Fourier Analysis

Workshop 1: Fourier Series

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1. By writing $\sin A$ and $\cos B$ in terms of exponentials, prove that

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B).$$

2. If $f(x)$ and $g(x)$ are periodic with fundamental period X, show that the following are also periodic with the same period:
(a) $h(x) = a\, f(x) + b\, g(x)$
(b) $j(x) = c\, f(x)\, g(x)$
where a, b, c are constants.

3. Find the fundamental periods for the following functions:
(a) $\cos 2x$
(b) $3 \cos 3x + 2 \cos 2x$
(c) $\cos^2 x$
(d) $|\cos x|$
(e) $\sin^3 x$.

4. Show that

$$\int_{-L}^{L} dx\ \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases}$$

5. (a) Sketch $f(x) = (1 + \sin x)^2$ and determine its fundamental period.
(b) Using a trigonometric identity for $\sin^2 x$ in terms of $\cos 2x$, write down the Fourier Series for $f(x)$ (don't do any integrals to obtain the coefficients).

6. Show that the Fourier Series expansion of the periodic function

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ +1 & 0 < x < \pi \end{cases}$$

is

$$f(x) = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{\sin[(2k+1)x]}{2k+1}.$$

7. Consider the function $f(x) = |\cos x|$.
(a) What is its fundamental period?
(b) Sketch the function for $-2\pi < x < 2\pi$
(c) Show that the Fourier Series expansion for $f(x)$ is

$$f(x) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{4m^2 - 1} \cos(2mx).$$