

Astrometric signatures of self-gravitating protoplanetary discs

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ABSTRACT

We use high-resolution numerical simulations to study whether gravitational instabilities within circumstellar discs can produce astrometrically detectable motion of the central star. For discs with masses of $M_{\text{disc}} = 0.1 M_*$, which are permanently stable against fragmentation, we find that the magnitude of the astrometric signal depends upon the efficiency of disc cooling. Short cooling times produce prominent filamentary spiral structures in the disc, and lead to stellar motions that are potentially observable with future high precision astrometric experiments. For a disc that is marginally unstable within radii of ~ 10 au, we estimate astrometric displacements of $10\text{--}10^2 \mu\text{arcsec}$ on decade time-scales for a star at a distance of 100 pc. The predicted displacement is suppressed by a factor of several in more stable discs in which the cooling time exceeds the local dynamical time by an order of magnitude. We find that the largest contribution comes from material in the outer regions of the disc and hence, in the most pessimistic scenario, the stellar motions caused by the disc could confuse astrometric searches for low-mass planets orbiting at large radii. They are, however, unlikely to present any complications in searches for embedded planets orbiting at small radii, relative to the disc size, or Jupiter-mass planets or greater orbiting at large radii.

Key words: accretion, accretion discs – astrometry – stars: formation – planetary systems: protoplanetary discs – stars: pre-main-sequence.

1 INTRODUCTION

High-precision astrometry is a powerful tool to search for companions to nearby stars. It also has the potential to discover significant numbers of extrasolar planetary systems. In this paper, we discuss the potential of astrometry as a probe of self-gravitating discs around pre-main-sequence stars.

A gaseous disc, with sound speed c_s , surface density Σ and epicyclic frequency κ , is described as self-gravitating if the Toomre (1964) Q parameter,

$$Q = \frac{c_s \kappa}{\pi G \Sigma}, \quad (1)$$

is of the order of unity. In discs where self-gravity is important, the outcome can either be fragmentation into one or more bound objects, or a quasi-steady state in which gravitational instabilities lead to the outward transport of angular momentum. Local simulations suggest that the boundary between these possibilities is set by the ratio of the local dynamical time-scale Ω^{-1} to the time-scale on which the disc radiates thermal energy. Fragmentation occurs whenever the cooling time $t_{\text{cool}} \lesssim 3\Omega^{-1}$, while longer

cooling times lead to stable angular momentum transport (Gammie 2001).

Circumstantial evidence suggests that self-gravity could play a role in protoplanetary discs as late as the optically visible Classical T Tauri phase, which lasts for several Myr (Haisch, Lada & Lada 2001). Evidence that relatively old discs may be self-gravitating comes, first, from models of FU Orionis outbursts, which require a low efficiency of disc angular momentum transport to reproduce the observed $\sim 10^2$ yr time-scales. If the viscosity is parametrized using the Shakura & Sunyaev (1973) prescription, $\nu = \alpha c_s h$, where h is the vertical scale height, FU Orionis models suggest a quiescent $\alpha \sim 10^{-4}$ (Bell & Lin 1994; Bell et al. 1995). For a given accretion rate, small values of α imply high surface densities, so that the disc would be self-gravitating at $r \sim 1$ au. Secondly, theory suggests that angular momentum transport ought to be suppressed in cool regions of the disc where the gas is poorly coupled to magnetic fields (Matsumoto & Tajima 1995; Gammie 1996; Fleming, Stone & Hawley 2000; Sano et al. 2000; Reyes-Ruiz 2001; Sano & Stone 2002). Again, this suggests that self-gravity may set in at radii of a few au as the first significant non-magnetic source of angular momentum transport (Armitage, Livio & Pringle 2001). Ascertaining when self-gravity is at work within the disc requires either the observation of spiral patterns using extremely high-resolution imaging, or detection of the

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astrometric motion of the stellar photocentre induced by the self-gravitating disc. It has been shown (Adams, Ruden & Shu 1989) that self-gravitating perturbations with $m = 1$ can force the central star to move from the centre of mass.

In this paper, we use numerical simulations to quantify the magnitude of the astrometric displacement. This has previously been studied by Boss (1998), who simulated a disc with a mass of $\approx 0.2M_*$ and found a large motion of the star, of the order of 0.1 au. This corresponds to milliarcsecond (mas) displacements at the distance of the nearest star-forming regions, which would be easily detectable by any of the forthcoming high-precision astrometry experiments. The disc simulated by Boss (1998), however, was highly unstable, and subsequently fragmented with the formation of substellar objects. Although promising for giant planet formation (Armitage & Hansen 1999; Boss 2000), prompt fragmentation implies that extremely fortuitous timing would be needed for the astrometric detection of self-gravitating discs. We concentrate instead on marginally unstable discs, which are not vulnerable to fragmentation and could potentially exist around many Classical T Tauri stars.

2 NUMERICAL SIMULATIONS

2.1 Smooth particle hydrodynamics code

The three-dimensional simulations presented here were performed using smooth particle hydrodynamics (SPH), a Lagrangian hydrodynamics code; see, for example, Benz (1990) and Monaghan (1992). Our implementation allows for the inclusion of point masses and for point-mass creation (Bate, Bonnell & Price 1995). In this simulation, the central star is modelled as a point mass on to which gas particles can accrete if they approach to within the sink radius. Although point-mass creation is allowed, the discs considered here are stable against fragmentation and the density never reaches values high enough for the creation of a point mass within the disc itself. A great saving in computational time is made by using individual, particle time-steps (Bate et al. 1995; Navarro & White 1993). The time-steps for each particle are limited by the Courant condition and a force condition (Monaghan 1992). Both point masses and gas use a tree to determine neighbours and to calculate gravitational forces (Benz 1990).

An advantage of using SPH for this calculation is that it is not necessary to impose any outer boundary conditions, and the SPH particles are free to move to radii greater than the initial disc radius. The outer edge of the disc is therefore free to distort and modes with $m = 1$ will not be affected or artificially driven by the outer boundary conditions (Heemskerk, Papaloizou & Savonjie 1992).

2.2 Initial conditions

We consider a system comprising a central star, modelled as a point mass with mass M_* , surrounded by a gaseous circumstellar disc with a mass of $0.1M_*$. The disc temperature is taken to have an initial radial profile of $T \propto r^{-0.5}$ (e.g. Yorke & Bodenheimer 1999) and the Toomre Q parameter is assumed to be initially constant with a value of 2. A stable accretion disc, where self-gravity leads to the steady outward transportation of angular momentum, should have a near constant Q throughout. A constant Q together with equation (1) then gives a surface density profile of $\Sigma \propto r^{-7/4}$, and hydrostatic equilibrium in the vertical direction gives a central density profile of $\rho \propto r^{-3}$.

The disc is modelled using 250 000 SPH particles, which are initially distributed according to the specified density profile between

inner and outer radii of r_{in} and r_{out} , respectively. The actual particle positions are chosen randomly, subject to the constraint that the centre of mass (and momentum) of the disc is initially coincident with that of the star. To achieve this, we simply add particles in point symmetric pairs about the central point mass.

The calculations performed here are essentially scale-free. In code units, we take $M_* = 1$, $r_{\text{in}} = 1$ and $r_{\text{out}} = 25$. To simplify the discussion we generally assume a physical mass scale of $1 M_{\odot}$ and a length-scale of 1 au. The circumstellar disc therefore has a mass of $0.1 M_{\odot}$, extends from 1 au to 25 au, and surrounds a star of mass $1 M_{\odot}$.

2.3 Cooling

Previous work has shown that the outcome of gravitationally unstable discs depends critically on the treatment of the energy equation (Pickett et al. 2000; Gammie 2001). Our global simulations adopt the same approach as was used for local models by Gammie (2001). We use an adiabatic equation of state, with adiabatic index $\gamma = 5/3$, and we allow the disc gas to heat up as a result of both PdV work and viscous dissipation. In the absence of cooling, viscous dissipation would act to heat up the disc, increase the Toomre Q parameter, and drive the disc towards stability. Cooling is implemented by adding a simple cooling term to the energy equation. Specifically, for a particle with internal energy per unit mass u_i ,

$$\frac{du_i}{dt} = -\frac{u_i}{t_{\text{cool}}} \quad (2)$$

where t_{cool} is a radially-dependent parameter which we specify for each run.

Although simple, this approach to the energetics of the disc can be related (at least approximately) to the real physics of an accretion disc. For an optically thick disc in equilibrium, the cooling time is the ratio of the thermal energy per unit area to the radiative losses per unit area. It can be shown – see, for example, Pringle (1981) – that in such a viscous accretion disc, the cooling time is given by

$$t_{\text{cool}} = \frac{4}{9\gamma} \frac{1}{\alpha\Omega} \quad (3)$$

where γ is the adiabatic index, Ω is the angular frequency, and α is the Shakura & Sunyaev (1973) viscosity parameter. Based on this, we assume radially-dependent cooling times that scale as Ω^{-1} ,

$$t_{\text{cool}} = \beta\Omega^{-1}, \quad (4)$$

with β being a constant. Gammie (2001) has shown, using a local model, that cooling times of $t_{\text{cool}} < 3\Omega^{-1}$ lead to fragmentation, while longer cooling times lead to a quasi-stable state. Because we are interested in the possibility of detecting, through the displacement of the central star, the presence of gravitational instabilities in quasi-stable circumstellar discs, we use β values greater than 3.

3 ASTROMETRIC SIGNAL

We consider a $0.1 M_{\odot}$ disc with an inner radius of 1 au and an outer radius of 25 au. The disc is initially gravitationally stable ($Q \sim 2$ at all radii) and undergoes cooling with a radially-dependent cooling time given by $t_{\text{cool}} = \beta\Omega^{-1}$. We consider cooling times of 5 and $10\Omega^{-1}$ which, according to Gammie (2001), should lead to quasi-stable discs that do not fragment. The $t_{\text{cool}} = 10$ and $5\Omega^{-1}$ simulations were run for 2.6 and 3.7 orbits of particles at the outer disc edge at $r_{\text{out}} = 25$. These run times are short (326 yr and 467 yr respectively for our assumed scaling) compared to the evolutionary

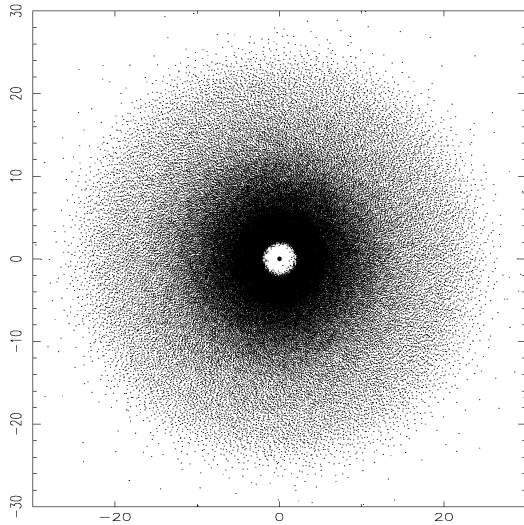


Figure 1. Face-on view of disc with $t_{\text{cool}} = 10\Omega^{-1}$ after a time of 326 yr. There is no noticeable structure in the disc.

time of the disc (i.e. the viscous time, r^2/ν) but are sufficient to reach a quasi-steady state in which heating and cooling are locally in balance.

For a cooling time of $10\Omega^{-1}$ (Fig. 1) there is no noticeable structure in the disc. For a cooling time of $5\Omega^{-1}$ (Fig. 2) there is significant structure in the disc but, as in Gammie (2001), there has been no fragmentation. The filamentary spiral structure which we observe is consistent (e.g. Nelson et al. 1998) with previous simulations of discs with masses significantly less than that of the central star. Although it is likely that the $m = 1$ mode is the dominant mode driving the stellar motion (Adams et al. 1989), it is not an obviously dominant mode in the disc.

Figs 3 and 4 show the surface densities of the $t_{\text{cool}} = 10$ and $5\Omega^{-1}$ simulations at the beginning and end of each run. Because we have

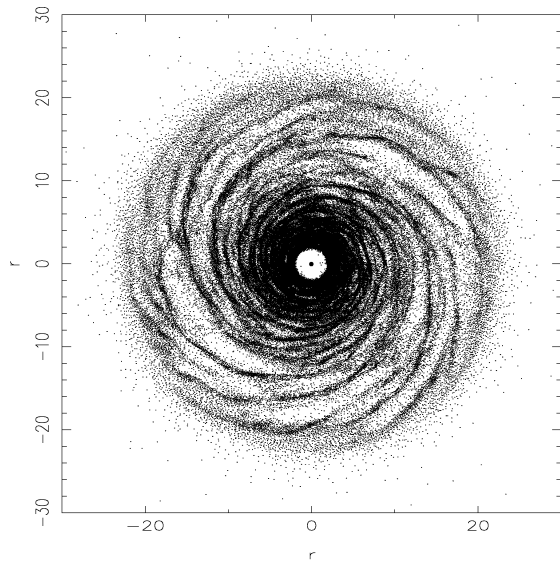


Figure 2. Face-on view of disc with $t_{\text{cool}} = 5\Omega^{-1}$ after a time of 467 yr. There is significant structure in the disc, but the maximum density at this time is not significantly different from the maximum density at the beginning of the simulation, and no fragmentation has occurred.

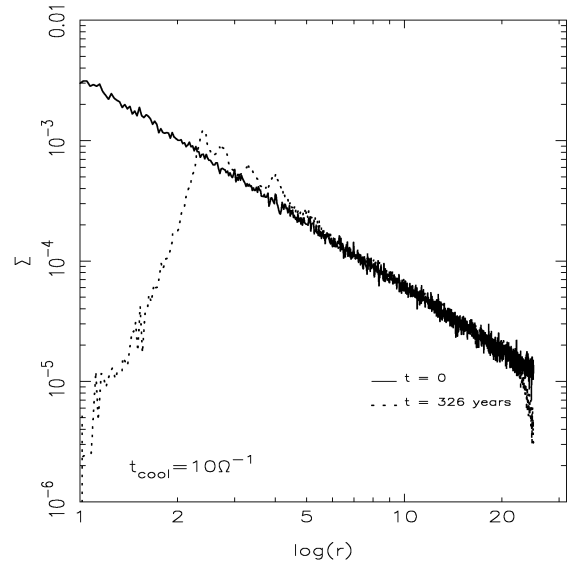


Figure 3. Surface density of disc with $t_{\text{cool}} = 10\Omega^{-1}$ at the beginning ($t = 0$) and end ($t = 326$ yr) of the simulation.

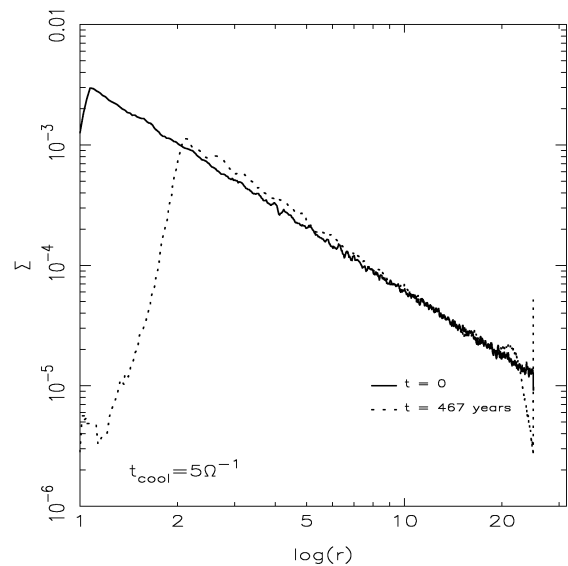


Figure 4. Surface density of disc with $t_{\text{cool}} = 5\Omega^{-1}$ at the beginning ($t = 0$) and end ($t = 467$ yr) of the simulation.

not attempted to model the inner boundary condition of the disc in any detail, there is rapid accretion and a drop in surface density close to the inner boundary. As discussed below, we find that the main contribution driving displacement of the star comes from material near the outer edge of the disc, so this numerical effect near the inner boundary is not a major concern. Apart from these particles with radii between 1 and 2 being accreted on to the central star, the surface density profile does not change significantly during the course of the simulation.

Figs 5 and 6 show the Toomre Q parameter at the beginning and end of each simulation. For $t_{\text{cool}} = 10\Omega^{-1}$ the value of Q at the end of the simulation is smaller than the initial value of 2 but still generally greater than 1. This disc is largely stable, although Q is small enough that some structure (not noticeable in Fig. 1) may exist. For $t_{\text{cool}} =$

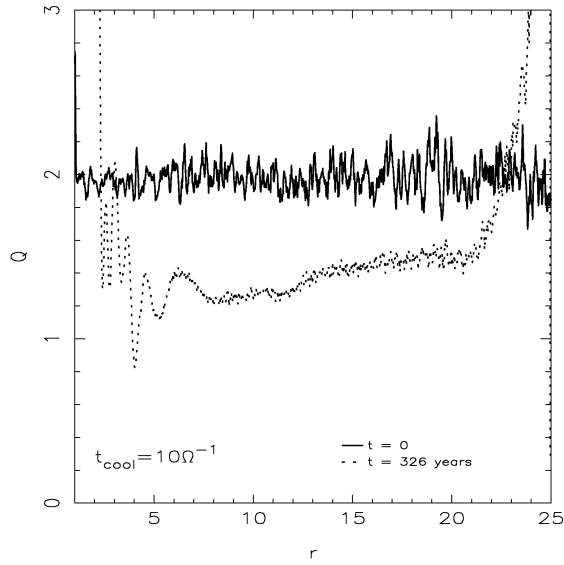


Figure 5. Toomre Q parameter for $t_{\text{cool}} = 10\Omega^{-1}$. At the beginning of the simulation ($t = 0$) Q has a constant value of 2. At the end ($t = 326$ yr) of the simulation Q largely lies between the initial value of 2 and the critical value 1.

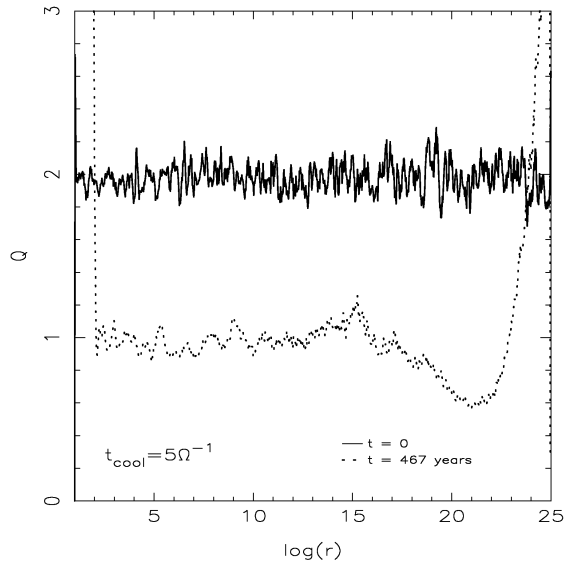


Figure 6. Toomre Q parameter for $t_{\text{cool}} = 5\Omega^{-1}$. At the beginning of the simulation ($t = 0$) Q has a constant value of 2. At the end ($t = 467$ yr) of the simulation Q has an almost constant value of 1 between radii of 2 and 15.

$5\Omega^{-1}$ the final Q value is close to 1 for radii between 2 and 15. The disc is quasi-stable with heating through gravitational instabilities balancing the cooling to give $Q \sim 1$.

Figs 7 and 8 show the displacement of the central star from the centre of mass of the star-disc system as a function of time, for the runs with $t_{\text{cool}} = 10$ and $5\Omega^{-1}$, respectively. The displacement grows approximately exponentially at early times, before saturating (Laughlin, Korchagin & Adams 1997) and reaching a plateau at a displacement which depends upon the assumed cooling time. For the more unstable disc, with $t_{\text{cool}} = 5\Omega^{-1}$, the saturation level of the

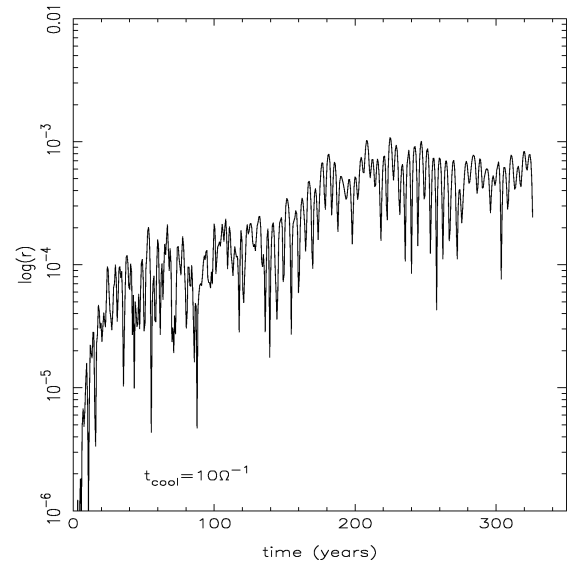


Figure 7. Time evolution of the distance of the central star from the centre of mass for $t_{\text{cool}} = 10\Omega^{-1}$. At the end of the simulation, the system seems to have settled into a steady state in which the central star is orbiting the centre of mass with an orbital radius of $\sim 10^{-3}$ radial units.

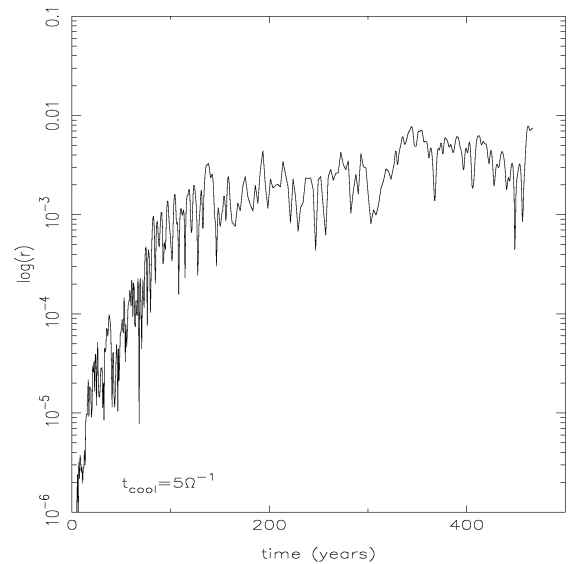


Figure 8. Time evolution of the distance of the central star from the centre of mass for $t_{\text{cool}} = 5\Omega^{-1}$. At the end of the simulation, the system seems to have settled into a steady state in which the central star is orbiting the centre of mass with an orbital radius of $\sim 5 \times 10^{-3}$ radial units.

stellar displacement is $\sim 5 \times 10^{-3}$ au, while for the more stable disc with $t_{\text{cool}} = 10\Omega^{-1}$ the plateau occurs at a lower level of $\sim 10^{-3}$ au. For a source at 100 pc, the above displacements will produce angular displacements of 0.1 and 0.02 mas, respectively. In both cases, the star is executing an approximately circular orbit around the centre of mass, with periods of approximately 50 yr and 35 yr for $t_{\text{cool}} = 5$ and $10\Omega^{-1}$, respectively.

Instead of the disc extending from 1 to 25 au, the scale-free nature of the simulation means that we could equally well assume,

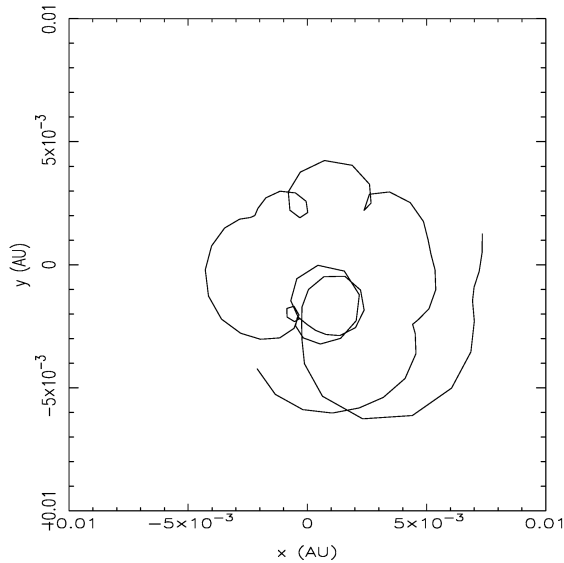


Figure 9. Projection of the position of the central star on to the orbital plane. The centre of mass is at $x = 0$, $y = 0$. The orbit is approximately circular and has substructure due to perturbations from higher-order modes.

without changing the mass scale, that it extends from 4 to 100 au, closer to a more commonly observed disc size (Padgett et al. 1999). The displacement of the central star from the centre of mass and the angular displacement would both be four times greater, and the period of the orbit would be eight times greater than that of a 25 au disc. A planet producing the same displacement with the same period as that calculated here would have a mass several times less than that of Jupiter. A Jupiter-mass planet producing the same displacement would be significantly closer to the central star and hence would have a correspondingly smaller period. It therefore seems unlikely (unless considering low-mass planets at large radii) that there could be much confusion between astrometric signals caused by planets and those resulting from disc instabilities.

In addition to a roughly circular motion, gravitational instability in the disc also generates lower amplitude, short-time-scale motions. This is illustrated in Fig. 9, which shows a projection of the position of the central star on to the orbital plane for the last 57 yr of the simulation with $t_{\text{cool}} = 5\Omega^{-1}$. The centre of mass is located at $x = 0$, $y = 0$. As well as the large orbit taking ~ 50 yr there are a number of smaller orbital structures as a result of the presence of higher-order modes in the disc.

For our surface density profile, most of the mass in the disc lies at relatively large radius (the enclosed mass scales as $r^{1/4}$). This behaviour is even stronger for the flatter surface density profiles (typically r^{-1} or $r^{-3/2}$) often considered as appropriate for protoplanetary discs. As expected given this mass distribution, the time-scale of the dominant circular motion of the star is of the same order as the dynamical time-scale in the *outer* disc. Assuming this to be true more generally, we can obtain estimates for the angular displacement $\Delta\theta$ and characteristic time-scale τ as a function of the outer radius, r_{grav} , within which a disc is self-gravitating. For a star at distance d we find

$$\Delta\theta \approx 100 \left(\frac{r_{\text{grav}}}{25 \text{ au}} \right) \left(\frac{d}{100 \text{ pc}} \right)^{-1} \mu\text{arcsec} \quad (5)$$

$$\tau \approx 50 \left(\frac{r_{\text{grav}}}{25 \text{ au}} \right)^{3/2} \text{ yr}, \quad (6)$$

where we have used the numbers from the $t_{\text{cool}} = 5\Omega^{-1}$ run for the estimate. We expect the exact numbers to depend upon the actual surface density distribution in the disc, and on the mass, so these figures should be regarded as order of magnitude estimates for discs with masses around a tenth of the mass of the star.

We can also compare the displacement caused by a gravitationally unstable disc with that generated by an orbiting planet within the disc. For a planet whose mass ratio to the central star is $q = M_p/M_*$, in a circular orbit at radius a , the displacement is

$$\Delta\theta = 50 \left(\frac{q}{10^{-3}} \right) \left(\frac{a}{5 \text{ au}} \right) \left(\frac{d}{100 \text{ pc}} \right)^{-1} \mu\text{arcsec}, \quad (7)$$

i.e. of similar magnitude to the result derived above. The *period* of the stellar oscillation caused by the planet, however, is substantially shorter than that generated by the disc. We conclude, therefore, that even in the worst case scenario where (i) the disc is gravitationally unstable, and (ii) the cooling time is within a factor of two of the fragmentation boundary, stellar displacements due to the disc are *not* likely to be confused with the signal from a Jupiter-mass planet. At substantially lower masses, of course, confusion would be possible. In this case, detection of the substructure in the orbit shown in Fig. 9 would be necessary to unambiguously determine whether an observed signal was of disc or planetary origin. The disc mass of $0.1M_*$ is also quite large, with most T Tauri stars having disc masses smaller than that used here (Beckwith et al. 1990). Not only will lower mass discs be less likely to be gravitationally unstable [see equation (1)], the amplitude of the displacement of the central star is likely to be smaller than that obtained here. This will further reduce the possibility of confusing stellar displacements resulting from the disc with that resulting from an orbiting planet.

4 CONCLUSION

We have used numerical simulations of discs around young stellar objects to quantify the astrometric displacement of the star caused by a self-gravitating disc. By modelling the energy balance of the disc using a cooling time formalism (Gammie 2001), we have shown that the magnitude of the displacement depends upon how stable the disc is against fragmentation into bound substellar objects. For a cooling time of $5\Omega^{-1}$ (within a factor of 2 of the fragmentation boundary at $\approx 3\Omega^{-1}$), a disc mass of $0.1M_*$, and a self-gravitating disc radius of the order of 10 au, we obtain relatively large displacements in the $10\text{--}10^2 \mu\text{arcsec}$ range (for a star at a distance of 100 pc). This magnitude of astrometric signal is potentially observable with any of several upcoming high-precision astrometric instruments, although the presence of a circumstellar disc is likely to complicate such observations, especially if time-dependent perturbations are present within the disc. Although the gravitationally driven instabilities modelled in this work are intrinsically time-dependent, their growth saturates (Laughlin et al. 1997) and their filamentary nature (Nelson et al. 1998) makes their structure approximately azimuthally symmetric. Their presence is therefore unlikely to further complicate observations. Discs with cooling times longer than $5\Omega^{-1}$, which are correspondingly more stable, have significantly smaller stellar motions. A detailed model for the angular momentum transport processes in the disc, together with an assessment of the heating and cooling processes at work, would be required to determine whether protoplanetary discs fall into the parameter regime that produces the largest displacements.

Our estimate for the angular displacement of the star caused by a self-gravitating disc is an order of magnitude smaller than that found by Boss (1998). Although there are numerous differences in the initial conditions and numerical techniques used, we believe that the smaller displacement that we obtain primarily reflects our choice of a more stable disc model. By design, our disc is permanently stable against fragmentation into substellar objects, whereas the disc simulated by Boss (1998) broke up towards the end of the simulation. Taken together, our results, plus those of Boss (1998), suggest that a relatively narrow window of self-gravitating disc conditions could lead to small but long-lived astrometric wobbles ($10\text{--}10^2 \mu\text{arcsec}$ at 100 pc). More dramatic displacements – perhaps of the order of a mas at the same distance – are also possible, but only as a precursor to fragmentation of the disc into substellar objects. Our simulations suggest that the disc-induced wobble could mimic that of a planet a few times smaller than Jupiter and orbiting at a large radius, relative to the disc size. Most T Tauri stars have disc masses smaller than that used in this simulation and would consequently induce a correspondingly smaller motion of the central star, reducing the likelihood of confusion between stellar motion resulting from disc instabilities and that resulting from an orbiting planet. The disc-induced stellar motions are therefore not likely to be a serious obstacle to the astrometric detection of planets orbiting at small radii (relative to the disc size). For planets orbiting at large radii, the planet mass which could be mimicked by a disc instability depends on the disc mass (relative to the central star) but is likely to be less than a Jupiter mass for most T Tauri disc systems.

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