Astrophysical Cosmology 4 2004/2005

Problem set 1

(1) The Robertson-Walker metric can be written as

$$c^2 d\tau^2 = c^2 dt^2 - R^2(t) \left(dr^2 + S_k^2(r)\, d\psi^2 \right),$$

where $S_k(r) = \sin r$ $(k = +1)$, r $(k = 0)$, or $\sinh r$ $(k = -1)$ and $d\psi$ is the angular separation between the two events under consideration.

(a) Show that, in the $k = +1$ case, the spatial part of this 3D metric is the same as that for a 2D space that forms the surface of a sphere.

(b) The proper time interval $d\tau$ is not in general the same as the cosmic time interval measured by an observer at the origin. By considering a pair of events with $dr = d\psi = 0$, explain why not.

(c) The element of radial proper distance is $R(t)\, dr$. How does this change with time? By considering events close to $r = 0$, deduce Hubble's law, and show that $H = \dot{R}/R$.

(d) Show that the curved $k = \pm 1$ cases are in practice indistinguishable from the flat $k = 0$ case provided R is large enough.

(2) Using the Robertson-Walker metric, show that the comoving separation between us and an object seen at redshift z is $r = \int c\, dt/R(t)$. Since r is independent of time, argue that the redshift is $1 + z = R(\text{now})/R(\text{emit})$.

(3) Derive the same relation by considering the infinitesimal Doppler shift caused when a photon travels a distance d, thus encountering an observer with relative velocity $\delta v = Hd$ (use $d = c\,\delta t$ and remember $H = \dot{R}/R$). Use the same approach to show that the 'peculiar' momentum of any particle decays $\propto 1/R$ (use a Lorentz transformation of the particle 4-momentum to get the change in momentum caused by δv).