MPhys Radiation and Matter 2016/17

Tutorial questions 2

(1) Verify the result for the oscillator Hamiltonian in section 1.5 of the notes.

(2) Argue that the local unobservability of phase in quantum mechanics requires the introduction of the electromagnetic field via a modification of derivatives: $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$, where q is the charge on a particle and $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$ is the momentum operator. Show how this yields the interaction Hamiltonian $H' = -(q/m)\mathbf{A} \cdot \mathbf{p}$.

(3) [harder] Show that if you treat the radiative interaction Lagrangian as involving a potential energy $V = q\phi - q\mathbf{v} \cdot \mathbf{A}$, then Euler's equation gives the expression for the Lorentz force in the form

$$\frac{d}{dt}(\mathbf{p} - q\mathbf{A}) = q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \; ,$$

where the generalized momentum is $\mathbf{p} = m\mathbf{v} + q\mathbf{A}$.

(4) Write down the matrix element that governs first-order atomic transitions, according to the golden rule. Show that, in the dipole approximation, this requires the matrix element of $\mathbf{p}$, and show how this can be expressed in terms of the matrix element of the position vector, $\mathbf{r}$.

(5) At what wavelengths will the approximation in which the electric dipole transition rate dominates cease to be good? (assuming a non-zero result from selection rules).

(6) According to the Golden Rule, the semiclassical transition rate between two states can be written as

$$\Gamma = \frac{\omega \, d\Omega}{2\pi\hbar c^3 m^2} \, N_{\mathbf{k},\alpha} \, \frac{e^2}{4\pi\epsilon_0} \, |M_{\mathrm{YX}}(\mathbf{k}, \alpha)|^2 \, .$$

Explain the meaning of the terms in this expression, and say how it is modified in the fully quantum case, especially considering spontaneous transitions.

(7) An electron is confined within an infinitely high cubical potential barrier of side L, and is placed in one of the first excited states. Use the dipole approximation to calculate the rate at which the electron makes spontaneous radiative transitions to the ground state (you may assume that $\int_0^\pi \theta \sin\theta \sin 2\theta \, d\theta = -8/9$).