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QUANTIFYING ORBITAL MIGRATION FROM EXOPLANET STATISTICS AND HOST METALLICITIES

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ABSTRACT

We investigate how the statistical distribution of extrasolar planets may be combined with knowledge of the host stars' metallicity to yield constraints on the migration histories of gas giant planets. At any radius, planets that barely manage to form around the lowest metallicity stars accrete their envelopes just as the gas disk is being dissipated, so the lower envelope of planets in a plot of metallicity versus semimajor axis defines a sample of nonmigratory planets that will have suffered less than average migration subsequent to gap opening. Under the assumption that metallicity largely controls the initial surface density of planetesimals, we use simplified core accretion models to calculate how the minimum metallicity needed for planet formation varies as a function of semimajor axis. Models that do not include core migration prior to gap opening (type I migration) predict that the critical metallicity is largely flat between the snow line and $a \approx 6$ AU, with a weak dependence on the initial surface density profile of planetesimals. When slow type I migration is included, the critical metallicity is found to increase steadily from 1 to 10 AU. Large planet samples that include planets at modestly greater orbital radii than present surveys therefore have the potential to quantify the extent of migration in both type I and type II regimes.

Subject headings: planetary systems: formation — planets and satellites: formation — solar system: formation

1. INTRODUCTION

The discovery of 51 Pegasi (Mayor & Queloz 1995) provided evidence for the potential importance of orbital migration (Goldreich & Tremaine 1980; Lin & Papaloizou 1986) in determining the structure of extrasolar planetary systems. For 51 Peg, and for other members of the class of hot Jupiters, in situ formation scenarios face obvious difficulties due to the predicted high temperature ($T > 10^3$ K) of the protoplanetary disk (Bell et al. 1997) at the radii—within 0.1 AU of the star—where the planets now orbit. This conclusion is largely borne out by detailed models of giant planet formation, which confirm that planet formation via core accretion is unlikely at such small distances from the star (Bodenheimer et al. 2000). For the much larger population of extrasolar giant planets that orbit at $a > 1$ AU, however, the situation is less clear. These planets, which are still orbiting their stars at much smaller radii than Jupiter's 5.2 AU, *could* potentially form in situ via core accretion (Bodenheimer et al. 2000), especially when recent downward revisions to both the radial location (Sasselov & Lecar 2000) and importance (Lodders 2003) of the snow line are considered. Alternatively, the typical formation radius for giant planets around roughly solar mass stars could fall beyond the radius of any observed extrasolar planet, implying a dominant role for migration in currently known systems.

Observationally, the statistical distributions of basic extrasolar planet properties [mass $M_p \sin(i)$, semimajor axis a, and eccentricity e] provide only conflicting clues as to the importance of migration. The low masses of some close-in planets point to relatively short residence times within the gaseous protoplanetary disk, since numerical simulations show that Saturn-mass planets accrete gas across gaps and grow to larger masses rather promptly (Lubow et al. 1999; Bate et al. 2003). If mass growth is ignored, however, the increasing fraction of known extrasolar planets with orbital radius is consistent with numerical models that assume that these planets all formed farther out and migrated inward due to planet-disk interactions (Armitage et al. 2002). Theoretically, it is well known that the timescale for core accretion to form planets at large radii $a \sim 20$ AU is long (Pollack et al. 1996)—at least in the simplest versions of the theory—but where in the inner disk planet formation is most favored is uncertain. Theoretical work has demonstrated that the timescale and outcome of core accretion can be modified by type I migration (Ward 1997; Tanaka et al. 2002) of giant planet cores (Hourigan & Ward 1984; Alibert et al. 2004), by gravitational interaction of cores with turbulent fluctuations in the disk (Rice & Armitage 2003), or by competition for planetesimals between several growing cores (Hubickyj et al. 2004). That this is by no means an exhaustive list of the possibilities illustrates that additional observational clues as to where and when giant planets form would be valuable.

In this paper, we investigate how models of giant planet formation via core accretion could be constrained by adding observational knowledge of the metallicity of the host star. The fraction of roughly solar-type stars that host known extrasolar planetary systems rises rapidly with stellar metallicity (Gonzalez 1998; Santos et al. 2001, 2004b; Fischer et al. 2004; Fischer & Valenti 2005), with several lines of evidence pointing to the high [Fe/H] being the cause, rather than the consequence, of planet formation (Kornet et al. 2005). The strikingly strong scaling of planet frequency with metallicity—which rises from almost zero frequency at $[Fe/H] < -0.5$ to $\approx$20% at $[Fe/H] \approx +0.5$—suggests that metallicity is a more important parameter in determining the probability of giant planet formation than intrinsic dispersion in either the gas disk mass or disk lifetime. If this is true, then the epoch of giant planet formation at a given radius in the disk (defined as the moment when the core accretes a significant gaseous envelope via runaway accretion) should correlate with the metallicity. High metallicity implies a larger surface density of planetesimals and much shorter planet formation timescales (Pollack et al. 1996). In particular, as illustrated in Figure 1, at any radius there should be a minimum threshold or critical metallicity, below which core accretion fails to reach

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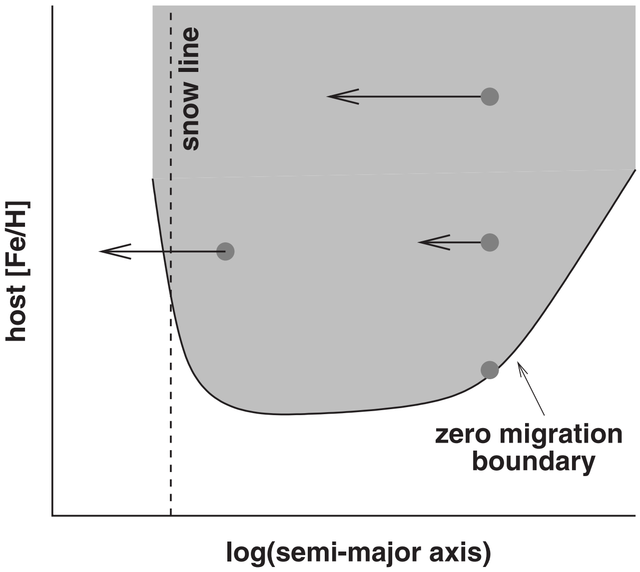

FIG. 1.—Illustration of the expected influence of core formation and orbital migration on the final distribution of extrasolar planets. In an idealized model, in which the initial mass and lifetime of the gas disk have a narrow range of values, giant planet formation will occur at a given radius provided that the host star's metallicity exceeds a threshold value. At this threshold value, runaway occurs and the gaseous envelope is accreted just as the protoplanetary disk is about to be dissipated, allowing no opportunity for type II migration. For higher host metallicities, type II migration is expected to become increasingly important. If the threshold metallicity is an increasing function of radius—in practice at radii outside the snow line—then inward migration *cannot* populate the unshaded region below the critical curve. The shape of this curve then defines a threshold for giant planet formation that is independent of the effects of orbital migration.

runaway within the lifetime of the gas disk. These "failed" gas giants potentially constitute a large population of $\sim 10\ M_\oplus$ planets much closer in than Uranus and Neptune in the solar system (Ida & Lin 2004). More importantly for our purposes, planets just *above* the critical metallicity form just as the gas is being dissipated. There is therefore little or no opportunity for these planets to migrate via type II gravitational interactions with the protoplanetary disk. Moreover, if the critical metallicity increases monotonically with increasing semimajor axis (and migration is inward, as is very probable at small radii), then there is no way to populate the region of parameter space below the threshold curve. Observational definition of the lowest metallicity stars that host planets at different radii can therefore probe the relative radial efficiency of the planet formation process, independent of the separate uncertainties that attend the migration process.

The outline of this paper is as follows. In § 2, we describe simplified models for gas giant planet formation that allow us to calculate the timescale for a single core to reach runaway gas accretion in the protoplanetary disk. In § 3, we compute the threshold metallicity as a function of orbital radius and discuss how it depends on the surface density profile of planetesimals and on the extent of type I migration of giant planet cores prior to the accretion of the envelope. The most significant differences between the models occur at relatively large radii that, although poorly sampled today, should be accessible to future astrometric and direct imaging surveys. In § 4, we analyze existing statistics of extrasolar planetary systems within the context of our theoretical model. Although useful constraints on planet formation are not possible with existing data, we demonstrate that one of the basic theoretical premises—that planets observed to lie near the threshold metallicity curve should have formed on average at later epochs—is consistent with the observed distribution of planetary masses around stars with different [Fe/H].

2. GIANT PLANET FORMATION MODEL

We assume that the massive planets observed in extrasolar planetary systems formed via core accretion, and that the dominant factor controlling whether planets form at a given radius prior to the dispersal of the gas disk is the metallicity of the gas that formed the star and disk. Observations suggest that in most cases the current photospheric stellar metallicity reflects the primordial value (Santos et al. 2004a), and hence we assume that the ratio of the surface densities of the planetesimals to that of the gas is linearly related to the stellar metallicity. Our goal is to compute as a function of radius the minimum metallicity that allows massive planets—defined as those with $M_{\rm pl} > 100\ M_\oplus$—to form within the lifetime of the gas disk. We describe here a simplified model for core accretion, similar to that developed by Ida & Lin (2004), that accomplishes this objective.

2.1. *The Gas Disk*

Within core accretion models of giant planet formation (Bodenheimer & Pollack 1986; Pollack et al. 1996), the time-dependent evolution of the gaseous component of the protoplanetary disk primarily affects the final masses of planets and the rate of type II migration subsequent to gap formation. The *threshold* for planet formation, i.e., whether a gas giant can form at all in a given disk, does depend upon the properties of the gas at late epochs (via the pressure and temperature of the disk, which provide boundary conditions for the planetary envelope), but this dependence is weak compared to the effect of changes in the planetesimal surface density. A minimal description of the gaseous disk therefore requires only specification of the surface density profile and lifetime. We adopt for the gaseous surface density a power law over the range of radii ($1\ {\rm AU} < a < 10\ {\rm AU}$) of interest

$$\Sigma_g = 7.2 \times 10^3 \left(\frac{a}{1\ {\rm AU}}\right)^{-\beta} {\rm g\ cm^{-2}}, \tag{1}$$

where a is the orbital radius and β is a parameter that specifies the mass distribution in the disk (note that we fix the gas surface density at a radius of 1 AU). We consider values of β in the range $1 \leq \beta \leq 2$, which includes the most commonly considered possibilities (Weidenschilling 1977; Bell et al. 1997; Kuchner 2004). This surface density profile is strictly the profile at the time when planetesimals form from the dust within the gas—since this is likely to occur at an early time it will differ little from the initial profile. We ignore star-to-star variations in the initial disk mass (Armitage et al. 2003). Provided that these are of less importance than (and uncorrelated with) variations in gas metallicity, they should only introduce scatter in the minimum metallicity required for planet formation.

For the lifetime of the gas disk, we take $\tau = 5 \times 10^6$ yr. This value is similar to observational estimates (Strom et al. 1989; Haisch et al. 2001). We assume that once $t > \tau$, the gas disk is promptly dispersed at all radii and the potential for further gas giant formation is quenched. Observations support the idea that dispersal of the gas occurs rapidly (Wolk & Walter 1996).

2.2. *The Planetesimal Disk*

We assume that the surface density of planetesimals, Σ_d, has a power-law distribution such that

$$\Sigma_d = 10 f_{\rm dust} \eta_{\rm ice} \left(\frac{a}{1\ {\rm AU}}\right)^{-\beta} {\rm g\ cm^{-2}}, \tag{2}$$

where β has the *same* value for the planetesimals as for the gas.[4] We consider β values of 1, 1.5, and 2. $f_{\rm dust}$ is a parameter that we vary to change the normalization of the planetesimal surface density (and stellar metallicity), and $\eta_{\rm ice}$ is a step function that represents the ice condensation/sublimation across the snow line that occurs, around a star of mass M_*, at $a_{\rm ice} = 2.7(M_*/M_\odot)^2$ AU. Since we are considering the formation of planets around solar-like stars, we assume $M_* = M_\odot$ and therefore $a_{\rm ice} = 2.7$ AU. Following Ida & Lin (2004) we take $\eta_{\rm ice} = 1$ when $a < a_{\rm ice}$, and $\eta_{\rm ice} = 4.2$ when $a > a_{\rm ice}$.

To convert between our parameter $f_{\rm dust}$ and the stellar [Fe/H], we note that if $f_{\rm dust} = 3$, then the metallicity is solar. If $f_{\rm dust}$ is greater (less) than 3, the metallicity is greater (less) than solar. The opacity of the gas is also important, since there is a weak dependence of the critical core mass, and a stronger dependence of the envelope's Kelvin-Helmholtz time, on κ. We assume that

$$\kappa = \frac{f_{\rm dust}}{3}\ {\rm cm^2\ g^{-1}}, \tag{3}$$

so that for solar metallicity the opacity is 1 cm^2 g^{-1}.

2.3. *Core Accretion Model*

To calculate the growth of a giant planet core within the above disk model and the subsequent accretion of a gaseous envelope, we employ a modified version of the scheme developed by Ida & Lin (2004). For each run, we start our calculation with a solid core of mass $M_{\rm core} = 10^{-3}\ M_\oplus$, density of 3.2 g cm^{-3}, and radius a. The core grows at the rate $\dot{M}_{\rm core} = \pi R_c^2 \Sigma_d \Omega F_g$, where R_c is the radius of the core, Ω is the angular frequency, and F_g is the gravitational enhancement factor (Greenzweig & Lissauer 1992). The core accretes planetesimals from a annular region around it known as the feeding zone, which has a full width of $\Delta a_c = 8 r_{\rm H}$, where

$$r_{\rm H} = \left(\frac{M_{\rm pl}}{3M_*}\right)^{1/3} a \tag{4}$$

is the Hill radius for a planet with mass $M_{\rm pl}$ around a star with mass M_*.

As the core grows we adjust, accordingly, the planetesimal surface density in the feeding zone. In this way the core self-consistently stops growing once it reaches its isolation mass. In our initial calculations we assume that the core's orbital radius remains fixed, although subsequently we relax this restriction and allow for orbital migration.

When the core's mass exceeds a critical value, $M_{\rm crit}$, it is no longer able to support a gaseous envelope in quasi-hydrostatic equilibrium, and runaway gas accretion occurs (Mizuno 1980; Bodenheimer & Pollack 1986; Papaloizou & Terquem 1999). This critical core mass depends on the planetesimal accretion rate, $\dot{M}_{\rm core}$, and on the opacity, κ, associated with the disk gas. We adopt a representative estimate for the critical core mass calculated by Ikoma et al. (2000):

$$M_{\rm crit} = 10\left(\frac{\dot{M}_{\rm core}}{10^{-6}\ M_\oplus\ {\rm yr^{-1}}}\right)^{0.25}\left(\frac{\kappa}{1\ {\rm cm^2\ g^{-1}}}\right)^{0.25} M_\oplus. \tag{5}$$

Once the core exceeds the critical core mass, the gaseous envelope contracts on a Kelvin-Helmholtz timescale, $\tau_{\rm KH}$. Ikoma et al. (2000) show that the Kelvin-Helmholtz timescale can be written as

$$\tau_{\rm KH} = 10^b\left(\frac{M_{\rm pl}}{M_\oplus}\right)^{-c}\left(\frac{\kappa}{1\ {\rm g\ cm^{-2}}}\right)\ {\rm yr}, \tag{6}$$

where the exact values of b and c depend on the choice of opacity table. Ikoma et al. (2000) found that $b \simeq 8$ and $c \simeq 2.5$, while Bryden et al. (2000) obtained a fit to the results of Pollack et al. (1996) with $b \simeq 10$ and $c \simeq 3$. We follow Ida & Lin (2004) and use $b = 9$ and $c = 3$. Once the critical core mass has been exceeded, we allow gas accretion at a rate

$$\frac{dM_{\rm pl}}{dt} = \frac{M_{\rm pl}}{\tau_{\rm KH}}, \tag{7}$$

where $M_{\rm pl}$ includes the mass of both the solid core and the gaseous envelope. It should be noted that in this model the core can continue to grow while gas is being added.

Equations (6) and (7) define a model for growth of a giant planet's envelope that is entirely demand-driven—there is no dependence whatsoever on the properties of the gas disk. This is reasonable for low-mass planets but will fail at higher masses when either the supply of gas becomes limited or the planet opens a gap. Since we are interested in planets that are forming just as the disk gas is dissipating, we assume that there is sufficient supply for these planets to reach masses of $\sim$100 $M_\oplus$. If runaway growth does occur in our model, we stop the calculations when $M_{\rm pl} > 100\ M_\oplus$. This mass threshold (which corresponds to 0.3 Jupiter masses) defines "success" in forming a giant planet.

The procedure for modeling planet growth is then as follows. At $t = 0$ we start with a $10^{-3}\ M_\oplus$ core located at a radius a. We calculate $\dot{M}_{\rm core}$, which we use to determine the core mass at the next time step. Simultaneously we determine $M_{\rm crit}$. If $M_{\rm core} > M_{\rm crit}$ then gas is accreted with the contraction timescale given by $\tau_{\rm KH}$. We stop the calculation once $t > 5 \times 10^6$ yr or once $M_{\rm pl} > 100\ M_\oplus$. For a given β and a given a we then determine the value of $f_{\rm dust}$ (which within our assumptions is a measure of the metallicity) for which a gas giant planet ($M_{\rm pl} > 100\ M_\oplus$) will form in exactly 5×10^6 yr. This value of $f_{\rm dust}$ ($f_{\rm dust,min}$) is the minimum value for which a gas giant planet can form prior to the dissipation of the gas disk, and it defines a group of planets that should undergo very little type II migration.

3. RESULTS

For a protoplanetary disk with a specified value of β, we use the model described above to calculate the minimum or threshold metallicity required to form a planet at radius a prior to the dissipation of the gas disk. We consider radii between 1 and 10 AU and assume, initially, that the core and growing planet suffer negligible type I migration during the formation process. Illustrative runs for a planet growing at 5 AU in a disk with $\beta = 1.5$ are shown in Figure 2. As the metallicity is increased (i.e., larger values of $f_{\rm dust}$) growth of the planet toward the critical core mass is accelerated. In this case values of $f_{\rm dust} = 2.05$ and $f_{\rm dust} = 2.35$ fail to yield a fully formed giant planet by our definition, as the planet mass after 5 Myr is less than 100 $M_\oplus$. Slightly higher metallicity ($f_{\rm dust} = 2.45$), however, results in runaway growth and produces a 100 $M_\oplus$ gas giant planet within 5 Myr.

[4] We note that although this is the simplest assumption, it might not be correct, in particular if there is significant radial migration of solids prior to planetesimal formation. Recent models of planetesimal formation via gravitational instability (Goldreich & Ward 1973; Youdin & Shu 2002; Youdin & Chiang 2004) require such migration.

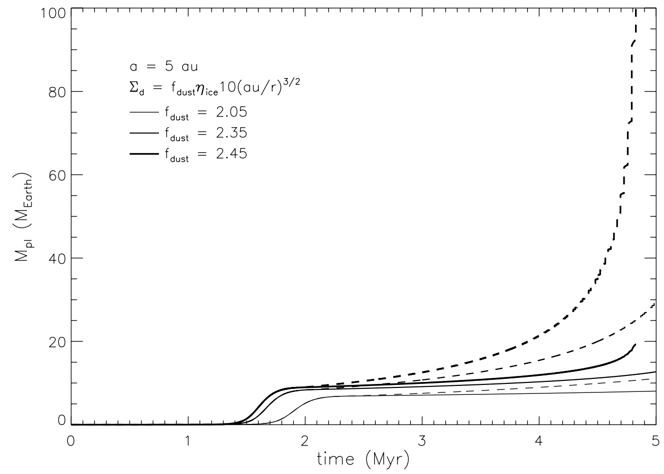

FIG. 2.—Growth curves for a core located at $a = 5$ AU and for three different planetesimal surface densities, as defined by f_{dust}. The different f_{dust} values are represented by different line thicknesses, and in each case the solid line shows the core mass, while the dashed line is the total planet mass (core + envelope). For $f_{dust} = 2.45$ a giant planet forms within 5 Myr, while for lower values of f_{dust} gaseous planet formation does not occur within the disk lifetime.

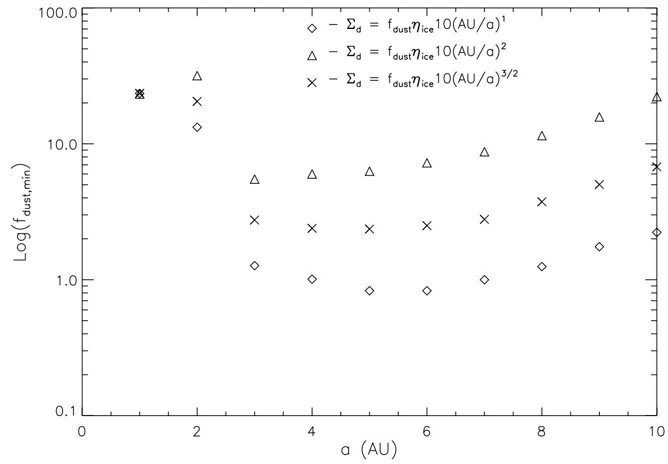

FIG. 3.—Minimum value of f_{dust} required to form a gaseous planet, in situ, within 5 Myr. This illustrates the difficulty in forming gaseous planets within the snow line (~2.7 AU), since the required planetesimal surface density is extremely high, and shows that even for reasonably steep surface density profiles, the radial dependence beyond the snow line is relatively weak out to ~7 AU.

By interpolating from a series of such runs in which the planet formation timescale brackets the assumed disk lifetime, we determine $f_{dust,min}$ as a function of orbital radius for each disk model. We consider surface density profiles of $\beta = 1$, 1.5, and 2, in each case normalizing the value of the planetesimal surface density at an arbitrary radius of 1 AU.

Figure 3 shows the derived values of $f_{dust,min}$ against radius for the three planetesimal surface density profiles that we consider. The most obvious feature in the plots is the sharp rise in the predicted threshold metallicity interior to the radius of the snow line. Although the *absolute* values of the threshold metallicity obviously scale with the assumed normalization (and hence the values agree for all disk models at 1 AU), for all models there is a jump of around an order of magnitude across the snow line. Since observationally the frequency of planets around stars significantly more metal-poor than the Sun is very low (Gonzalez 1998; Santos et al. 2001, 2004b; Fischer et al. 2004; Fischer & Valenti 2005), this implies that gaseous planets cannot form in situ within the snow line (here at 2.7 AU) unless the metallicity is significantly higher than solar. This result is largely consistent with Ida & Lin (2004), who found that gaseous planets would only form within the snow line for large planetesimal surface densities. Although there are hints of a correlation between planet orbital period and host [Fe/H] (Sozzetti 2004), it is clear that the observed distribution of extrasolar planets in the *a*-[Fe/H] plane does not resemble Figure 3. There are a number of planets with semimajor axes $a < 1$ AU around metal-poor stars, and no clear sign of a jump at any plausible snow line radius. At relatively small orbital radii, then, migration appears to be necessary in order to explain the observed statistics of extrasolar planets.

Figure 3 also shows the behavior of the threshold metallicity with orbital radius beyond the snow line. The minimum metallicity required for gas giant formation depends only weakly on radius out to ~6 AU for all of the surface density profiles that we have considered, and for the flattest profile ($\beta = 1$) there is little dependence out to larger radii of around 10 AU. This is consistent with the model developed by Pollack et al. (1996), which predicts that Jupiter forms at 5.2 AU in 8 Myr with a local planetesimal surface density of 10 g cm^{-2}, while Saturn forms within a comparable timescale (10 Myr) if the local planetesimal surface density at 9.5 AU is 3 g cm^{-2}. There is a weak trend for planet formation to be favored at smaller radii around lower metallicity stars (Pinotti et al. 2005) if $\beta = 2$, but in general we would expect that there should not be a strong radial dependence over the range of orbital radii currently accessible to radial velocity surveys (i.e., the threshold metallicity for planet formation at 3 AU should be similar to that at 6 AU). At larger radii, however, there is a significant dependence that varies between models. The predicted threshold metallicity rises by a factor of ≈2 between 6 and 10 AU if $\beta = 1$, whereas for a steeper $\beta = 2$ profile the rise is closer to a factor of 4. Within the context of core accretion models, measurement of the lower envelope of detected planets in the *a*-[Fe/H] plane at fairly large radii can therefore constrain some inputs to the formation model. Astrometric surveys appear to offer the best possibilities for assembling large enough planet samples at the desired radii.

3.1. *Core Migration*

The above calculations, like the baseline core accretion models presented by Pollack et al. (1996), assume that the core grows at fixed orbital radius. If the core remains embedded within a gaseous disk, however, analytic calculations (Ward 1997) show that differential torques arising from the core's gravitational interaction with the gas should induce rapid type I migration. The influence of torques from turbulent fluctuations in the disk surface density may also drive orbital drift (Nelson & Papaloizou 2004; Laughlin et al. 2004). Migration in either regime can accelerate gaseous planet formation (Hourigan & Ward 1984; Alibert et al. 2004; Rice & Armitage 2003) but can also prevent planet formation if the cores migrate into the central star prior to the accretion of the envelope and subsequent gap opening. Here, we consider the possible influence of type I core migration on the expected distribution of massive planets in the orbital radius–host metallicity plot.

Preliminary calculations showed, as expected, that the survival prospects for cores allowed to migrate at the analytic type I rate (Ward 1997) are slim. To allow the core to survive, we therefore assume that the migration, on average, must be slower than the canonical type I migration rate by about an order of magnitude. This assumption is similar to that made by Alibert et al. (2004) and may be justified, in part, by numerical simulations that suggest that the true type I migration rate may be significantly slower than previously assumed (Miyoshi et al. 1999;

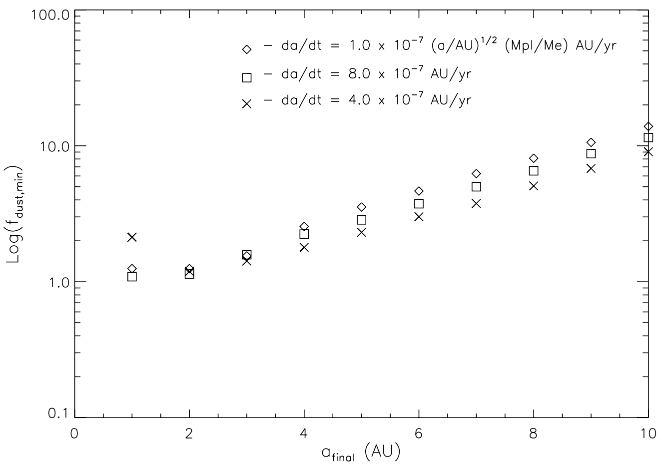

FIG. 4.—Minimum value of $f_{\rm dust}$ required to form a gaseous planet if the core is assumed to migrate to a final radius $a_{\rm final}$. We consider three different migration rates and find that in all cases gaseous planets can have final semimajor axes within the snow line for $f_{\rm dust}$-values that are significantly smaller than that required when core migration is ignored. This figure also suggests that if core migration plays a role in gaseous planet formation the metallicity required may increase with increasing $a_{\rm final}$.

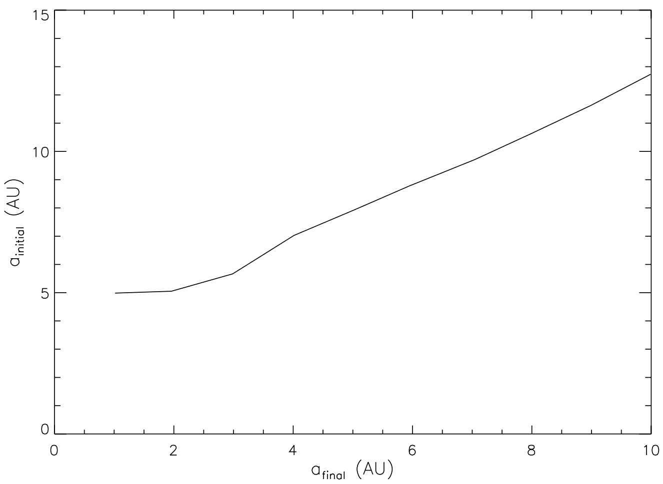

FIG. 5.—Initial semimajor axis ($a_{\rm initial}$) against final semimajor axis ($a_{\rm final}$) for the nonconstant migration rate. For planets that remain beyond the snow line the change in semimajor axis is almost constant, despite the radial dependence of the migration rate. For planets that end up within the snow line, $a_{\rm initial}$ appears almost constant suggesting that the surface density discontinuity at the snow line can produce large changes in $a_{\rm final}$ for small changes in $a_{\rm initial}$.

Jang-Condell & Sasselov 2005). Specifically, we consider three different core migration rates,

$$\dot{a} = 4 \times 10^{-7} \text{ AU yr}^{-1}, \tag{8}$$

$$\dot{a} = 8 \times 10^{-7} \text{ AU yr}^{-1}, \tag{9}$$

$$\dot{a} = 1 \times 10^{-7} \left(\frac{M_{\rm pl}}{M_\oplus}\right) \left(\frac{a}{\rm AU}\right)^{1/2} \text{ AU yr}^{-1}. \tag{10}$$

These prescriptions are intended only to sample a range of migratory behavior that allows giant planets to form without being consumed as cores by the star. We do not suggest that any of these rates is representative of the actual core migration rate, although the latter rate has a mass dependence that matches the standard type I migration rate. Since we adjust the planetesimal surface density in the feeding zone as the core grows, it is straightforward to add core migration to the model described in § 2. As before, we vary $f_{\rm dust}$ to determine the threshold metallicity required to form a planet at *final* radius $a_{\rm final}$ in 5×10^6 yr. This requires only an additional iteration to determine the (initially unknown) value of $a_{\rm initial}$ that yields such a planet. Henceforth we consider only surface density profiles of $\beta = 1.5$.

Figure 4 shows $f_{\rm dust,min}$ against $a_{\rm final}$ for the three migration rates that we considered (eq. [10]). The results for all three migration prescriptions are qualitatively similar. The prominent jump in threshold metallicity at the snow line, seen in the calculations with a static core, is erased in the case of a migrating core that can accrete planetesimals across a much wider range of orbital radii. These smaller threshold values interior to the snow line appear to be in better accord with observations. Moreover, at larger radii there is now a steady increase in the predicted threshold metallicity with radius, even at radii accessible to ongoing radial velocity surveys for extrasolar planets. If type I migration is implicated in the formation of gas giants, we would therefore expect to see a steady rise in the minimum host metallicity as the orbital radius of the planet increases.

Figure 5 shows $a_{\rm initial}$ against $a_{\rm final}$ for the nonconstant migration rate and illustrates the amount of migration that has taken place. For planets that remain beyond the snow line ($a_{\rm final} >$ 2.7 AU), the change in semimajor axis is largely independent of radius ($\Delta a \sim 3$ AU). The radial dependence of the migration rate appears to be balanced by the radial dependence of planet's growth rate. For planets that end up within the snow line ($a_{\rm final} <$ 2.7 AU), $a_{\rm initial}$ appears to be almost constant, with $\Delta a \sim 4$ AU for $a_{\rm final} = 1$ AU and $\Delta a \sim 3$ AU for $a_{\rm final} = 2$ AU. This suggests that the surface density discontinuity at the snow line can produce a large change in $a_{\rm final}$ for a small change in $a_{\rm initial}$. Although our chosen migration rates are somewhat arbitrary, these results suggest that migration rates of between 2 and 4 AU in 5×10^6 yr can produce planets within the snow line around stars with reasonably low metallicities and can result in an increase in the threshold metallicity with radius.

By focusing on the shape of the critical metallicity curve, we have deliberately avoided detailed discussion of the *relative number* of planets expected to populate different regions of the [Fe/H]-a-$M_{\rm pl}$ space. This depends on the fraction of plausible initial conditions that yield each specific outcome (Armitage et al. 2002; Ida & Lin 2004). We note, however, that such statistical considerations may yield additional evidence for type I migration of gas giant cores. In particular, Ida & Lin (2004) predict a dearth of planets with masses between 10 and 100 $M_\oplus$ within the snow line, a region they term the "planet desert." In their model, a planet can only fall within the desert if its growth is halted during the rapid envelope growth phase. This is an unlikely outcome since this phase lasts for such a short time. Equivalently, in the absence of core migration the range of metallicity values $\Delta f_{\rm dust}$ that populate the desert region is small compared to $f_{\rm dust,min}$, and as a consequence few planets are predicted in the desert.

When core migration is included, this prediction of an unoccupied planet desert can be modified, though the results do depend on the details of the migration history. As shown in Figure 4, $f_{\rm dust,min}$ at small orbital radii is greatly reduced in the presence of migration. Compared to this threshold value, we find a reasonably large range of $f_{\rm dust}$ values that result in planet masses that fall within the "planet desert." For the nonconstant migration rate, $f_{\rm dust} = 1.3$ yields a 100 $M_\oplus$ planet at 1 AU within 5 Myr, while $f_{\rm dust} = 1.1$ produces a 48 $M_\oplus$ planet after 5 Myr. Roughly 20% of stars with metallicities close to the threshold value would then be candidates for forming planets in the otherwise

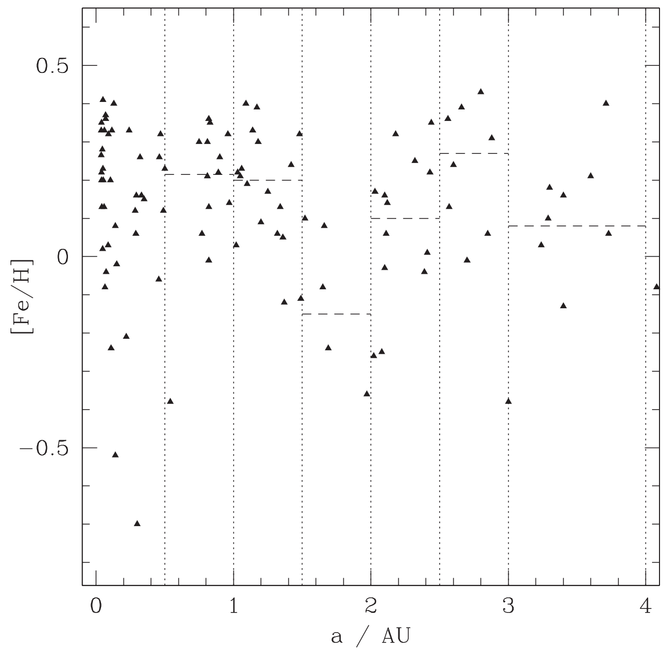

FIG. 6.—Distribution of extrasolar planets in the a-[Fe/H] plane. Although there are insufficient data to strongly constrain the relationship between semimajor axis and metallicity, there is a hint that the metallicity required for giant planet formation increases with increasing radius. The divisions within the figure are used to divide the planets within each radius bin into high- and low-metallicity samples.

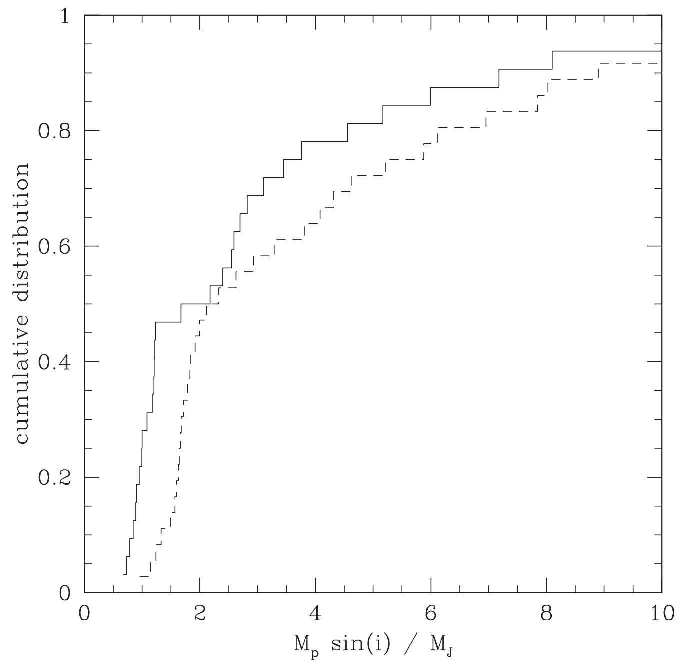

FIG. 7.—Mass distribution of the planets in the high- and low-metallicity samples defined by the divisions in Fig. 6. The two distributions are consistent with the expectation that the planets around lower metallicity stars (*solid line*) have, on average, lower masses. This is consistent with the idea that planets around lower metallicity stars are likely to have formation times comparable to the disk lifetimes and are unlikely to undergo significant type II migration.

(in the absence of migration) unoccupied region of phase space. Unfortunately, the range of $f_{\rm dust}$ values that resulted in planets with masses within the desert region was smaller for the constant migration rates than for the nonconstant migration rate. This suggests that for planets to lie within the desert, the core migration rate must be such as to maintain a large planetesimal accretion rate, $\dot{M}_{\rm core}$, resulting in a large critical core mass (see eq. [5]). Therefore, although the existence of planets within the desert region may point to the role of type I migration, more detailed modeling will be required to establish this clearly.

4. APPLICATION TO CURRENT DATA

To compare these expectations with current observations of extrasolar planetary systems, we use the metallicity data published by Santos et al. (2004b). The Santos et al. (2004b) catalog includes [Fe/H] measurements, derived using uniform analysis methods, for 98 planet host stars. For our purposes we include all planets in multiple planet systems and take the average value of [Fe/H] for those stars with more than one derived abundance. HD 47536b and its host star are excluded due to the large uncertainty in the mass of that planet, leaving 110 massive planets for which we have the host star metallicity, together with the planet mass $m_p \sin(i)$, semimajor axis, and eccentricity. Figure 6 shows the distribution of these planets in the a-[Fe/H] plane. Although there are some hints that this distribution is not merely a scatter plot (for example, there are no observed extrasolar planets at $a > 1$ AU around stars with $[{\rm Fe/H}] < -0.4$), the lower envelope of the distribution that has been the focus of the theoretical discussion is obviously poorly defined in current data due to the small number of planets observed that are unambiguously outside the snow line. Nevertheless, we can use the data for the more limited purpose of testing whether some of our basic assumptions are consistent with observations.

Theoretically, we expect that planets observed near the critical metallicity line formed late in the lifetime of the gaseous protoplanetary disk. We would expect them to have lower masses—since there was less time available to accrete gas before the disk was dissipated—and possibly different eccentricity. To test whether these expectations are consistent with current data, we divide the planets with semimajor axis $a > 0.5$ AU (i.e., excluding the hot Jupiters, but probably including some planets that are now seen interior to the snow line) into high and low host star metallicity samples. Some care is necessary, because the existing sample of planets discovered via radial velocity is biased against the discovery of low mass planets at large radii. Specifically, for a fixed number of radial velocity measurements with a given noise level (and some implicit assumptions as to the time sampling of the survey), the minimum detectable planet mass will scale with orbital radius roughly as $[m_p \sin(i)]_{\rm min} \propto a^{1/2}$. This detection bias means that simply dividing the sample of planets into two—those with host stars above and below some fixed value of [Fe/H]—risks mixing a variation of host metallicity with planet orbital radius into a spurious trend of planet mass with host metallicity.

To avoid such potential biases, we construct matched samples of planets around high- and low-metallicity host stars. We first bin planets according to their semimajor axis (in 0.5 AU increments between 0.5 and 3 AU, plus one final bin from 3 to 4 AU). Within each radial bin, the minimum detectable planet mass in a radial velocity survey varies by much less than the intrinsic dispersion in observed planet masses. We then split the planets *within each radial bin* into subsamples of high- and low-metallicity host stars, with (as far as is possible) equal numbers of planets in each. Our final sample of planets around relatively low- (high-) metallicity stars then consists of all the planets below (above) the dashed lines shown in Figure 6.

The mass distribution of the planets in the high- and low-metallicity samples, defined as above, is shown in Figure 7. The two distributions are consistent with the expectation that the planets around lower metallicity stars have, on average, lower masses. Formally, a Kolmogorov-Smirnov test shows that the probability that the two distributions are drawn from the same parent distribution is $P_{\rm KS} = 6 \times 10^{-3}$, so the statistical evidence in support of the existence of the hypothesized "no-migration" curve in the a-[Fe/H] plane is currently suggestive rather than overwhelming. However, the fact that some indications of a signal are present in current data does suggest that the larger planet samples that will plausibly be accumulated in the near future should suffice to test some of the ideas outlined in § 3.

Mindful of the fact that in some theoretical models orbital migration is associated with concurrent eccentricity growth (Papaloizou et al. 2001; Murray et al. 2002; Goldreich & Sari 2003; Ogilvie & Lubow 2003), we have tested whether the eccentricity distribution of the high and low metallicity samples is significantly different. Unlike in the case of the $m_p \sin(i)$ distributions, the eccentricity distributions of the two samples are statistically indistinguishable. Within the context of our model there is no evidence that relatively early planet formation, followed by significant radial migration, promotes growth of the final eccentricity.

5. SUMMARY

In this paper we have investigated how, with the addition of knowledge of the host stars' metallicity, the statistics of extrasolar planets can be used to constrain models for giant planet formation. Our main results are as follow.

1. By studying planets at a particular radius around the lowest metallicity host stars, it is possible to isolate a subsample that will have suffered, on average, less type II migration than the typical planet at that radius. This separation will only be clean outside the snow line, and only if metallicity is the most important random variable affecting the timescale for core formation. Comparison of such a nonmigratory sample with planets around metal-rich stars could constrain the amount of mass accreted onto the planet during type II migration.

2. If giant planet cores grow in place, it is difficult to explain the presence of massive extrasolar planets around relatively low metallicity hosts within the snow line. Static core growth models predict a threshold metallicity for planet formation that is roughly flat within 6 AU but that rises to larger radius. The functional dependence varies according to the slope of the planetesimal surface density distribution.

3. If giant planet cores suffer type I migration as they grow, the threshold metallicity rises smoothly beyond $a \approx 2$ AU. The details of the migration are relatively unimportant, provided that the overall rate is slow compared to the canonical analytic predictions. Under limited conditions, it is possible to populate what would otherwise be a desert in the distribution of planets with masses $10\ M_\oplus < M_{\rm pl} < 100\ M_\oplus$ at small orbital radii (Ida & Lin 2004).

Existing observations appear to be consistent with the basic theoretical premise of this paper—that host metallicity is the dominant factor controlling the timescale for massive planet formation. This hypothesis is motivated by the observed dependence of planet frequency on stellar [Fe/H] (Gonzalez 1998; Santos et al. 2001, 2004b; Fischer et al. 2004; Fischer & Valenti 2005) and implies a planet mass–metallicity correlation that is seen, albeit at low significance, in the data. This leaves us hopeful that with larger samples of massive extrasolar planets useful constraints on planet formation models will be attainable.

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