friedmann's equation, deduce the differential relation between comoving distance and redshift:

$$R_0\,dr = \frac{c}{H_0}\,\left[(1-\Omega)(1+z)^2 + \Omega_v + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]^{-1/2}\,dz.$$

(2) Integrate this expression for the case of the $\Omega = 1$ Einstein–de Sitter universe. In this model, calculate the apparent angle subtended by a galaxy of proper diameter 30 kpc, as a function of redshift (recall $c/H_0 = 3000\,h^{-1}\,\mathrm{Mpc}$). Show that there is a critical redshift at which this angle has a minimum value.

(3) From the relation between distance and redshift, deduce the differential relation between redshift and time. Integrate this for the case of the $\Omega = 1$ Einstein–de Sitter universe. A galaxy is observed at redshift 1.5 to contain stars that are 3.5 Gyr old: assuming $\Omega = 1$, deduce a limit on the Hubble constant (remember $H_0^{-1} = 9.78h^{-1}$ Gyr).

(4) Show that the result of integrating dr/dz between ∞ and z is the comoving horizon length. For redshifts below about 10^4, the universe may be assumed to be matter dominated: deduce an expression for the horizon length as a function of z that is valid for $10^4 \gtrsim z \gtrsim 1$.

The microwave background was emitted at $z \simeq 1100$. Calculate the horizon size at this epoch. If $\Omega = 1$, what angle on the sky does this length subtend? Is it surprising that the microwave background radiation is uniform to 1 part in 1000?