

STATISTICAL PHYSICS

Ising Model, Landau Theory, Universality

Tutorial Sheet 7

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

7.1 Order parameter of an antiferromagnet [s]

The simplest representation of an antiferromagnet is afforded by the Ising model with a nearest neighbour interaction parameter J which is *negative*.

- By considering a one-dimensional system identify the two possible ground states and hence the two phases which one might expect to coexist at low enough temperatures. Find a suitable order parameter which differentiates between the two low temperature phases and is 0 in the high temperature disordered phase
- Generalise to a three-dimensional system

7.2 Ising Model Representation of Binary Alloy [s]

The simplest model of a binary alloy envisages an assembly consisting of $N/2$ atoms of species A and $N/2$ atoms of species B, distributed over sites forming a body centred cubic lattice. The energy of interaction of neighbouring pairs of atoms is ϵ_{AA} , ϵ_{AB} or ϵ_{BB} according to the pair involved. Show that the canonical partition function is that of the Ising model in zero field and with interaction parameter

$$J = -\frac{1}{4}(\epsilon_{AA} + \epsilon_{BB} - 2\epsilon_{AB})$$

7.3 Mean-Field Theory of Ising Model Revisited [s]

- Review the derivation given in lectures of a mean field free energy function per spin

$$f(m) = -\frac{zJ}{2}m^2 - hm + kT \left[\left(\frac{1+m}{2} \right) \ln \left(\frac{1+m}{2} \right) + \left(\frac{1-m}{2} \right) \ln \left(\frac{1-m}{2} \right) \right]$$

- Show that minimising $f(m)$ leads to the Weiss mean-field equation for the magnetisation
- Expand $f(m)$ in powers of m keeping terms up to order m^4 . You will need the expansion

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

7.4 Landau Theory [r/s]

Check and complete the Landau theory calculations, given in lectures, for the critical exponents β , γ , δ and α of the Ising model. For the latter you will need to first show the result $c_h = -T(\partial^2 f / \partial T^2)_h$.