

ON THE FORMATION TIMESCALE AND CORE MASSES OF GAS GIANT PLANETS

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ABSTRACT

Numerical simulations show that the migration of growing planetary cores may be dominated by turbulent fluctuations in the protoplanetary disk, rather than by any mean property of the flow. We quantify the impact of this stochastic core migration on the formation timescale and core mass of giant planets at the onset of runaway gas accretion. For standard solar nebula conditions, the formation of Jupiter can be accelerated by almost an order of magnitude if the growing core executes a random walk with an amplitude of a few tenths of an AU. A modestly reduced surface density of planetesimals allows Jupiter to form within 10 Myr, with an initial core mass below $10 M_{\oplus}$, in better agreement with observational constraints. For extrasolar planetary systems, the results suggest that core accretion could form massive planets in disks with lower metallicities, and shorter lifetimes, than the solar nebula.

Subject headings: accretion, accretion disks — planetary systems: formation — planets and satellites: formation — planets and satellites: individual (Jupiter) — solar system: formation

1. INTRODUCTION

The core accretion model (Mizuno 1980; Pollack et al. 1996) provides the most popular explanation for the origin of the solar system’s gas giants and is consistent with the higher frequency of extrasolar planets found to be orbiting metal-rich stars (Laughlin 2000; Murray & Chaboyer 2002; Santos et al. 2003). In the simplest version of this model, a core grows at a fixed orbital radius from the binary accretion of solid planetesimals (Safranov 1969). Initially, this core is surrounded by a near-hydrostatic gaseous envelope, with most of the luminosity being provided by ongoing planetesimal accretion. Growth continues until a critical mass is exceeded. Above the critical mass there is no stable core-envelope solution, and more rapid accretion of the bulk of the planetary envelope ensues.

Observational constraints from the solar system pose two possible problems for core accretion models. First, although Jupiter can form within the lifetimes of protoplanetary disks (Haisch, Lada, & Lada 2001), it is hard to form Uranus and Neptune in their present locations rapidly enough. This has prompted suggestions that the outer giant planets may have migrated outward from birthplaces closer to the Sun (Thommes, Duncan, & Levison 1999). Second, upper limits to the core mass of Jupiter, derived from Galileo data, are *smaller* than most theoretical estimates. Guillot (1999) obtains a firm constraint of $M_{\text{core}} \leq 14 M_{\oplus}$, which is reduced to $10 M_{\oplus}$ using a more model-dependent approach. This is only marginally consistent with the 10–30 M_{\oplus} core predicted by Pollack et al. (1996) and has led to renewed interest in models for massive planet formation via disk instability (Boss 1997).

The possibility that orbital migration (Goldreich & Tremaine 1980) of planetary cores might reduce the accretion timescale and ameliorate these problems was recognized by Hourigan & Ward (1984). In a laminar disk flow, however, gravitational torques from the disk induce a rapid, uniformly inward drift (Ward 1997). The benefit of a higher accretion rate must there-

fore be balanced against the reduced residence time of the planet in the disk. In this Letter, we point out that this trade-off may not be necessary in a turbulent disk. Numerical simulations (Laughlin, Steinacker, & Adams 2003; Nelson & Papaloizou 2003a) of the migration of low-mass planets within turbulent magnetized disks (which extend work by Nelson & Papaloizou 2003b; Winters, Balbus, & Hawley 2003) show that for low enough masses, the *sense* as well as the rate of migration is determined by turbulent fluctuations in the disk. As a result, the planet random walks in orbital radius as it grows. This behavior was suggested as a likely consequence of migration in a magnetized disk by Terquem (2003). Here we quantify how random walk migration affects massive planet formation.

2. METHOD AND ASSUMPTIONS

We begin our calculations by assuming that a $0.1 M_{\oplus}$ solid core, with density $\rho_{\text{core}} = 3.2 \text{ g cm}^{-3}$, has formed at 5 AU in a disk with a local surface density of planetesimals of $\sigma_p = 10 \text{ g cm}^{-2}$ and a surface density profile of $\sigma_p \propto r^{-3/2}$. According to Safranov (1969), the solid core grows at the rate $\dot{M}_{\text{core}} = \pi R_c^2 \sigma_p \Omega F_g$, where R_c is the effective, or capture, radius, Ω is the angular frequency, and F_g is the gravitational enhancement factor (Greenzweig & Lissauer 1992). We denote the total planet mass (core plus envelope) by M_p .

The planetesimal accretion produces a core luminosity of $L_{\text{core}} = GM_{\text{core}}\dot{M}_{\text{core}}/R_{\text{core}}$. With this we can construct quasi-equilibrium models of the envelope using standard stellar structure equations. The equations of hydrostatic equilibrium, mass conservation, and radiative transfer are

$$\frac{dP}{dR} = -g\rho, \quad (1)$$

$$\frac{dM}{dR} = 4\pi R^2 \rho, \quad (2)$$

$$\frac{dT}{dR} = \frac{-\nabla GM_p T}{R^2 P} \quad (3)$$

(Papaloizou & Terquem 1999; Hansen & Kawaler 1994). Here P is the pressure, ρ is the density, T is the temperature, R is

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the distance from the planet center, g is the acceleration due to gravity, M is the mass enclosed within R , and ∇ is the smaller of the radiative or adiabatic temperature gradients. The inner boundary is located at the core radius, and, as in Pollack et al. (1996), the outer boundary is located at the smaller of the Hill radius, $R_{\text{Hill}} = a(M_p/3M_{\text{star}})^{1/3}$, or the accretion radius, $R_a = GM_p/c_s^2$, where a is the planet's orbital radius and c_s is the local gas sound speed in the disk. The envelope is taken to be a hydrogen and helium mixture, with mass fractions of 0.7 and 0.28, respectively, and with an equation of state given by Chabrier et al. (1992). Opacities from Bell & Lin (1994) are used in calculating the radiative temperature gradient. The adiabatic temperature gradient is found from the equation of state tables (Chabrier et al. 1992).

To solve equations (1)–(3), we need to calculate the planetesimal accretion rate, which depends on σ_p , F_g , and R_c . The planetesimal surface density—which is more precisely the time-dependent mean surface density within the planet's feeding zone—and the gravitational enhancement factor both depend only on M_p and can be calculated using expressions given by Pollack et al. (1996) and Greenzweig & Lissauer (1992). For the capture radius we adopt a simpler approach than Pollack et al. (1996) and take R_c to be the radius at which a planetesimal with a radius of 100 km, falling radially through the planet's envelope, has intercepted a mass of 1/10 of its own mass. Since this depends on the planet's structure, we iterate at each time step to find a self-consistent solution. If more than one solution is possible (with different planet masses), we choose between them by requiring that the total planet mass cannot decrease with increasing time. We then advance to the next time step by incrementing the core mass and by updating $\sigma_p(r)$ to account for the mass of planetesimals accreted on to the core in the current time step. We neglect both the shepherding effect of a migrating core upon the planetesimals (Tanaka & Ida 1999), which would reduce the accretion rate, and the possibility of diffusion of the planetesimals due to the same fluctuating potential that causes core migration (which might increase the accretion rate).

3. RESULTS

Our model for planetary growth is a slightly simplified version of that used by Pollack et al. (1996). If we assume, as they did, that the core remains at a fixed radius, almost identical growth rates are obtained. Figure 1 shows the core mass and total planet mass against time for a core fixed in radius at 5 AU. There is an initially rapid growth phase (their phase 1), in which the core grows to a mass of $\sim 10 M_{\oplus}$ in less than 1 Myr, followed by a second phase, (phase 2) in which the core grows slowly to a mass of $\sim 17 M_{\oplus}$. Once the core reaches this mass, the gaseous envelope grows extremely rapidly (phase 3), producing a final planet mass, in this case, of $\sim 100 M_{\oplus}$ and a total growth time of just over ~ 7 Myr.

If the disk at 5 AU is turbulent,⁴ the latest simulations of migration suggest that the core may actually undergo a random walk in orbital radius. After N steps of size Δa , the typical distance moved will be $\Delta a\sqrt{N}$. We find that the most interesting behavior occurs for planets that wander by a few tenths of an AU per megayear, which can be accomplished by moving the

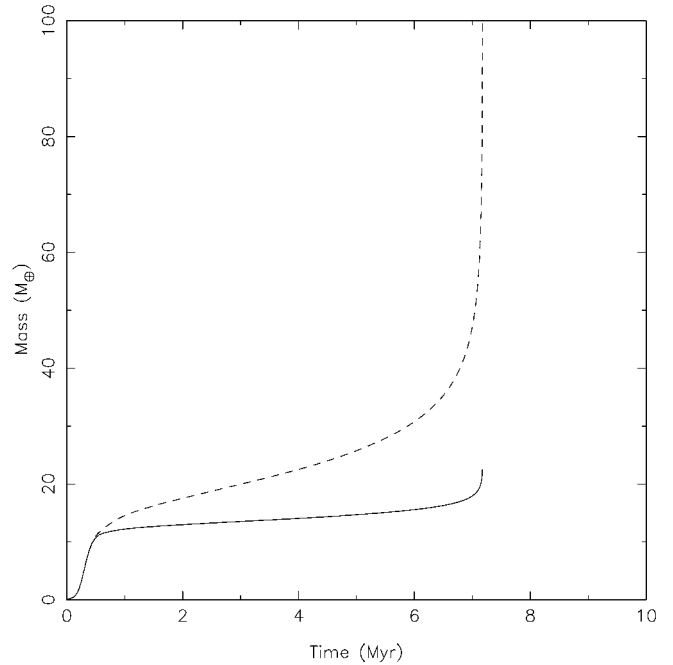


FIG. 1.—Core mass (solid line) and total planet mass (dashed line) for a core fixed in radius at 5 AU in a disk with $\sigma_p = 10 \text{ g cm}^{-2}$.

core either in or out by a distance $\Delta a = 0.01$ AU every $dt = 2000$ yr, with the direction of motion determined randomly. This “coarse-grained” approach ignores smaller time-scale fluctuations, but we have verified that including these (while keeping the overall amount of migration constant) does not substantially alter the results. Figure 2 shows the core mass and total planet mass as a function of time for a simulation in which the core started at 5 AU but then random walked through the disk. We also plot the migration track of the core for this specific realization. The onset of rapid growth of the envelope occurs in a time of less than 1 Myr, an order of magnitude less than that obtained for a core fixed in radius.

These results can be understood in terms of the relationship between M_{core} and M_p (Papaloizou & Terquem 1999). For a given planetesimal accretion rate there is a critical core mass M_{crit} , above which the core cannot support a stable envelope. Below this mass there are generally two solutions, a high-mass and low-mass solution. For $M_{\text{core}} \ll M_{\text{crit}}$, the low-mass solution has a small envelope mass, so that $M_p \approx M_{\text{core}}$. As the core mass approaches M_{crit} , the envelope mass increases, ultimately contributing $\sim 30\%$ of the total planet mass. Moreover, the critical mass increases with increasing planetesimal accretion rate. As a result, for a given core mass, the low-mass solution decreases with increasing accretion rate, while the high-mass solution increases with increasing accretion rate.

When the core is fixed in radius, the planetesimal accretion rate initially increases with time as the core and planet grow. This growth, however, depletes the planetesimals in the feeding zone (Pollack et al. 1996), resulting in the accretion rate reaching a peak and then declining to an approximately constant value. For the parameters of the calculation shown in Figure 1, this plateau is at $(7\text{--}8) \times 10^{-7} M_{\oplus} \text{ yr}^{-1}$. At this stage the core mass is $\sim 10 M_{\oplus}$, well below the critical core mass for this planetesimal accretion rate of $\sim 17\text{--}18 M_{\oplus}$ (Papaloizou & Terquem 1999). The core and planet therefore grow slowly,

⁴ There is uncertainty as to whether magnetohydrodynamic turbulence can be sustained at radii of a few AU because of the low-ionization fraction near the disk midplane (Gammie 1996). The disk may only be fully turbulent at late epochs, when the gas surface density is low enough that cosmic rays can ionize the interior.

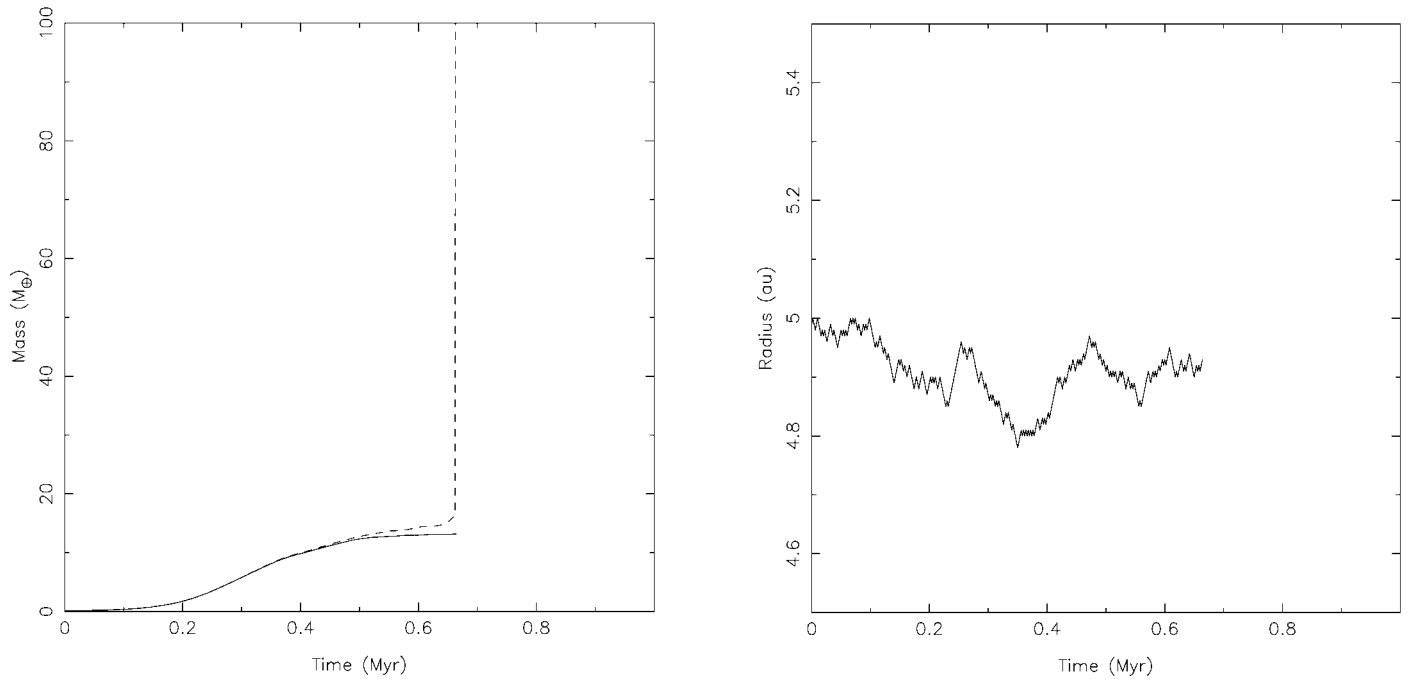


FIG. 2.—*Left*: Core mass (solid line) and total planet mass (dashed lines) for a core that is allowed to random walk through the disk. *Right*: Specific migration track for this realization of the model.

taking $\sim 6\text{--}7$ Myr, until the core mass nears the critical core mass. Once at the critical core mass, the low-mass envelope solution is no longer valid. However, since the accretion rate depends on the total planet mass, a high-mass solution is possible. The envelope mass is therefore determined by the high-mass solution, and the planet and core grow rapidly. This halts when the planetesimal accretion rate saturates and the total planet mass exceeds the high-mass solution for that accretion rate. This occurs, in this case, at a total planet mass of $\sim 100 M_{\oplus}$.

When the core is able to random walk through the disk, the result is quite different. The random walk allows the core to not only sample a larger region of the disk but also prevents a single region from being depleted to the same extent as when the core is stationary. The planetesimal accretion rate can therefore be maintained at a much higher value, and the core grows more rapidly (Hourigan & Ward 1984; Ward 1989). This is a generic feature of *any* migration model, and it may be offset, in part, by the tendency of a migrating core to shepherd planetesimals away from the feeding zone (Tanaka & Ida 1999). What is more novel about the random walk case is that the planetesimal accretion is also highly variable, and this is what ultimately leads to the rapid growth phase in this scenario.

At early times, as in the stationary case, the core mass is significantly lower than the local M_{crit} , the total planet mass is dominated by the core, and the planet grows via the accretion of planetesimals. In this regime, when M_{core} is less than about $10 M_{\oplus}$, the variable accretion rate occasioned by migration simply means that the planet grows at a variable rate. If, however, the core mass approaches the local critical core mass, the envelope mass can become quite significant. Once M_{core} reaches $10\text{--}12 M_{\oplus}$, it is possible for the core to migrate into a region in which the accretion rate is low enough for the core mass to be reasonably close to the local critical core mass. The planet then acquires an envelope that contributes a nonnegligible por-

tion of the total planet mass. If, subsequently, the core random walks into a region in which the accretion rate is considerably higher, the low-mass stable envelope solution (Papaloizou & Terquem 1999) would again have the planet mass dominated by the core. This would, however, require a decrease in the total planet mass. Such a decrease is unlikely to be physical, since it would require shedding a bound envelope whose binding energy has presumably already been radiated as an additional contributor to the planetary luminosity. Accordingly, we assume that the low-mass solution remains inaccessible, in which case the envelope grows rapidly, reaching, in this case, a mass of $\sim 200 M_{\oplus}$.

Within this model, the core mass at which the rapid growth phase occurs depends on the specific random walk of the core. If the core is able to stay in a region of high accretion rate for a long time, the critical core mass will be higher than if the core, fairly early on, random walks into and then out of a region with a low accretion rate. From additional runs, we find typical cores masses are between 12 and $15 M_{\oplus}$. We have also experimented with altering the rate of migration. If the planet executes a random walk with a larger amplitude—as some of the simulations suggest—the end result is rapid formation of a gas giant with a massive core. For example, a run with $\Delta a = 0.02$ AU and $dt = 10^2$ yr led to rapid accretion after 0.5 Myr, with a core mass of $25 M_{\oplus}$. We expect qualitatively similar results if the migration includes a steady radial drift in addition to the fluctuating component, since steady drift would similarly allow the core to sample a very wide range of disk radii.

These results suggest that under standard solar nebula conditions we can form Jupiter rather rapidly, but with a core mass that may still be too large (Guillot 1999). However, since the derived formation time for Jupiter at 5 AU is smaller than the lifetime of the solar nebula, it is possible to trade off some of the reduction in formation time for a smaller core mass. Fig-

4. DISCUSSION

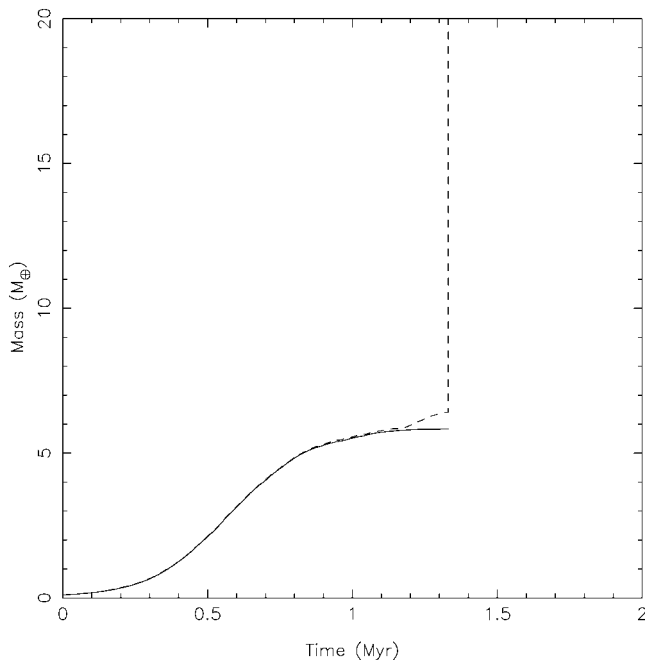


FIG. 3.—Core mass (solid line) and total planet mass (dashed line) for a core allowed to random walk through a disk with an initial planetesimal surface of $\sigma_p = 5 \text{ g cm}^{-1}$. Reducing σ_p leads to a smaller core mass at the conclusion of the calculation, while still allowing planet formation within the typical lifetime of protoplanetary disks.

ure 3 shows that if we reduce the initial planetesimal surface density at 5 AU to 5 g cm^{-2} , the lower time-averaged accretion rate allows the onset of rapid growth to occur at a smaller final core mass. In this model the growth time is now longer ($\sim 1.5 \text{ Myr}$), but the core mass at the end of the calculation has been reduced to only $\sim 6 M_{\oplus}$. Depending on the additional mass of heavy elements accreted during the subsequent phases of growth, this could be consistent with the apparently low core mass of Jupiter.

If gas giants form within turbulent regions of the protoplanetary disk, numerical simulations (Nelson & Papaloizou 2003a; Laughlin et al. 2003) show that fluctuating disk torques can cause growing cores to wander in orbital radius. We have incorporated a simple treatment of this random walk migration into models for the formation of gas giants via core accretion, and we find that for standard solar nebula conditions the formation timescale of Jupiter can be reduced by almost an order of magnitude. Potentially, this could allow massive planets to form via core accretion at greater orbital radii, or in disks with smaller planetesimal surface densities, than previously suspected. For the solar system, a modestly *smaller* surface density of planetesimals (5 g cm^{-2}) allows for the timely formation of Jupiter with an initial core mass less than $10 M_{\oplus}$, in better agreement with observational constraints.

For extrasolar planetary systems the main implication is that for protoplanetary disks of modest mass and solar metallicity, the formation timescale for giant planets at 5 AU is significantly less than the typical disk lifetime (Haisch et al. 2001). At smaller radii—closer to the snow line (Sasselov & Lecar 2000)—the timescale would be shorter still. This implies that giant planets could form in less favorable conditions, either in clusters in which the disk lifetime was shorter or around lower metallicity stars with smaller reservoirs of planetesimals (although if planets are common in M4, as suggested by Sigurdsson et al. 2003, their formation by core accretion is still problematic). The observed preponderance of metal-rich stars as planetary hosts may then arise from a combination of an enhanced *probability* of forming multiple planets at high metallicity, coupled with frequent destruction of planets via type II inward migration (Armitage et al. 2002; Trilling, Lunine, & Benz 2002).

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