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Constraints on the ionization sources of the high-redshift intergalactic medium

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ABSTRACT

Constraints on the ionization structure of the intergalactic medium (IGM) are derived as directly imposed by observations in conjunction with the results of numerical simulations for structure formation. Under the assumption that the population of sources dominating the ultraviolet (UV) ionizing background at $z < 6$ is the same population which reionized the IGM, it is shown that consistency with measurements of the mean Lyα transmitted flux at high redshifts requires the epoch of hydrogen reionization of the IGM to have occurred at a redshift $z_{\rm ri} < 11$, independent of the space density of the sources. The upper limit on the reionization redshift depends only on the shape of the UV spectra of the sources. Consistency with constraints on the reionization epoch from the *Wilkinson Microwave Anisotropy Probe* requires the sources of photoionization to have had hard spectra, such as quasi-stellar objects (QSOs) or Population III stars. The only ways to escape these conclusions are either (i) the sources which dominate the photoionization background at $z < 6$ are the remnants of a population that was a much more prodigious source of ionizing photons at earlier times, or (ii) the sources responsible for the photoionization of the IGM are an unknown population that contributes negligibly to the ionization of the IGM at $z < 6$. The evolution of the QSO luminosity function over the range $3 < z < 6$ is estimated from recent QSO counts, and the fraction of ionizing photons arising in QSOs is evaluated under the assumptions of either pure luminosity evolution or pure density evolution. It is shown that QSOs dominate the UV ionizing background for $z < 3.5$, but that it is unlikely that more than one-half to one-third of the ionizing background radiation originates in QSOs in the redshift range $4.5 < z < 6$. QSOs acting alone could not have reionized the hydrogen in the IGM prior to $z \approx 4$. If the QSOs had hard spectra, however, they may have reionized the helium in the IGM as early as $z \approx 5$. The possibility that the IGM was reionized by low-luminosity active galactic nuclei is discussed.

Key words: methods: numerical – intergalactic medium – quasars: absorption lines – quasars: general – X-rays: diffuse background.

1 INTRODUCTION

The ionization structure of the intergalactic medium (IGM) at high redshift has been the focus of much recent attention. On the basis of measurements of the mean transmitted Lyα flux in spectra of the highest known redshift quasi-stellar objects (QSOs) by the Sloan Digital Sky Survey (SDSS), it has been suggested that the reionization of the IGM had just completed at $z \gtrsim 6$ (Becker et al. 2001). By contrast, measurements of fluctuations in the cosmic microwave background (CMB) by the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) suggest that the IGM was reionized earlier, at $z > 11$ (2σ lower limit; Kogut et al. 2003). The most obvious candidate sources of reionization are stars and QSOs. An assessment based on the results of the *Hubble Ultra-Deep Field* (*UDF*) and the Great Observatories Origins Deep Survey (GOODS) by Bunker et al. (2004) suggests that the galaxies identified at $z \approx 6$ are too few to account for the reionization of the IGM by $z = 6$. On the other hand, Yan & Windhorst (2004) and Stiavelli, Fall & Panagia (2004) dissent from this view, arguing that, with plausible assumptions, these sources are adequate. Yan & Windhorst further argue that galaxies appear to be the only sources of reionization. They claim that QSOs contribute negligibly to the overall ionizing photon budget, falling short of the required numbers by more than two orders of magnitude. Meiksin & White (2004) argue instead for a much smaller discrepancy for the quasars. The range of views in the literature highlights the sensitivity of conclusions regarding reionization to prevailing uncertainties, in particular the luminosity function of the sources, their nature – especially their emergent ultraviolet (UV) spectra – and the structure of the IGM.

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A question distinct from the sources responsible for reionization is the nature of the sources which dominate the UV photoionizing background at $z < 6$, when the IGM is clearly ionized. The populations need not be the same, as the ionizing contribution from the sources responsible for reionizing the IGM may decline with time, while other sources begin to dominate the UV background of the post-reionization Universe. For instance, the mean escape fraction of ionizing photons from galaxies at high redshifts is currently unclear, and could in principle be sufficiently high to enable galaxies to reionize the IGM. Steidel, Pettini & Adelberger (2001) report a detection of UV ionizing photons from the brightest Lyman break galaxies in a sample at $z \approx 3$, suggesting an escape fraction of about 10 per cent, although this difficult measurement has not yet received independent confirmation. Attempts to quantify the mean escape fraction have generally resulted only in upper limits. For instance, Fernandez-Soto, Lanzetta & Chen (2002) constrain the mean escape fraction for galaxies in the range $1.9 < z < 3.5$ to less than 4 per cent, which may be too small for galaxies to maintain the ionization of the IGM. Applying an escape fraction of $\sim$10 per cent to the full range of galaxy luminosities, Steidel et al. obtain a value for the global emissivity of ionizing radiation that is inconsistent with the emissivity required by numerical simulations of the IGM to reproduce the measured values of the mean transmitted Lyα flux at $z \approx 3$. The high emissivity obtained by Steidel et al. would result in a much higher Lyα transmitted flux through the IGM than measured, unless the sources were very short-lived (e.g. bursting at just $z \approx 3$; Meiksin & White 2003). This suggests that the escape fraction found by Steidel et al. may be anomalously high. While an escape fraction of 1–3 per cent may be just sufficient for galaxies to maintain the ionization of the IGM at $z \approx 3$, it is unclear whether this fraction is sufficient for galaxies to meet the requirements for reionizing the IGM (see Stiavelli et al. 2004).

Constraints on the epoch of reionization and on the level of ionization in the post-reionization IGM have usually been treated as separate issues. In this paper, it is shown that the two are closely related. Any model that meets the requirements for the reionization epoch implies a level of ionization of the IGM just after reionization is complete. This level must be consistent with constraints imposed by measurements of the mean Lyα transmitted flux through the IGM. In particular, if the high Lyα optical depth measurements at $z \approx 6$ designate the tail end of the reionization epoch, then the rate of UV ionizing photon production by the sources must 'gracefully exit' the reionization epoch at the level required to match the measured Lyα optical depth measurements at $z \lesssim 6$.

A second purpose of this paper is to provide a statistical assessment of the contribution of QSOs to the photoionizing background at $z > 3$, and to the overall budget of ionizing photons at $z \approx 6$, using the current best estimates of the faint (Hunt et al. 2004) and bright (Fan et al. 2001, 2004) ends of the QSO luminosity–redshift distribution. Estimates of the UV background based on QSO counts are not very well constrained at high redshifts ($z > 3$) due to the low sampling of the luminosity–redshift plane by existing surveys. The uncertainty is sufficiently large to allow for a substantial contribution to the UV metagalactic background from QSOs at high redshifts (Meiksin & White 2004).

2 IONIZATION POROSITY OF THE INTERGALACTIC MEDIUM

The epoch of reionization by sources of number density $n_{\rm S}$ may be characterized by the time at which the porosity $Q = n_{\rm S}\langle V_{\rm I}\rangle$ exceeds unity, where $\langle V_{\rm I}\rangle$ is the average volume of the Universe ionized per source. At earlier stages, the filling factor of the remaining neutral gas is $f = 1 - Q$. For any given evolutionary scenario of the sources, the porosity is determined by the total rate of production $\dot{N}_{\rm S}$ of ionizing photons per unit volume, which in turn scales like the total emissivity of the sources. On the other hand, for a uniform medium, the neutral fraction, and hence the Lyα optical depth τ_α, of the IGM scales inversely with the total source emissivity. As a consequence, the product of the two will reflect only the evolution and spectral shape of the sources (and the properties of the IGM), but will be independent of their actual numbers. For any given source history and spectral shape, the Lyα optical depth just following the epoch of reionization is thus well specified and must be consistent with measured values. Turning this argument around, if the evolution of the emissivity and attenuation length are known, or reasonable guesses made based on observations or numerical simulations, then measurements of the Lyα optical depth may be used to provide a direct estimate of the porosity of the IGM, and hence of the epoch of reionization.

In reality, the IGM is not uniform, but clumped into the Lyα forest, so that the relation between the porosity and the measured mean Lyα optical depth is indirect. (Here, the mean Lyα optical depth is defined as $\bar{\tau}_\alpha = -\log\langle f_\alpha\rangle$, where $\langle f_\alpha\rangle$ is the mean transmitted Lyα flux from a background source.) It is necessary to appeal to models of structure formation to establish the relation. Within the context of the models, the intimate relation between the porosity and mean transmitted Lyα flux persists. Setting the source emissivity at a level to reionize the IGM at a given epoch fixes the post-reionization photoionization rate of the IGM, and so determines the mean transmitted Lyα flux just after reionization is completed (for any given cosmological model). Any reionization model must yield predictions consistent with these measured values. For instance, if the predicted photoionization rate were too high, the predicted mean transmitted Lyα flux levels would exceed those measured. Constraints on the reionization epoch and the post-reionization Lyα transmitted flux values must be satisfied simultaneously.

The results of a Λ cold dark matter (CDM) simulation run to investigate the effects of radiative transfer on the Lyα forest (Meiksin & White 2004) are used to quantify the emissivity required to maintain the ionization of the IGM over the redshift range $3 < z < 6$ at a level consistent with current measurements of the mean transmitted Lyα flux. The simulation was run using a pure particle mesh (PM) dark matter code, and it was assumed that the gas and dark matter have the same spatial distribution, which is found to give an adequate description of the structure of the IGM (Meiksin & White 2001; Meiksin & White 2003). The parameters used for the simulation are $\Omega_{\rm M} = 0.30$, $\Omega_\Lambda = 0.70$, $\Omega_{\rm b} = 0.045$, $h = H_0/100$ km s^{-1} Mpc^{-1} $= 0.70$, $\sigma_8 = 0.92$ and slope of the primordial density perturbation power spectrum $n = 0.95$. These values will be used throughout. The model is consistent with existing large-scale structure, Lyα forest flux distribution, cluster abundance and *WMAP* constraints (Meiksin & White 2004). The simulation was run using 512^3 particles and a 1024^3 force mesh, in a cubic box with (comoving) side length $25\,h^{-1}$ Mpc, adequate for obtaining converged estimates of the Lyα pixel flux distribution, the photon attenuation length and the required mean emissivity at the Lyman edge (Meiksin & White 2003, 2004). A description of the parallel PM code is given in Meiksin & White (2003).

In the limit of a medium with a photon attenuation length at the Lyman edge short compared with the horizon scale, the

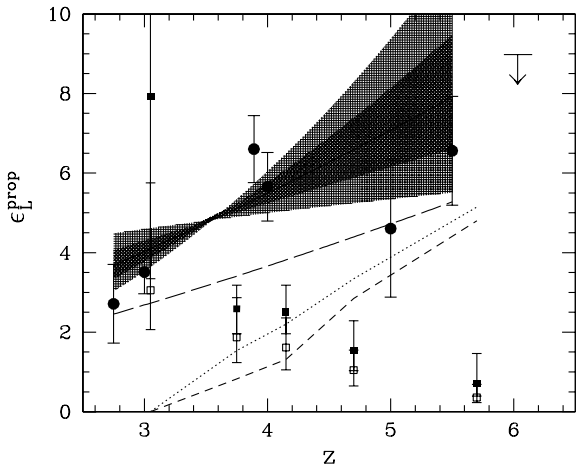

Figure 1. Metagalactic emissivity (proper) at the Lyman edge (in units of 10^{26} erg s^{-1} Hz^{-1} Mpc^{-3}), as a function of redshift. The data points (filled circles and upper limit point) are the emissivity values required in a ΛCDM model of the IGM to match the measured values of the transmitted Lyα flux through the IGM. The shaded regions show the 1σ (dark) and 2σ (light) ranges for a power-law fit (solid line) to the evolution in emissivity. The long-dashed line shows the required source emissivity, allowing for a boost in the total emissivity by 50 per cent from diffuse emission by the IGM. The squares show the estimated contribution from QSOs assuming either a PLE model (filled squares) or a PDE model (open squares) for the QSO luminosity function (see Section 3). The short-dashed and dotted lines show the deficit left by these (respective) QSO models from the required source emissivity (long-dashed line). The deficit must be made up by sources other than the QSOs. (A metagalactic spectral index of $\alpha_{MG} = 1$ has been assumed.)

photoionization rate Γ is related to the emissivity ϵ_L at the Lyman edge by

$$\Gamma \approx \frac{\epsilon_L \sigma_L}{h_P(3+\alpha_{MG})} r_0 \tag{1}$$

where σ_L is the photoelectric cross-section at the threshold frequency ν_L, h_P is the Planck constant, r_0 is the photon attenuation length, and the spectrum of the ionizing background is taken to vary at frequencies above the threshold as a power law $\epsilon_\nu = \epsilon_L(\nu/\nu_L)^{-\alpha_{MG}}$. Fig. 1 shows the resulting constraints on the metagalactic emissivity using the data in table 3 of Meiksin & White (2004). These data were based on ΛCDM model requirements to match the mean transmitted Lyα flux measurements from a variety of sources (see Meiksin & White 2004, appendix B). It is emphasized that the emissivity and attenuation length are solved for self-consistently in the model. Simulation-based estimates of the required emissivity in the literature usually rely on an assumed attenuation length (or, equivalently, effective optical depth at the Lyman edge) based on statistical representations of the measured Lyα forest absorber properties (such as H I column density and Doppler parameter). Uncertainties in the measured distributions, however, produce uncertainties in the derived attenuation length as large as a factor of 2–3 (Meiksin & Madau 1993; Meiksin & White 2003, 2004). This uncertainty translates directly into a comparable uncertainty in the required source emissivity. For this reason, it is crucial that the inferred emissivity from a simulation be consistent with the attenuation length that the same simulation predicts.

A least-squares fit to the emissivity values from Meiksin & White (2004) over $2.75 \leqslant z \leqslant 5.5$ gives the comoving emissivity

$$\epsilon_L \approx A_{MG}\left(\frac{3+\alpha_{MG}}{3}\right)(1+z)^\gamma\, h\,\mathrm{erg\,s^{-1}\,Hz^{-1}\,Mpc^{-3}}, \tag{2}$$

where $A_{MG} = 8.4^{+(9.7,31)}_{-(4.5,6.5)} \times 10^{25}$ and $\gamma = -1.6 - 0.6\log(A_{MG}/8.4 \times 10^{25})$. The errors on A_{MG} are the 1σ and 2σ limits. (The fit is acceptable at the 13 per cent confidence level according to the χ^2 test.) The upper limit at $z \approx 6$ is consistent with this fit, as shown in Fig. 1.

The total rate at which ionizing photons are generated by sources, per unit comoving volume, is related to the total (comoving) emissivity of the sources, ϵ_L^S, by

$$\dot{N}_S = \int_{\nu_L}^{\infty} \frac{\epsilon_L^S}{h_P}\left(\frac{\nu}{\nu_L}\right)^{-\alpha_S}\frac{d\nu}{\nu} \approx \frac{\epsilon_L}{h_P\alpha_S}$$
$$= A_S\left(\frac{3+\alpha_{MG}}{3\alpha_S}\right)(1+z)^\gamma\, h\,\mathrm{ph\,s^{-1}\,Mpc^{-3}}, \tag{3}$$

where $A_S = 1.3^{+(1.5,4.6)}_{-(0.7,1.0)} \times 10^{52}$, $\gamma = -1.6 - 0.6\log(A_S/1.3 \times 10^{52})$, α_S is the spectral index of the sources at frequencies above the photoelectric threshold ($L_\nu \propto \nu^{-\alpha_S}$), and the 1$\sigma$ and 2σ error ranges are provided. Here the approximation $\epsilon_L^S = \epsilon_L$ has been made at the Lyman edge. This neglects the additional diffuse emission arising from the IGM itself, which may enhance the total metagalactic photoionization rate over the direct rate from the sources by as much as an additional 50 per cent (Meiksin & Madau 1993; Haardt & Madau 1996). The source emissivity may thus be overestimated by this amount. In the spirit of providing an upper limit to the reionization redshift consistent with the measurements of the mean transmitted Lyα flux, the more generous estimate of the source emissivity is adopted here.

The evolution of $\alpha_S\dot{N}_S$ (comoving) is shown in Fig. 2 (solid curve), using the fit to ϵ_L above. Allowing for a boost of 50 per cent in the total emissivity by diffuse emission from the IGM reduces the required photon emission rate from sources by the same amount (long-dashed line). These rates are compared with the minimum rate $n_H t_H^{-1}$ at which ionized photons must be produced per unit volume over a Hubble time, $t_H \approx (2/3)(H_0\Omega_M^{1/2})^{-1}(1+z)^{-3/2}$

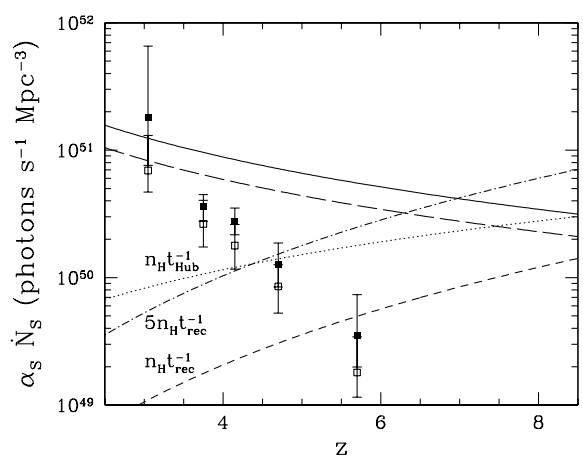

Figure 2. The evolution of the comoving production rate of ionizing photons. The required rate inferred from measurements of the mean transmitted Lyα flux is shown by the solid line. The long-dashed line shows the required source rate if diffuse recombination radiation from the IGM boosts the total metagalactic photoionization rate by 50 per cent. Also shown are the minimal ionizing photon production rate $n_H t_H^{-1}$ required to ionize the IGM over a Hubble time (dotted line), and the rate of recombinations for clumping factors of $C = 5$ (dot-dashed line) and $C = 1$ (short-dashed line). These latter three rates are shown for $\alpha_S = 1$. For alternative values, they should be multiplied by the factor α_S. Also shown are the predicted contributions from QSO sources, assuming either a PLE model (filled squares) or PDE model (open squares) for the QSO luminosity function (see Section 3).

(for $z \gg 1$), to reionize the IGM. The two rates cross at $z \approx 6$–13, depending on the value of the spectral index. This coincidence in values is expected immediately following reionization, and provides independent support (but not conclusive evidence) that reionization occurred during this time. The smooth evolution of the required emissivity for $z < 6$ (Fig. 1), reflected by the smoothness of the fit $\dot{N}_{\rm S}$ curve in Fig. 2, suggests extrapolation to higher redshifts is reasonable. Doing so results in too low an emission rate of ionizing photons compared with the required minimum number at redshifts above $z \approx 11$ for $\alpha_{\rm S} = 0.5$, and above $z \approx 8.5$ for $\alpha_{\rm S} > 1$.

For $z < 15$, the full recombination time at the average density of the IGM exceeds the Hubble time, and the effect of recombinations will be small. Because the IGM is non-uniform, however, the reionization rate will be much higher in clumpy regions, while smaller in underdense regions. The volume-averaged recombination time will then be reduced by the 'clumping factor' $\mathcal{C} = \langle n_{\rm H}^2 \rangle / \langle n_{\rm H} \rangle^2$ (Shapiro & Giroux 1987; Meiksin & Madau 1993). Clumping will slow down the growth of ionization regions due to recombinations within them, requiring the recombined gas to be reionized again and so drawing on the overall ionizing photon budget. The value of the clumping factor is unknown. Numerical simulations prior to reionization suggest values of $\mathcal{C} \sim 30$ at $z = 8$–10 and $\mathcal{C} > 100$ by $z = 6$ (Springel & Hernquist 2003). It is unclear, however, that these are the appropriate values to use. Once the gas is ionized, simulations suggest a clumping factor of $\mathcal{C} \approx 3$–4 (at $z = 3$; Sokasian, Abel & Hernquist 2003). Of particular relevance is the number of photons per atom required to photoionize a clump (or minihalo) of gas, which requires a detailed numerical computation beyond the scope of current full cosmological simulations. The computations of Shapiro, Iliev & Raga (2004) and Iliev, Shapiro & Raga (2004) have begun to examine this problem. They find an average of two to five ionizing photons are required per hydrogen atom, depending on redshift, with fewer photons required the harder the spectrum and the fainter the mean flux incident on a minihalo.

Fig. 2 shows the required minimum photon production rate, $\mathcal{C} n_{\rm H} t_{\rm rec}^{-1}$, assuming the IGM is recombination-dominated, with clumping factors of $\mathcal{C} = 1$ and 5. Here, $t_{\rm rec}$ is the full hydrogen recombination time at the average density of the IGM, $t_{\rm rec} = 1/n_{\rm e}\alpha_{\rm B}(T)$, where $n_{\rm e}$ is the average electron number density in the IGM and $\alpha_{\rm B}(T)$ is the radiative recombination rate to the excited levels of H I. (This will be taken at the reference temperature of $T = 2 \times 10^4$ K for numerical values below.) For $\mathcal{C} = 5$, the recombination time is shorter than the Hubble time for $z > 4.4$, and nearly half the Hubble time at $z = 7$. For this case, recombinations will play an important role in the reionization of the IGM. On the other hand, for $\mathcal{C} = 3$ the recombination time is shorter than the Hubble time only for $z > 6.6$. The role recombinations played in the reionization of hydrogen in the IGM is therefore unclear.

Madau, Haardt & Rees (1999) considered an IGM that was recombination-dominated, with clumping factors of up to $\mathcal{C} = 30$. Yan & Windhorst (2004) based their assessment for whether a class of objects could reionize the IGM on this value. Such a large value, however, is inconsistent with the estimated values of $\dot{N}_{\rm S}$ from measurements of the mean transmitted Lyα flux in Fig. 2. Scaling the curve for $n_{\rm H} t_{\rm rec}^{-1} (\mathcal{C} = 1)$ up by a factor of 30 boosts the volume-averaged recombination rate to a value that exceeds the estimated upper limit on the source rate by a factor of 3 at $z = 6$, even allowing for a hard source spectrum of $\alpha_{\rm S} = 0.5$. (The discrepancy is even larger for softer sources.) This suggests that $\mathcal{C} < 10$ is a more realistic upper limit. (The limit is even more restrictive for softer sources.)

The epoch of reionization is quantified using the ionization porosity parameter. The H II ionization porosity, $Q_{\rm H\,II}$, evolves according to (Meiksin & Madau 1993; Madau et al. 1999)

$$\frac{{\rm d}Q_{\rm H\,II}}{{\rm d}z} = \left[\frac{\dot{N}_{\rm S}(z)}{n_{\rm H}(0)} - \frac{\mathcal{C}(z) Q_{\rm H\,II}}{t_{\rm rec}(z)} \right] \frac{{\rm d}t}{{\rm d}z}, \tag{4}$$

where $n_{\rm H}(0)$ is the comoving number density of hydrogen atoms. If the recombination time is long compared with the Hubble time, the recombination term in equation (4) may be neglected. Without a detailed knowledge of the clumping factor and its evolution, including feedback effects, a computation still beyond the capability of current large-scale gravity+hydrodynamics+radiative transfer simulations, the role of recombinations cannot be determined in detail. Here, $\mathcal{C} = 1$ will generally be assumed, so that only an upper redshift limit to the epoch of reionization is estimated. Significant clumping will serve to further delay complete reionization ($Q > 1$). The effect of clumping on slowing down the reionization process may be particularly important for He II reionization (Madau & Meiksin 1994).

Using equation (3) in equation (4) and integrating gives, in the absence of recombinations,

$$Q_{\rm H\,II}(z) = \frac{\dot{N}_{\rm S}(z) t_{\rm H}(z)}{n_{\rm H}(0)[1 - (2/3)\gamma]} \left[1 - \left(\frac{1+z}{1+z_{\rm i}} \right)^{(3/2)-\gamma} \right], \tag{5}$$

where $z_{\rm i}$ is the turn-on redshift of the sources. In the presence of recombinations (and for a redshift-independent clumping factor and IGM temperature), the solution is

$$Q_{\rm H\,II}(z) = \frac{\dot{N}_{\rm S}(z) t_{\rm H}(z)}{n_{\rm H}(0)} \tau_{\rm H}^{1-(2/3)\gamma}(z) {\rm e}^{\tau_{\rm H}(z)} \times \int_{\tau_{\rm H}(z)}^{\tau_{\rm H}(z_{\rm i})} {\rm d}\tau \, {\rm e}^{-\tau} \frac{1}{\tau^{2-(2/3)\gamma}}, \tag{6}$$

where $\tau_{\rm H}(z)$ is the Hubble time measured in units of the recombination time (including the clumping factor). $\tau_{\rm H}(z) = \mathcal{C} t_{\rm H}(z)/t_{\rm rec}(z) \simeq 0.016\mathcal{C}(1+z)^{3/2}$, and is equivalent to the mean number of recombinations in the IGM over a Hubble time. Provided 2–$2\gamma/3$ is not a positive integer, the integral may be expressed in terms of the incomplete gamma function $\Gamma(a, x) = \int_x^\infty {\rm d}t {\rm e}^{-t} t^{a-1}$. For $\gamma > -3$ [$\gamma \neq 0, \pm (3/2)$], and $z_{\rm i} \to \infty$,

$$Q_{\rm H\,II}(z) = \frac{\dot{N}_{\rm S}(z) t_{\rm H}(z)}{n_{\rm H}(0)[1 - (2/3)\gamma]} \left\{ 1 + \frac{3\tau_{\rm H}}{2\gamma} + \frac{3\tau_{\rm H}}{2\gamma[1 + (2/3)\gamma]} \times \left[\tau_{\rm H} - {\rm e}^{\tau_{\rm H}} \tau_{\rm H}^{-(2/3)\gamma} \Gamma\left(2 + \frac{2}{3}\gamma, \tau_{\rm H} \right) \right] \right\}. \tag{7}$$

When $2 - 2\gamma/3$ is a positive integer, the solution is

$$Q_{\rm H\,II}(z) = \frac{\dot{N}_{\rm S}(z) t_{\rm H}(z)}{n_{\rm H}(0)} {\rm e}^{\tau_{\rm H}(z)} E_{2-2\gamma/3}(\tau_{\rm H}), \tag{8}$$

where $E_n(x) = \int_1^\infty {\rm d}t {\rm e}^{-xt} t^{-n}$ is an exponential integral. For the special case of a constant comoving source emissivity ($\gamma = 0$), equation (8) recovers equation (22) of Madau et al. (1999). In the absence of recombinations, $\tau_{\rm H} \to 0$ and equation (5), for $z_{\rm i} \gg z$, is recovered. The asymptotic series representation of equation (6) for $\tau_{\rm H}(z) > 1$ (and $z_{\rm i} \gg z$) is

$$Q_{\rm H\,II}(z) \sim \mathcal{C}^{-1} \frac{\dot{N}_{\rm S}(z) t_{\rm rec}(z)}{n_{\rm H}(0)} \sum_{n=0}^{\infty} (-1)^n \frac{[2 - (2/3)\gamma]_n}{\tau_{\rm H}^n(z)}, \tag{9}$$

where $[a]_n \equiv \Gamma(a+n)/\Gamma(a) = a(a+1)\ldots(a+n-1)$. The first term ($n = 0$) is the approximation equation (23) of Madau et al. (1999), valid in the limit $\tau_{\rm H} \gg 1$. Similar expressions follow for

the He III porosity, $Q_{\rm He\,III}$, if $n_{\rm H}(0)$ is replaced by $n_{\rm He}(0)$, and $\dot{N}_{\rm S}$ and $t_{\rm rec}$ refer to the production rate of He II ionizing photons and the recombination rate to He II, respectively.

The key assumption is now made that the sources of ionization at $z < 6$ are drawn from the same population as that responsible for the reionization of the IGM. Under this assumption, the porosity $Q_{\rm H\,II}$ may be estimated from equation (3) for $\dot{N}_{\rm S}$, extrapolated to $z > 6$. This seems a reasonable assumption to make, as the inferred emissivity in Fig. 1 evolves smoothly over $3 < z < 6$ in spite of the strong downward turn in the measured mean Lyα transmitted flux at $z > 5$. To make the estimate, however, it is first necessary to be more specific concerning the factor $(3 + \alpha_{\rm MG})/\,\alpha_{\rm S}$ in equation (3).

When QSOs are the sources of reionization, the UV metagalactic background hardens at high redshifts beyond the input spectral shape because high-energy photons travel further than photons near the photoelectric threshold before being absorbed by the IGM. Here $\alpha_{\rm MG} \approx 1$ is adopted based on the results of Haardt & Madau (1996), including re-emission from the IGM. Although the spectral index should be computed self-consistently from the models considered here, the uncertainty in this value introduces only a small uncertainty in the total photoionization rate. It is noted that $\alpha_{\rm MG}$ may be double this if young star-forming galaxies with a Salpeter initial mass function (IMF) and solar abundances are the sources of ionization, as they have much softer spectra than QSOs.

More critical is the value of $\alpha_{\rm S}$, which sets the total number of ionizing photons released by a source in terms of its specific luminosity at the photoelectric threshold. The value for a QSO is uncertain. Results based on *Hubble Space Telescope* (*HST*) data give $\alpha_{\rm S} = 1.76 \pm 0.12$ (Telfer et al. 2002). More recent results based on *Far-Ultraviolet Spectroscopic Explorer* (*FUSE*) data give instead $\alpha_{\rm S} = 0.56^{+0.28}_{-0.38}$ (Scott et al. 2004). The origin of the discrepancy is unclear. The *FUSE* sample is at lower redshift and the sources have lower luminosities, so the difference may reflect a redshift and/or luminosity dependence. The corrections due to intervening absorption by the IGM, however, are significantly smaller for the *FUSE* sample than for the *HST* sample, so that the *FUSE* result may be a more reliable estimate of the spectral index of the mean QSO spectrum.

For a non-power-law source such as a galaxy, an effective spectral index $\alpha_{\rm S}^{\rm eff}$ may be defined according to

$$\alpha_{\rm S}^{\rm eff} \equiv \frac{L_{\rm L}}{h_{\rm P}\dot{N}_{\rm S}}, \tag{10}$$

where $L_{\rm L}$ is the specific luminosity of the source at the Lyman edge. The effective spectral index is shown in Fig. 3 for starburst galaxies for both a fading sudden burst and a constant star formation rate, using Starburst99 (Leitherer et al. 1999), updated with the stellar spectra of young stars from Smith, Norris & Crowther (2002). A Salpeter IMF is assumed, and solar abundances. While the spectral index of a fading burst quickly climbs to values exceeding 10 (after 4×10^7 yr), in the case of continuous star formation, the effective spectral index instead reaches an asymptotic value of $\alpha_{\rm S}^{\rm eff} \approx 2.3$. For continuous star formation with Population (Pop) II abundances ($Z = 0.05\,{\rm Z}_\odot$), the asymptotic value of the effective spectral index is $\alpha_{\rm S}^{\rm eff} \approx 1.8$. This is comparable to the effective spectral index for a blackbody spectrum with a temperature of $T = 5 \times 10^4$ K, used by Stiavelli et al. (2004) to represent a galaxy dominated by Pop II stars. They represented Pop III stars (essentially no metals) by a blackbody with $T = 10^5$ K, for which $\alpha_{\rm eff} \approx 0.5$.

One implication of the rapidly climbing value of the effective index in the case of a fading burst is that the ionizing efficiency of the galaxy declines rapidly. If star formation in a bursting galaxy were episodic, with a duty cycle of 50 per cent and a cycle time longer than 2×10^7 yr, the net contribution of the population to the number of ionizing photons is reduced by 50 per cent from the case of continuous star formation. This complication is not considered here.

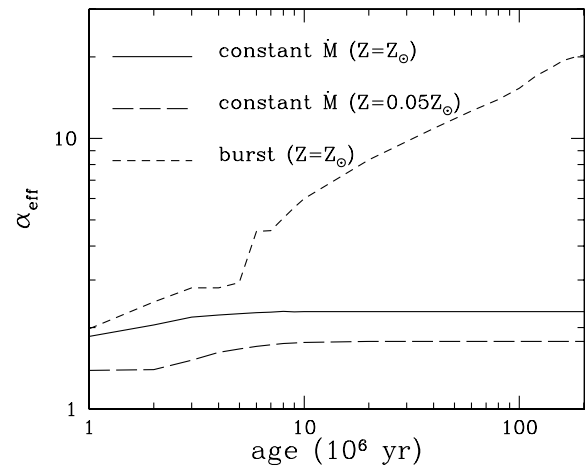

Figure 3. The evolution of the effective spectral index $\alpha_{\rm eff}$ (see text) for a starburst of solar metallicity with a constant star formation rate (solid line) or a sudden burst of star formation (short-dashed line). Also shown is the evolution assuming a metallicity $Z = 0.05\,{\rm Z}_\odot$ and a constant star formation rate (long-dashed line).

Because of the uncertainty in the spectral index, three values are considered here for the spectrum near the hydrogen Lyman edge: a hard spectrum with $\alpha_{\rm S} = 0.5$ corresponding to a hard spectrum QSO or Pop III dominated galaxy, $\alpha_{\rm S} = 1.8$ corresponding to a soft spectrum QSO or Pop II dominated galaxy, and $\alpha_{\rm S} = 2.3$ corresponding to a starburst with solar metallicity.

In Fig. 4, the evolution of the porosity parameter is shown for hard and soft source spectra, and presuming the sources turned on at very high redshifts ($z_{\rm i} \to \infty$). Within the observational uncertainties, the

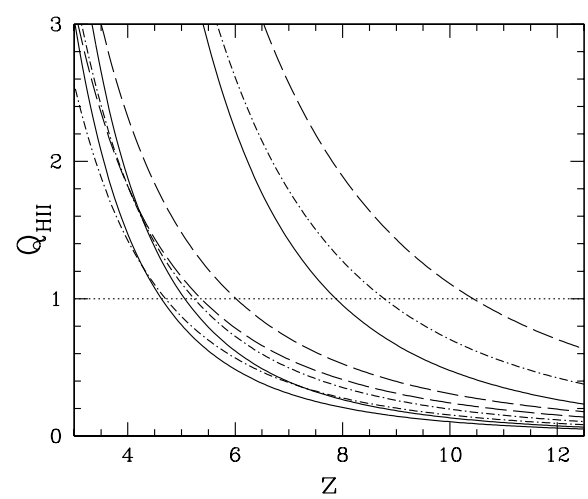

Figure 4. The evolution of the ionization porosity parameter $Q_{\rm H\,II}$ for hydrogen reionization based on the metagalactic ionizing emissivity estimated from measurements of the mean transmitted Lyα flux. The solid lines show the expected evolution assuming a source spectral index of, from right to left, $\alpha_{\rm S} = 0.5$, 1.8 and 2.3. The dashed lines show the corresponding evolution for the 2σ upper limits on the estimated emissivity. A clumping factor of $C = 1$ is assumed for the above curves. Also shown are the corresponding 2σ upper limits for $C = 3$ (dot-dashed curves).

earliest epoch of reionization for hard sources ($\alpha_S = 0.5$), such as QSOs or Pop III stars, is $z_{ri} < 11$ (2σ upper limit). For soft sources ($\alpha = 2.3$) such as starburst galaxies, $z_{ri} < 5.5(2\sigma)$. Intermediate sources with $\alpha_S = 1.8$, such as soft spectrum QSOs or Pop II stars, may ionize the IGM only at $z_{ri} < 6(2\sigma)$. While hard spectrum sources are well able to photoionize the IGM prior to $z = 6$, reionization by softer spectrum sources such as soft spectrum QSOs or starbursts (unless dominated by Pop III stars) is more problematic.

The estimate provided neglects several factors which will further delay the epoch of reionization. In particular, reionization will be delayed if the galaxies turn on later (finite z_i) or if the clumping factor is large. The source emissivity at the Lyman edge was also equated to the total required metagalactic emissivity, neglecting the additional ionizing photons arising from the IGM itself. Taking this into account could reduce the estimated source emissivity by as much as 50 per cent (see above), and so lower the overall predicted rate of ionizing photons by this same amount, delaying reionization even further.

The 2σ upper limits on z_{ri} for soft and intermediate spectrum sources lie well below the *WMAP* 1σ and 2σ lower limits of $z_{ri} > 14$ and 11, respectively (Kogut et al. 2003). The prediction for $\alpha_S = 0.5$ is just barely compatible with the *WMAP* 3σ lower limit of $z_{ri} > 8$. It should be noted that Spergel et al. (2003) find the broader reionization redshift constraint $z_{ri} = 17 \pm 4$ after combining with other data sets. This still disagrees with the intermediate and soft spectra predictions at the 3σ level, but agrees with the prediction for $\alpha_S = 0.5$ at the $\gtrsim 2\sigma$ level. The results suggest that if the ionization of the IGM at $z < 6$ is provided by soft or intermediate spectrum sources, such as starbursts dominated by Pop II stars, they are not members of the same population that reionized the IGM, unless the population as a whole was a much more prodigious source of ionizing radiation at earlier times ($z > 6$). Sources with a hard spectrum ($\alpha_S = 0.5$), such as Pop III stars or hard spectrum QSOs, are preferred.

Gnedin (2004) similarly found that for his numerical simulations to match the mean Lyα transmitted flux measurements at $z < 6$, the epoch of reionization (defined differently in terms of the volume-weighted mean IGM ionization fraction) must have occurred at $z_{ri} \approx 6$. The results here are much more generic and not dependent on any particular prescription for cosmic star formation, as in Gnedin's simulations. Gnedin finds that the predicted reionization epoch is determined by a single parameter, the UV emissivity parameter. Setting this parameter to match the mean Lyα optical depth measurements at $z < 6$ requires reionization to have occurred at $z \approx 6$. The one-parameter nature of Gnedin's models appears to be a consequence of the assumed nature of the ionizing sources formed as part of the growth of cosmological structure. The analysis here shows that reionization is determined by three parameters, expressed by A_S, γ and α_S. The constraint on the reionization epoch found here follows only from the assumption that the population of sources which dominates the ionization background at $z < 6$ is the same population that reionized the IGM. The Lyα optical depth measurements at $z < 6$ are used to determine A_S and γ, leaving α_S free to set the reionization epoch.

In the next section, the QSO contributions to the metagalactic ionizing background and to the total budget of ionizing photons are estimated based on the QSO counts in recent surveys.

3 CONTRIBUTION OF QSOS TO THE IONIZING PHOTON BUDGET

The QSO luminosity function for $z < 2$ is well established. The QSO population detected by the Two-Degree Field (2dF; Croom et al. 2004) is well fit at least up to $z < 2.1$ by a pure luminosity evolution (PLE) model with the double power-law form (Boyle, Shanks & Peterson 1988)

$$\phi(M, z) = \frac{\phi^*}{10^{0.4(1-\beta_1)[M-M^*(z)]} + 10^{0.4(1-\beta_2)[M-M^*(z)]}}, \tag{11}$$

where ϕ^* is a fixed spatial normalization factor and $M^*(z)$ is an evolving break magnitude.

For $z > 3$, the model parameters are less clear because surveys have generally only detected QSOs brighter than the break magnitude. The most complete of these surveys is the QSO survey at $z > 3.6$ carried out as part of the SDSS (Fan et al. 2001, 2004). An exception is the survey of Steidel et al. (2002), in which active galactic nuclei (AGNs) dimmer than the break magnitude were discovered as part of a Lyman break galaxy survey at $z \simeq 3$.

In this section, the contribution of QSOs to the ionization rate of the IGM over the redshift range $3 < z < 6$ is estimated based on the findings of Fan et al. (2001, 2004), and as also constrained by the low-luminosity AGN data of Steidel et al. (2002). This is done for both PLE and pure density evolution (PDE) models, for which M^* is fixed in equation (11) but ϕ^* is allowed to evolve, as an attempt to bracket the uncertainty in the evolution.

The luminosity function parameters in equation (11) are estimated using a maximum-likelihood procedure. For any given set of parameters, the predicted numbers of QSOs in a set of absolute magnitude and redshift bins are given by $\mu_{ij} = \phi(M_i, z_j)\, V^j_{\rm eff}(M_i)\, {\rm d}M_i$ for magnitude bin i and redshift bin j, where $V^j_{\rm eff}(M_i)$ is the effective survey volume probed for QSOs with absolute magnitudes within the bin centred on M_i and redshifts in the bin centred on z_j. All absolute magnitudes are taken at the published values at 1450 Å, and a flat cosmological model with $\Omega_M = 0.35$ and $h = 0.65$ is used for the fits for consistency with Fan et al. (2001, 2004). [The magnitudes and effective volumes from Steidel et al. (2002) and Hunt et al. (2004) are transformed to this cosmology.] The likelihood of the model compared with the actual numbers N_{ij} of QSOs detected is then given by

$$\mathcal{L} = \prod_{ij} \frac{{\rm e}^{-\mu_{ij}}\, \mu_{ij}^{N_{ij}}}{N_{ij}!}, \tag{12}$$

where Poisson probabilities are required because of the low number of objects per bin.

The likelihoods are computed on a multidimensional (linear) grid in β_1, β_2, ϕ^* and M^*. (This corresponds to assuming a uniform prior for each parameter.) Here β_1 is assumed to correspond to the low-luminosity end of the QSO luminosity function and β_2 to the high end. For the PLE models, the same ϕ^* is used for all redshift bins, but M^* is stepped over separately for each redshift bin. Conversely, for the PDE models the same M^* is adopted for all redshifts, but ϕ^* is allowed to vary for each redshift bin. Over 10^9 models are constructed for the PLE case and 10^8 for the PDE case.

The parameter values for the maximum-likelihood fit of the PLE model are $\beta_1 = 1.24$, $\beta_2 = 2.70$ and $\phi^* = 3.25 \times 10^{-7}$ Mpc^{-3} mag^{-1} (comoving). The values for $M^*(z)$ are given in Table 1. The parameters for the PDE fit are $\beta_1 = 1.12$, $\beta_2 = 2.90$ and $M^* = -25.60$. The values for $\phi^*(z)$ are given in Table 1.

The goodness-of-fit of the maximum-likelihood models is estimated using the χ^2 test for the predicted versus detected number of QSO sources for all 15 available bins in redshift and magnitude (one of which is a cumulative magnitude bin), and allowing for seven degrees of freedom (after accounting for the eight model parameters). For the PLE model, $\chi^2 = 6.8$ and $p(\chi^2 > 6.8) = 0.45$. For the PDE model, $\chi^2 = 8.5$ and $p(\chi^2 > 8.5) = 0.29$. Both fits are acceptable.

Table 1. Redshift-dependent parameters for the QSO luminosity function maximum-likelihood estimates. The two leftmost columns of values provide the parameters for the maximum-likelihood model for the PLE and PDE cases, as indicated. The two rightmost columns provide the marginalized parameter values, and their 1σ errors, for the PLE and PDE cases. All magnitudes are at 1450 Å. The normalization $\phi^*(z)$ is in units of 10^{-7} Mpc^{-3} mag^{-1} (comoving). The fits are performed for a flat cosmology with $\Omega_{\rm M} = 0.35$ and $h = 0.65$.

Redshift	PLE: M^* (ML)	PDE: ϕ^* (ML)	PLE: M^* (margin.)	PDE: ϕ^* (margin.)
$z = 3$	-27.2	6.0	$-26.2^{+1.6}_{-1.8}$	$6.1^{+3.2}_{-2.7}$
$3.6 < z < 3.9$	-24.7	1.2	$-24.0^{+0.7}_{-0.8}$	$1.1^{+5.0}_{-0.3}$
$3.9 < z < 4.4$	-24.4	0.74	$-23.6^{+0.7}_{-0.9}$	$1.3^{+2.4}_{-0.8}$
$4.4 < z < 5.0$	-23.8	0.27	$-22.9^{+0.9}_{-1.0}$	$0.43^{+1.23}_{-0.16}$
$z > 5.7$	-22.9	0.08	$-21.4^{+0.9}_{-1.4}$	$0.09^{+0.34}_{-0.03}$

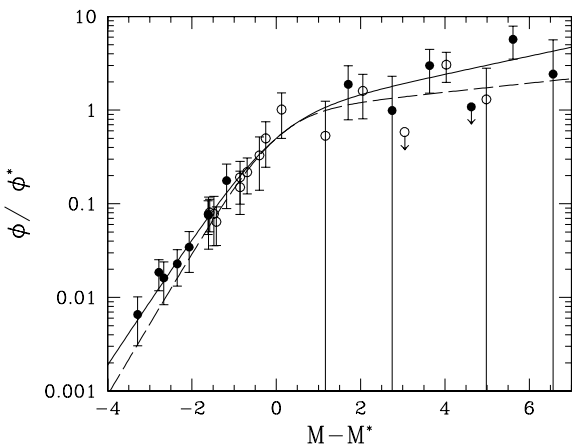

Figure 5. The maximum-likelihood fits to the QSO luminosity function for the PLE (solid line) and PDE (dashed line) models. The measured luminosity function values have been renormalized by ϕ^* and M^* for the PLE model (solid points) and the PDE model (open points). The error bars are Poisson, based on the maximum-likelihood fits.

The maximum-likelihood fits are shown in Fig. 5, along with the (renormalized) measured values of the luminosity function and their Poisson errors, which are based on the fit models.

The maximum-likelihood parameter values, each marginalized over all other model parameters, and with their equivalent 1σ error ranges, are, for the PLE model, $\beta_1 = 1.24^{+0.12}_{-0.12}$, $\beta_2 = 2.2^{+0.3}_{-0.2}$ and $\phi^* = 2.8^{+3.0}_{-1.8} \times 10^{-7}$ Mpc^{-3} mag^{-1} (comoving). The marginalized values for $M^*(z)$ are given in Table 1. For the PDE model, the values are $\beta_1 < 1.1$, $\beta_2 = 2.8^{+0.4}_{-0.5}$ and $M^* = -25.4^{+0.8}_{-0.6}$. The marginalized values for $\phi^*(z)$ are given in Table 1. The upper limit on β_1 results because smaller values were not considered.

The results on β_1 and β_2 agree well with the findings of Hunt et al. (2004) and Fan et al. (2001, 2004). Hunt et al. obtain $\beta_1 = 1.24 \pm 0.07$ at $z \approx 3$. For $3.6 < z < 5.0$, Fan et al. (2001) obtain $\beta_2 = 2.58 \pm 0.23$, while at $z > 5.7$, Fan et al. (2004) obtain $\beta_2 = 3.2^{+0.8}_{-0.7}$. (All errors are 1σ.) The PLE model estimate of β_2 found here disagrees with the $z > 5.7$ value at the 98 per cent confidence level, suggesting some departure from PLE, but the evidence is not conclusive.

For $0.4 < z < 2.1$, Croom et al. (2004) obtain $\beta_1 = 1.0 \pm 0.1$ and $\beta_2 = 3.25 \pm 0.05$ for a PLE model with a break magnitude varying as an exponential of look-back time. (The results for a quadratic redshift dependence are similar.) There is again only marginal agreement with β_2. There is greater disagreement over the normalizations. Croom et al. report $\phi^*{}_B = 1.8 \times 10^{-6}$ Mpc^{-3} mag^{-1} for the B-band luminosity function. This value (including an adjustment for the small differences in assumed cosmologies) exceeds the values for ϕ^* found for the PLE model here by a factor of $\sim$6, significant at the 5σ level. However, it should be noted that the normalizations may only be fairly compared provided the ratio of B-band to 1450-Å luminosities was itself luminosity-independent. A steep correlation between spectral slope and B-band magnitude for QSOs could in principle account for the difference in normalizations. Currently there is no evidence for a correlation sufficiently steep. It thus appears there has been substantial density evolution from $z \approx 3$ to $z \lesssim 2$.

For each set of luminosity function parameters, the integrated emissivity values at the Lyman edge for the PLE and PDE models are computed from $\epsilon_{\rm L} = \int \phi(M) L_{\rm L}(M)\,{\rm d}M$, where $L_{\rm L}(M)$ is the frequency-specific QSO luminosity at the Lyman edge corresponding to a given absolute magnitude M at 1450 Å, and assuming $L_\nu \propto \nu^{-0.5}$ between 1450 Å and the Lyman break. The resulting marginalized maximum-likelihood emissivity values (transformed to a flat cosmology with $\Omega_{\rm M} = 0.3$ and $h = 0.7$) are shown in Fig. 1 for the PLE model (filled squares) and the PDE model (open squares), along with their 1σ errors. The corresponding marginalized estimates for the ionizing photon production rates ($\alpha_{\rm S}\dot{N}_{\rm S} = \epsilon_{\rm L}/h_{\rm P}$) are shown in Fig. 2.

As shown in Fig. 1, for $z \lesssim 3$, all of the required emissivity is accounted for by QSOs. At $z > 4.5$, however, it appears QSOs are inadequate sources of ionizing photons, falling short by factors of at least 2–3, depending on the amount of diffuse radiation. Additional sources are required to make up the difference. Within the uncertainties, the QSO deficit could be as high as a factor of 30 by $z \approx 5.5$.

The deficit of ionizing photons from QSOs is apparent in Fig. 2 as well. For $z > 5.5$, QSOs fall short of the ionizing photon production rate required for consistency with the measured mean transmitted Lyα flux values (solid and long-dashed lines) by at least a factor of 5. The 1σ upper limit on the QSO rate for the PLE case at $z = 5.7$, however, comes close to the bare minimum rate required to ionize all the baryons over a Hubble time, assuming a hard ($\alpha_{\rm S} = 0.5$) source spectrum. While QSOs could not have ionized the IGM acting alone, they still may have provided a non-negligible contribution of ionizing photons during the reionization epoch.

4 DISCUSSION

Any assessment of the contribution of a population of sources to the ionizing photon budget of the IGM depends on several unknown factors. Principal among these are the spectral index $\alpha_{\rm MG}$ of the UV background, the contribution of Lyman limit systems to the attenuation length at the Lyman edge, the amount of diffuse emission arising from the IGM, and the clumping factor of the IGM. In this section, the role each of these factors may play is discussed.

4.1 Sources of ionization of the intergalactic medium

Comparison of the results of numerical simulations of the IGM with measurements of the mean transmitted Lyα flux through the IGM constrains the ionization rate per hydrogen atom (or per He II atom

for He II Lyα). This is related to the emissivity by a factor of $3 + \alpha_{\rm MG}$ (equation 1). The results of Haardt & Madau (1996) suggest $\alpha_{\rm MG} \approx 1$ shortward of the hydrogen Lyman edge, but this is due in large part to the contribution of redshifted He II recombinant Lyα and two-photon emission. If most of the helium is still in the form of He II, the spectrum may be harder, possibly even with $\alpha_{\rm MG} < 0$. In this case, the required emissivity could be reduced by as much as a factor of 2, bringing it much more in line with the expected QSO emissivity.

Meiksin & White (2004) appealed to the uncertainty in the values of $\alpha_{\rm MG}$ and the QSO break magnitude (M^*), adjusting these to allow QSOs to dominate the UV ionizing background at high redshifts. The assessment here does not rule out this possibility. However, because QSOs acting alone would not be able to photoionize the IGM by $z = 6$, it seems likely the UV background will have a large contribution from other sources, such as galaxies. (Although this need not be the case if reionization occurred at $z \gg 6$.) While this would suppress the correlations in the UV background predicted by Meiksin & White (2004), and the predicted enhancement of the Lyα flux correlations in the IGM, the suppression may be by no more than a factor of a few. In this case, the contribution of QSOs to the UV background may still be detectable through the fluctuations they induce.

The ionization rate predicted for a given emissivity also depends on the attenuation length r_0. The values used here were computed self-consistently from numerical simulations, but these simulations do not include radiative transfer and therefore are not able to accurately predict the contribution of Lyman limit systems. These are accounted for by direct observations for $z \lesssim 4$, but their abundances are not known at higher redshifts. The attenuation length may be $\sim$30 per cent smaller than the values adopted here, resulting in an increase in the required emissivity by the same amount. This would result in a further reduction in the fraction of the ionizing background provided by QSOs. A similar effect may arise if the emissivity of the sources rapidly declines with redshift, in which case the effective path length over which the emissivity is integrated to obtain the ionization rate may be comparable or even shorter than the attenuation length, which also would permit a higher emissivity at a given redshift (Meiksin & White 2003).

If a substantial fraction of the required emissivity ($\sim$50 per cent) arises from diffuse recombination radiation within the IGM, then the requirements for the source emissivity are reduced by the same amount. The fractional contribution of QSOs to the required source emissivity would then increase. The difference between the required source emissivity and the predicted emissivity from QSOs for the PLE and PDE cases is shown in Fig. 1, assuming the diffuse emission from the IGM contributes at 50 per cent the source level. QSOs provide more than half of the required source emissivity up to $z \lesssim 4.5$, and less beyond. The curves showing the difference describe the emissivity contributed by other sources, presumably galaxies.

4.2 Sources of reionization of the IGM

The reionization redshifts estimated here rest on the assumption that the population of sources which dominates the UV ionizing background at $z < 6$ is the same population responsible for reionization. This is to be expected if reionization occurred near $z \approx 6$, as argued by Becker et al. (2001), but there is nothing to preclude a much earlier reionization epoch ($z \gg 6$), or even a series of several reionization epochs followed by recombination. On the other hand, the mild evolution in the emissivity required to match the measured Lyα optical depths at $z < 6$ suggests there was no abrupt change in the nature of the sources dominating the UV background at $z \approx 6$, so that the emissivity may be reasonably extrapolated to higher redshifts.

The estimated reionization redshifts were all upper limits, neglecting any excess clumping ($\mathcal{C} > 1$) in the reionized gas. If the clumping factor is moderate ($\mathcal{C} \approx 2$–4), as suggested by recent numerical computations, then recombinations may not delay the reionization epoch very significantly. Fig. 4 shows the evolution of the porosity for $\mathcal{C} = 3$, assuming the 2σ upper limit on the emissivity. For $\alpha_{\rm S} = 2.3$ and 1.8, reionization now occurs well below $z = 6$. For $\alpha_{\rm S} = 0.5$, the 2σ upper limit of $z_{\rm ri} < 9$ is still compatible with the SDSS data (Becker et al. 2001), but barely with the *WMAP* 2σ lower limit of Spergel et al. (2003). The expected value of the reionization redshift for $\alpha_{\rm S} = 0.5$ and $\mathcal{C} = 3$ is $z_{\rm ri} = 7.1$, which is compatible with the *WMAP* 2.5σ lower limit of Spergel et al. For higher values of the clumping factor ($\mathcal{C} \approx 5$–10), reionization for the various spectral index cases considered here would occur substantially later.

If the reionization of minihalos doubles the reionization requirements (Iliev et al. 2004; Shapiro et al. 2004), then reionization for $\alpha_{\rm S} = 0.5$ is not expected to occur until $z_{\rm ri} \lesssim 6.2$. While this is consistent with the data of Becker et al. (2001), it is consistent with the *WMAP* lower limit on $z_{\rm ri}$ of Spergel et al. (2003) at only the 2.7σ level.

The estimates here assumed the sources turned on at infinitely high redshifts ($z_{\rm i} \gg z_{\rm ri}$). Reionization would occur later for finite values of $z_{\rm i}$.

The estimates for the reionization epoch were also based on the assumption that all the required emissivity was provided by the sources, while in fact diffuse emission from the IGM could contribute an additional $\sim$50 per cent. In this case, the expected source contribution would be reduced by 50 per cent, and $(1 + z_{\rm ri})$ reduced by about 15 per cent. For $\alpha_{\rm S} = 0.5$, this would push the 2σ upper limit on $z_{\rm ri}$ down from 11 to 9.

Reionization may have occurred earlier if the emissivity values used here are underestimated, as would be the case if the attenuation length of Lyman photons were shorter than the values used here (see Section 4.1). However, even a doubling of the emissivities would increase the expected values of $(1 + z_{\rm ri})$ by only about 20 per cent, or $z_{\rm ri} \approx 10$ for $\alpha_{\rm S} = 0.5$. For $\alpha_{\rm S} = 1.8$ and 2.3, the expected redshifts for the reionization epoch may be increased to $z_{\rm ri} \approx 6$, still too low to match the *WMAP* constraints, but consistent with the SDSS Lyα optical depth measurements. At much higher redshifts, the reionization epoch will be limited by the effect of recombinations even for small clumping factors.

In the scenarios above, the expected redshift of He II reionization is always above $z_{\rm ri} > 11$ for $\alpha_{\rm S} = 0.5$ (even for $\mathcal{C} = 3$), so that helium may have been ionized very early if the source spectra continue as hard power laws shortward of the He II Lyman edge. If the spectrum softens to $\alpha_{\rm S} > 1.8$ shortward of the He II Lyman edge, however, He II reionization would not have occurred prior to $z_{\rm ri} \lesssim 6$ (except if the total emissivity is doubled), but generally prior to $z_{\rm ri} \gtrsim 3$, unless He II ionizing photons are produced at a negligible rate, as is expected if the ionizing sources are star-forming galaxies (Smith et al. 2002).

QSOs are found to provide too few ionizing photons at $z > 5$ to reionize the hydrogen in the IGM. This may be quantified in terms of the ionization porosity. Because of the rapid decline in the photon production rate, equation (6) no longer applies. Instead, the decline in Fig. 2 is better modelled as an exponential in redshift: $\alpha_{\rm S}\dot{N}_{\rm S} = A_{\rm S}{\rm e}^{-\gamma z}$. The solution of equation (4) (with recombinations

neglected) is

$$Q_{\rm H\,II} = \frac{\dot{N}_{\rm S}(z)t_{\rm H}(z)}{n_{\rm H}(0)}\left[1 - 2y + 2{\rm e}^{y}y^{3/2}\Gamma\left(\frac{1}{2}, y\right)\right], \qquad (13)$$

where $y = \gamma(1+z)$, and $z_{\rm i} \rightarrow \infty$ is assumed. A similar expression follows for $Q_{\rm He\,III}$ on replacing $n_{\rm H}(0)$ by $n_{\rm He}(0)$ and $\dot{N}_{\rm S}$ by $\dot{N}_{\rm S}/4^{\alpha_{\rm S}}$.

Because of the broad, non-Gaussian probability distribution for $\dot{N}_{\rm S}$ from QSOs, the probability distribution for the reionization redshift is determined from a maximum-likelihood estimate using the probability distributions found in Section 3 for the QSO emissivity. For the PLE case, the parameters of the maximum-likelihood model are $A_{\rm S} = 5.7 \times 10^{52}$ ph s^{-1} and $\gamma = 1.26$. For the PDE case, $A_{\rm S} = 6.1 \times 10^{52}$ ph s^{-1} and $\gamma = 1.37$. The marginalized maximum-likelihood values for the parameters are, for the PLE case, $A_{\rm S} = 5.2^{+6.2}_{-3.1} \times 10^{52}$ ph s^{-1} and $\gamma = 1.65 + 0.10\log(A_{\rm S}/5.2 \times 10^{52})$. For the PDE case, $A_{\rm S} = 5.7^{+7.4}_{-3.3} \times 10^{52}$ ph s^{-1} and $\gamma = 1.74 + 0.12\log(A_{\rm S}/5.7 \times 10^{52})$.

The resulting probability distributions for the reionization redshifts, for both H II and He III, are shown for the PLE and PDE cases in Fig. 6. Results for both $\alpha_{\rm S} = 1.8$ and 0.5 are provided. For $\alpha_{\rm S} = 1.8$, the hydrogen and helium reionization redshifts nearly coincide (because the coefficients for $Q_{\rm H\,II}$ and $Q_{\rm He\,III}$ are nearly identical for this value of the spectral index), and correspond to reionization redshifts $z_{\rm ri} \approx 3$. While inconsistent with the H I data, measurements of the mean transmitted He II Lyα flux reveal the absence of a measurable flux by $z \approx 3.5$ (Zheng et al. 2004). Any future detections of a finite He II Lyα optical depth at $z > 3.5$ would suggest that the QSO spectra are harder than $\alpha_{\rm S} = 1.8$ shortward of the He II Lyman edge. For $\alpha_{\rm S} = 0.5$, helium reionization may occur as early as $z \approx 5$ by QSOs alone. By contrast, even for this hard a spectrum, QSOs alone are unable to reionize the hydrogen in the IGM any earlier than $z \approx 4$.

If helium was not fully ionized until $z \approx 3$, then the UV background may have undergone a second change in its spectral shape shortward of the hydrogen Lyman edge due to the release of (redshifted) recombinant He II photons and two-photon emission (Haardt & Madau 1996). (The first change may have been at $z < 4.5$ when QSOs began to dominate the source emissivity.) There would of course also be a marked change shortward of the He II Lyman edge. In addition to the indirect effect on the temperature of the IGM, these changes may be detectable through their effects on the relative abundances of ionization states of metal ions (e.g. Si IV/C IV), for which there is some evidence at $z \approx 3.1$ (Songaila & Cowie 1996).

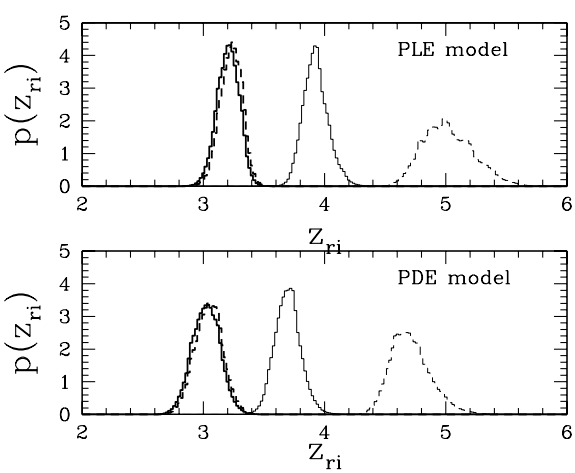

Figure 6. The probability distributions for the epoch of reionization for hydrogen (solid lines) and helium (dashed lines) assuming a QSO spectral index of $\alpha_{\rm S} = 1.8$ (heavy lines) or 0.5 (light lines).

4.3 Did low-luminosity AGNs reionize the IGM?

The implication that the reionization sources had hard spectra ($\alpha_{\rm S} = 0.5$) would suggest that star-forming galaxies (or star clusters) dominated by Pop III stars are the only viable candidates for reionizing the IGM. QSOs, the only other hard spectra sources known, decline too rapidly in their number at high redshifts. However, reionization solely by Pop III stars has its own difficulties, as the stars would rapidly pollute their environment with metals, so that reionization would be accomplished predominantly by Pop II stars (Schneider et al. 2002; Ricotti & Ostriker 2004). As shown above, Pop II stars are just barely able to reionize the IGM by $z = 6$ without violating the emissivity requirements needed to match the measurements of the mean transmitted Lyα flux at $z < 6$. They fall well short of the *WMAP* reionization requirement. Pop III stars (or Pop II stars) could only have reionized the IGM if their emissivity rapidly declined from the time of reionization to $z < 6$.

Madau et al. (2004) discuss miniquasars as an alternative reionization source, arguing that they may arise naturally in the same systems forming Pop III stars through accretion on to black holes formed from collapsed massive stars. However, Dijkstra, Haiman & Loeb (2004) claim that this and any scenario in which reionization was performed by quasar-like objects are excluded by constraints from the soft X-ray background. In this section, the possibility that low-luminosity AGNs reionized the IGM is re-explored.

From an energetic perspective, the requirements that galaxies harbouring active nuclei at high redshifts must satisfy for reionization are readily met. A black hole of mass $10^{6.5}$ M$_\odot$ would shine with an Eddington luminosity of $L_{\rm Edd} \simeq 5 \times 10^{44}$ erg s^{-1}, producing a specific luminosity at the Lyman edge of $L_{\rm L} \sim 0.1L_{\rm Edd}/\nu_{\rm L} \approx 10^{28}$ erg s^{-1} Hz^{-1} (allowing for a contribution near the Lyman edge to the bolometric luminosity of 10 per cent). At $z = 6$, equation (2) gives a total (comoving) emissivity of $\epsilon_{\rm L} \approx 3 \times 10^{24}$ erg s^{-} Hz^{-1} Mpc^{-3}. The corresponding comoving space density of galaxies harbouring such black holes is then $n_{\rm g} \sim 2 \times 10^{-4}$ Mpc^{-3}, only a small percentage of galaxies today. The magnitude at 1450 Å would then be $M_{1450} \approx -19$ (assuming $f_\nu \sim \nu^{-0.5}$). This corresponds to a z' magnitude of $\gtrsim$27 at $z \sim 6$ and $\gtrsim$28 at $z \sim 8$. The number of sources per unit redshift per arcmin^2 at $6 < z < 8$ would then be $\mathrm{d}N/\mathrm{d}z \approx 1/2$. It is possible that as many as a dozen of such objects appear as i'-dropouts in the *UDF* (see, for example, Bunker et al. 2004). At such low luminosities, however, they would have escaped detection in the GOODS fields and in the $z \approx 3$ survey of Steidel et al. (2002). It does imply a sharp turn up at faint magnitudes in the number of sources compared with the Hunt et al. (2004) luminosity function at $M_{1450} = -20$. However, the comoving density of the objects may have declined substantially by $z \approx 3$, as they are no longer required to maintain the ionization of the IGM, because the known QSOs are adequate (Fig. 1). The excess towards fainter magnitudes may then be only mild.

The low luminosities and abundance of the objects also place a minimal demand on the number of ionizing photons required per hydrogen atom to reionize the IGM. The typical (proper) distance between the sources at $z = 11$ is $\sim$1 Mpc, corresponding to a flux of ionizing photons of $F \sim 10^{53}$ ph s^{-1} Mpc^{-2}, or $F_0 = 0.01$–0.1 in the units of Iliev et al. (2004). Such low fluxes require only one to two ionizing photons per hydrogen atom to photoionize a minihalo (Iliev et al. 2004) as a consequence of photoevaporation.

The objects would also be consistent with constraints imposed by the soft X-ray background. The estimate of Dijkstra et al. (2004) was based on assuming a mean requirement of 10 ionizing photons per hydrogen atom to reionize the IGM. They then argued that any quasar-like sources which produced sufficient photons to reionize the IGM would exceed by a factor of about 4–6 the component of the residual soft X-ray background unaccounted for by discrete sources. As this factor scales linearly with the number of ionizing photons required per atom, a requirement of only two photons per atom would bring the level into agreement with the residual background. (This argument may apply to miniquasars as well.)

However, even without appealing to the decrease in the number of required ionizing photons per atom, the soft X-ray background constraint may be avoided because of the presumed hard spectrum ($\alpha_S = 0.5$) of the sources. Dijkstra et al. (2004) based their estimates for QSOs on the spectrum of Sazonov, Ostriker & Sunyaev (2004), who assumed a mean QSO energy spectrum varying as (their equation 14) $\langle F(E)\rangle \sim E^{-0.6}$ for $1 \leqslant E < 10$ eV, and $\langle F(E)\rangle \sim E^{-1.7}$ $e^{E/2\,\mathrm{keV}}$ for 10 eV $\leqslant E < 2$ keV, based on the *HST* data of Telfer et al. (2002). However, adopting $\alpha_S = 0.5$ at $E > 10$ eV can increase the number of ionizing photons while leaving the X-ray constraints unchanged. For instance, adopting the form (with E in units of keV)

$$\langle F_E\rangle = 210 \begin{cases} 0.63E^{-0.6} & 0.001 \leqslant E < 0.01 \\ E^{-0.5}e^{-E/0.4} & 0.01 \leqslant E < 2 \end{cases} \tag{14}$$

preserves the optical to X-ray spectral index α_{ox} between 2500 Åand 2 keV (as well as the normalization at 2 keV), but produces three times as many hydrogen ionizing photons as the corresponding model of Sazonov et al. (2004). (It also predicts $L_L \simeq 0.1 L_{bol}/\nu_L$, used above to relate L_L to L_{Edd}.) This again would bring the predicted contribution of the sources to the soft X-ray background into agreement with the residual component. It thus appears that hard spectrum quasar-like objects may reionize the IGM without exceeding the soft X-ray background limits.

5 CONCLUSIONS

A relation between the mean transmitted Lyα flux and ionization porosity of the IGM is exploited to obtain generic limits on the epoch of reionization. Under the assumption that the sources which dominate the UV ionization background at $z < 6$ are members of the same population of sources that reionized the IGM, it is shown that the epoch of reionization must have occurred at $z_{ri} < 11$ (2σ upper limit) for hard spectrum ($\alpha_S = 0.5$) sources such as QSOs, AGNs and Pop III dominated star-forming galaxies, $z_{ri} < 6$ (2σ) for moderate spectrum ($\alpha_S = 1.8$) sources such as soft spectrum QSOs, AGNs and Pop II dominated star-forming galaxies, and $z_{ri} < 5.5$ (2σ) for soft spectrum ($\alpha_S = 2.3$) sources such as star-forming galaxies with solar abundances.

After reionization, it is possible that no single population of sources dominated the UV ionizing background down to $z \approx 2$. Instead, the UV ionizing background may have undergone multiple qualitative changes in its spectral shape as the nature of the dominant ionizing sources changed. An assessment of the contribution of QSOs to the budget of ionizing photons based on recent high-redshift QSO surveys suggest that QSOs alone are adequate for providing the ionization rate required to reproduce the mean Lyα optical depth measurements at $z \lesssim 3$. QSOs produce at least half of the required emissivity at $z \approx 3.5$, but fall short by a factor of at least 2–3 for $4.5 < z < 6$. As a consequence, the UV metagalactic background shortward of the Lyman edge may have undergone a qualitative change in its spectral shape in the interval $3.5 < z < 4.5$. QSOs produce too few hydrogen ionizing photons to reionize the IGM any earlier than $z_{ri} \approx 4$, even assuming a hard spectral index ($\alpha_S = 0.5$). For a moderate spectral index ($\alpha_S = 1.8$), QSOs alone are unable to ionize the helium in the IGM any earlier than $z \approx 3$, but helium reionization may occur as early as $z \approx 5$ for QSOs with a hard spectral index ($\alpha_S = 0.5$) – unless the clumping factor is large. If QSOs do not ionize He II until $z \approx 3$, then a second qualitative change will result in the spectral shape of the UV background.

Both PLE and PDE models are found to provide acceptable fits to the QSO counts over the redshift range $3 < z < 6$. However, comparison with PLE models for $z < 2$ suggests substantial density evolution in the QSO luminosity function between $z > 3$ and $z \lesssim 2$. Predicted break magnitudes for the models at $z > 3$ suggest that the breaks would be clearly detectable in the PDE case for a survey 1 mag deeper than the SDSS QSO survey and 3 mag deeper for the PLE case.

Consistency with the *WMAP* 2σ lower limit of $z_{ri} > 11$ (Kogut et al. 2003) suggests that the sources which reionized the IGM must have had hard spectra ($\alpha_S \approx 0.5$), like hard spectra QSOs or AGNs, or Pop III stars, unless reionization occurred very early ($z \gg 6$), in which case the ionization state of the IGM at $z < 6$ would not be expected to yield any constraints on the nature of the reionization sources. An assessment of the contribution of ionizing photons from QSOs based on recent surveys shows that they fall short of the minimal requirement for reionizing the IGM by a factor of several. This suggests that (i) galaxies were much more prodigious sources of ionizing photons at $z > 6$ than at $z < 6$, either because of a higher star formation rate or a higher escape fraction of ionizing photons, (ii) star formation in galaxies at $z > 6$ was dominated by the production of Pop III stars, producing a hard spectrum, (iii) at least a small fraction of galaxies harboured a low-luminosity AGN at $z > 6$, so that they were hard spectrum sources at these epochs, or (iv) reionization was produced by an as yet unidentified population of sources (such as high-redshift miniquasars).

Another possibility is that numerical simulations of the structure of the IGM at high redshifts are seriously in error. The high-precision agreement with measurements of the Lyα forest in spectra at moderate redshifts ($z < 4$) (Meiksin, Bryan & Machacek 2001) does not preclude the possibility that energetic events such as galactic winds or QSOs at higher redshifts substantially disturbed the gas at earlier times, so that the simulations do not provide valid constraints on the source emissivity required to match the mean Lyα transmitted flux measurements at $z > 5$. Currently, there is no direct evidence that this may be the case.

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