STATISTICAL PHYSICS

Radial distribution function; Debye-Hückel Theory **Tutorial Sheet 5**

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

5.1 **Debye screening length** [**r**]

In the Debye Hückel theory one represents the density of free charges by a continuous distribution $n(r)$, which is a smooth function. This is a 'continuum approximation' as it ignores the granularity of charges. Show that the continuum approximation is valid provided that

$$q^3 n^{\frac{1}{2}} \left(\frac{\beta}{\epsilon}\right)^{\frac{3}{2}} \ll 1$$

where $\beta = 1/kT$ and n is the density of particles. What does this correspond to physically?

Hint: You should compare the microscopic length (typical separation between charges) ignored in the continuum approximation to the lengthscale predicted by the theory (Debye screening length).

5.2 **Multi species plasma** [**s**] Consider a plasma containing more than one species of mobile ion, with no fixed charges but overall charge neutrality (for example, an ionized gas). By considering the distribution of ions around a point charge (of arbitrary sign) show that the Debye screening length λ_D obeys

$$\lambda_D^{-2} = \frac{\sum_i q_i^2 n_i}{\epsilon k T}$$

where q_i is the charge of species i and n_i is the particle density (at infinity) of species i.

5.3 **Radial distribution Function** [**s**] For a plasma containing two ionic species with opposite charges but the same density $n(\infty)$, calculate the radial distribution functions $g_{++}(r)$, $g_{-+}(r)$, where $n(\infty)g_{ij}(r)$ is the conditional probability density for finding a particle of type i in a small volume at distance r from one of type j. You may assume Debye-Hückel theory is valid and should use the results of the previous question.

5.4 **Semi-infinite plasma** [**s**] A semi-infinite sample of a one-component plasma ends at a flat wall which carries a positive surface charge density σ. The one-component plasma is made up of free positive charges of far concentration $n(\infty)$ and a fixed background concentration $n(\infty)$ of negative charges. Assuming the Debye-Hückel equation applies, calculate and sketch the potential ϕ as a function of z, the distance from the wall. Sketch also, on the same horizontal axis, the density of free charges in the plasma.

5.5 **Virial equation of state** [**c**] Complete the proof, outlined in the notes, of the virial equation of state

$$P = \rho kT - \frac{\rho^2}{6}\int_0^\infty \left(r\frac{d\phi}{dr}\right) g(r)\, 4\pi r^2 dr$$

where $g(r)$ is the radial distribution function and $\phi(r)$ is the interatomic potential and here ρ is the particle density.