FoMP: Vectors, Tensors and Fields 2009/2010

Problem Sheet 2: vector geometry and vector angular momentum

2.1 Show that the perpendicular distance of the point with position vector $\underline{d}$ from the plane $\underline{r} \cdot \underline{a} = b$ is $|b - \underline{d} \cdot \underline{a}|/a$.

2.2 Solve the following equations for $\underline{r}$, and give a geometrical interpretation to your result: $\underline{a} \cdot \underline{r} \; = \; \ell \quad ; \quad \underline{b} \times \underline{r} \; = \; \underline{c}$, where $\underline{a} \cdot \underline{b} \neq 0$.

2.3 Show that the solution of the vector equation

$$k\underline{r} + (\underline{r} \times \underline{a}) \;=\; \underline{b} \, ,$$

where $k \neq 0$, is

$$\underline{r}(k^2 + a^2) \;=\; (\underline{a} \times \underline{b}) + k\underline{b} + \frac{(\underline{a} \cdot \underline{b})\underline{a}}{k} \; .$$

[Hint: Take vector and scalar products of the original equation with $\underline{a}$]

2.4 (i) A system of particles is subject to rotation described by an angular velocity vector $\underline{\omega}$. Draw a diagram to show that the velocity vector of a particle at position $\underline{r}$ is $\underline{v} = \underline{\omega} \times \underline{r}$, and show that this rotational velocity is perpendicular to the radius vector.

(ii) Any given velocity vector can be decomposed into rotational and radial components: write down expressions for these two components.

(iii) A particle rotates with period T in a circle of radius R in the $x - y$ plane, with the centre of the circle at $(x, y) = (X, 0)$. Write down an explicit expression for $\underline{r}(t)$; hence obtain the velocity $\underline{v}(t)$. What is the rotational component of the velocity vector?