MPhys Radiation and Matter 2016/17

Tutorial questions 4

(1) An interstellar nebula emits Lyman alpha radiation with a peak brightness temperature of 30 K at the line centre, and the line is observed to have a full width at half maximum of 10 $\mathrm{km\,s^{1}}$. Assuming Doppler broadening, show that there are two temperatures for the nebula that can account for these observations, and calculate the optical depth in each case. Suggest ways by which these different explanations might be distinguished.

(2) A simplified galaxy model consists of a uniform slab of stars mixed with absorbing material (dust). Use the equation of radiative transfer to calculate the surface brightness observed as a function of inclination and show that it is independent of inclination in the limit that the amount of dust becomes very large. Specifically, let the slab have a thickness L, and let τ denote the optical depth for a light ray passing perpendicularly through the slab. Assume the emissivity of starlight, ϵ, and the density of dust to be constant throughout the slab.

How does the surface brightness vary with inclination in the optically thin limit? Show that this is consistent with conservation of energy.

Radiative transfer can also include scattering, which conserves photons but redirects them. Explain why the equation of radiative transfer would be harder to solve in this case. Consider the case where the above slab is optically thick due to scattering, and assume that the result is isotropic emergent radiation. Use conservation of energy to calculate the surface brightness.

(3) Define collision cross section σ, collision rate coefficient $\langle \sigma v \rangle$ and critical density $n_{\rm crit}$.

Prove that in a gas of two-level atoms with a Maxwellian velocity distribution the rate coefficient for upward collisions is equal to that for downward collisions multiplied by the Boltzmann ratio of level occupancy. Hence show that the ratio of level occupancies as a function of density and temperature is given by

$$\frac{n_{\rm U}}{n_{\rm L}} = \frac{(g_{\rm U}/g_{\rm L}) \exp[-(E_{\rm U} - E_{\rm L})/kT]}{1 + n_{\rm crit}/n} .$$

This gas is now irradiated with an isotropic flux corresponding to a photon state occupation number N_k at the relevant frequency. Assuming $g_{\rm U} = g_{\rm L}$, show that there are approximately equal numbers of atoms in the upper and lower states when either $N_k \gg 1$, and $N_k \gg n/n_{\rm crit}$ or $n \gg n_{\rm crit}$ and $kT \gg \Delta E$.