MPhys Radiation and Matter 2016/17

Tutorial questions 3

(1) (a) Define specific intensity of radiation, I_ν. For unpolarized radiation, show that specific intensity is related to photon occupation number, $N_{\mathbf{k},\alpha}$, via

$$I_\nu = (4\pi\hbar\nu^3/c^2)\, N_{\mathbf{k},\alpha}.$$

Confirm that $N_{\mathbf{k},\alpha} = 1/(e^{h\nu/kT} - 1)$ in black body radiation, and thus that its specific intensity is $B_\nu(T) = (2h\nu^3/c^2)/(e^{h\nu/kT} - 1)$ and its energy density is $(8\pi h\nu^3/c^3)/(e^{h\nu/kT} - 1)$.

(2) Derive the equation of radiative transfer in the form $dI_\nu/d\ell = -\kappa_\nu I_\nu + \mathcal{E}_\nu$. If the radiation passes through material with two non-degenerate energy levels, discuss the contribution of stimulated and spontaneous transitions to the opacity and emissivity. Under what circumstances will I_ν be constant along light rays?

(3) The equation of radiative transfer says that specific intensity is unchanged along light rays in empty space, so that e.g. the observed surface brightness of the Sun is independent of the distance to the observer. Show that this is consistent with the inverse-square law.

(4) Show that if a source is masering, its level populations appear to be those at equilibrium at a negative temperature. Show that the specific intensity in such a source grows initially exponentially; what limits this growth?

(5) Show that the brightness temperature (the temperature of a black body giving an identical specific intensity) of a masering source can be *much* greater than the kinetic temperature in the source.

(6) A non-masering source emits photons from a transition between two levels, U and L. The levels have populations in the ratio $R = n_{\rm U}/n_{\rm L}$. Show that the specific intensity cannot exceed the black body specific intensity at a temperature at which the levels would be at thermodynamic equilibrium in the ratio R, *ie* at a temperature given by $(E_{\rm U} - E_{\rm L})/kT = -\ln(Rg_{\rm L}/g_{\rm U})$ [Hint: write down the specific intensity in the optically thick case, and express $\mathcal{E}_\nu$ and κ_ν in terms of $n_{\rm U}, n_{\rm L}$].

(7) A line is emitted from atoms in a gas at temperature T which collisionally excites the levels U, L to Boltzmann equilibrium. Describe the changing shape of the line profile as the optical depth at the centre of the line is increased from small values.